CHAPTER 0
INTRODUCTION

0.1 IDENTIFICATION AND ADAPTIVE CONTROL

Most current techniques for designing control systems are based on a good understanding of the plant under study and its environment. However, in a number of instances, the plant to be controlled is too complex and the basic physical processes in it are not fully understood. Control design techniques then need to be augmented with an identification technique aimed at obtaining a progressively better understanding of the plant to be controlled. It is thus intuitive to aggregate system identification and control. Often, the two steps will be taken separately. If the system identification is recursive—that is the plant model is periodically updated on the basis of previous estimates and new data—identification and control may be performed concurrently. We will see *adaptive control*, pragmatically, as *a direct aggregation of a (non-adaptive) control methodology with some form of recursive system identification.*

Abstractly, system identification could be aimed at determining if the plant to be controlled is linear or nonlinear, finite or infinite dimensional, and has continuous or discrete event dynamics. Here we will restrict our attention to finite dimensional, single-input single-output linear plants, and some classes of multivariable and nonlinear plants. Then, the primary step of system identification (structural identification) has already been taken, and only parameters of a fixed type of model need to be determined. Implicitly, we will thus be limiting ourselves to *parametric system identification*, and *parametric adaptive control.*

Applications of such systems arise in several contexts: advanced flight control systems for aircraft or spacecraft, robot manipulators, process control, power systems, and others.

Adaptive control, then, is a technique of applying some system identification technique to obtain a model of the process and its environment from input-output experiments and using this model to design a controller. The parameters of the controller are adjusted during the operation of the plant as the amount of data available for plant identification increases. For a number of simple PID (proportional + integral + derivative) controllers in process control, this is often done manually. However, when the number of parameters is larger than three or four, and they vary with time, automatic adjustment is needed. The design techniques for adaptive systems are studied and analyzed in theory for *unknown* but *fixed* (that is, time invariant) plants. In practice, they are applied to *slowly time-varying* and *unknown* plants.

Overview of the Literature

Research in adaptive control has a long and vigorous history. In the 1950s, it was motivated by the problem of designing autopilots for aircraft operating at a wide range of speeds and altitudes. While the object of a good fixed-gain controller was to build an autopilot which was insensitive to these (large) parameter variations, it was frequently observed that a single constant gain controller would not suffice. Consequently, gain scheduling based on some auxiliary measurements of airspeed was adopted. With this scheme in place several rudimentary model reference schemes were also attempted—the goal in this scheme was to build a self-adjusting controller which yielded a closed loop transfer function matching a prescribed reference model. Several schemes of self-adjustment of the controller parameters were proposed, such as the sensitivity rules and the so-called M.I.T. rule, and were verified to perform well under certain conditions. Finally, Kalman [1958] put on a firm analytical footing the concept of a general self-tuning controller with explicit identification of the parameters of a linear, single-input, single-output plant and the usage of these parameter estimates to update an optimal linear quadratic controller.

The 1960s marked an important time in the development of control theory and adaptive control in particular. Lyapunov's stability theory was firmly established as a tool for proving convergence in adaptive control schemes. Stochastic control made giant strides with the understanding of dynamic programming, due to Bellman and others. Learning schemes proposed by Tsypkin, Feldbaum and others (see Tsypkin [1971] and [1973]) were shown to have roots in a single unified framework of recursive equations. System identification (off-line) was thoroughly researched and understood. Further, Parks [1966] found a way of redesigning the update laws proposed in the 1950s for model reference schemes so as to be able to prove convergence of his controller.

In the 1970s, owing to the culmination of determined efforts by several teams of researchers, complete proofs of stability for several adaptive schemes appeared. State space (Lyapunov based) proofs of stability for model reference adaptive schemes appeared in the work of Narendra, Lin, & Valavani [1980] and Morse [1980]. In the late 1970s, input output (Popov hyperstability based) proofs appeared in Egardt [1979] and Landau [1979]. Stability proofs in the discrete time deterministic and stochastic case (due to Goodwin, Ramadge, & Caines [1980]) also appeared at this time, and are contained in the textbook by Goodwin & Sin [1984]. Thus, this period was marked by the culmination of the analytical efforts of the past twenty years.

Given the firm, analytical footing of the work to this point, the 1980s have proven to be a time of critical examination and evaluation of the accomplishments to date. It was first pointed out by Rohrs and coworkers [1982] that the assumptions under which stability of adaptive schemes had been proven were very sensitive to the presence of unmodeled dynamics, typically high-frequency parasitic modes that were neglected to limit the complexity of the controller. This sparked a flood of research into the robustness of adaptive algorithms: a re-examination of whether or not adaptive controllers were at least as good as fixed gain controllers, the development of tools for the analysis of the transient behavior of the adaptive algorithms and attempts at implementing the algorithms on practical systems (reactors, robot manipulators, and ship steering systems to mention only a few). The implementation of the complicated nonlinear laws inherent in adaptive control has been greatly facilitated by the boom in microelectronics and today, one can talk in terms of custom adaptive controller chips. All this flood of research and development is bearing fruit and the industrial use of adaptive control is growing.

Adaptive control has a rich and varied literature and it is impossible to do justice to all the manifold publications on the subject. It is a tribute to the vitality of the field that there are a large number of fairly recent books and monographs. Some recent books on recursive estimation, which is an important part of adaptive control are by Eykhoff [1974], Goodwin & Payne [1977], Ljung & Soderstrom [1983] and Ljung [1987]. Recent books dealing with the theory of adaptive control are by Landau [1979], Egardt [1979], Ioannou & Kokotovic [1984], Goodwin & Sin [1984], Anderson, Bitmead, Johnson, Kokotovic, Kosut, Mareels, Praly, & Riedle [1986], Kumar and Varaiya [1986], Polderman [1988] and Caines [1988]. An attempt to link the signal processing viewpoint with the adaptive control viewpoint is made in Johnson [1988]. Surveys of the applications of adaptive control are given in a book by Harris & Billings [1981], and in books edited by Narendra & Monopoli [1980] and Unbehauen [1980]. As of the writing of this book, two other books on adaptive control by Astrom & Wittenmark and Narendra & Annaswamy are also nearing completion.

In spite of the great wealth of literature, we feel that there is a need for a "toolkit" of methods of analysis comparable to non-adaptive linear time invariant systems. Further, many of the existing results concern either algorithms, structures or specific applications, and a great deal more needs to be understood about the dynamic behavior of adaptive systems. This, we believe, has limited practical applications more than it should have. Consequently, our objective in this book is to address fundamental issues of stability, convergence and robustness. Also, we hope to communicate our excitement about the problems and potential of adaptive control. In the remainder of the introduction, we will review some common approaches to adaptive control systems and introduce the basic issues studied in this book with a simple example.

0.2 APPROACHES TO ADAPTIVE CONTROL

0.2.1 Gain Scheduling

One of the earliest and most intuitive approaches to adaptive control is gain scheduling. It was introduced in particular in the context of flight control systems in the 1950s and 1960s. The idea is to find auxiliary process variables (other than the plant outputs used for feedback) that correlate well with the changes in process dynamics. It is then possible to compensate for plant parameter variations by changing the parameters of the regulator as functions of the auxiliary variables. This is illustrated in Figure 0.1.

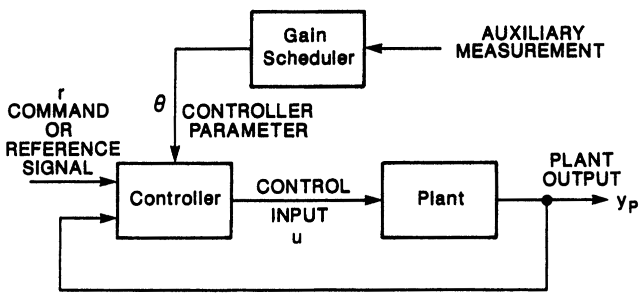

Figure 0.1: Gain Scheduling Controller

The advantage of gain scheduling is that the parameters can be changed quickly (as quickly as the auxiliary measurement) in response to changes in the plant dynamics. It is convenient especially if the plant dynamics depend in a well-known fashion on a relatively few easily measurable variables. In the example of flight control systems, the dynamics depend in relatively simple fashion on the readily available dynamic pressure—that is the product of the air density and the relative velocity of the aircraft squared.

Although gain scheduling is extremely popular in practice, the disadvantage of gain scheduling is that it is an *open-loop* adaptation scheme, with no real "learning" or intelligence. Further, the extent of design required for its implementation can be enormous, as was illustrated by the flight control system implemented on a CH-47 helicopter. The flight envelope of the helicopter was divided into *ninety* flight conditions corresponding to thirty discretized horizontal flight velocities and three vertical velocities. Ninety controllers were designed, corresponding to each flight condition, and a linear interpolation between these controllers (linear in the horizontal and vertical flight velocities) was programmed onto a flight computer. Airspeed sensors modified the control scheme of the helicopter in flight, and the effectiveness of the design was corroborated by simulation.

0.2.2 Model Reference Adaptive Systems

Again in the context of flight control systems, two adaptive control schemes other than gain scheduling were proposed to compensate for changes in aircraft dynamics: a series, high-gain scheme, and a parallel scheme.

Series High-Gain Scheme

Figure 0.2 shows a schematic of the series high-gain scheme.

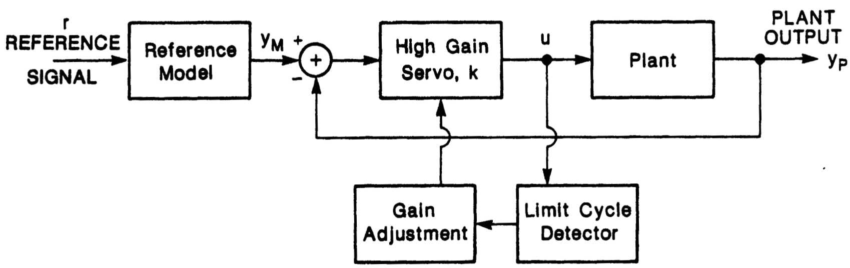

Figure 0.2: Model Reference Adaptive Control—Series, High-Gain Scheme

The reference model represents a pilot's desired command-response characteristic. It is thus desired that the aircraft response, that is, the output y_p, matches the output of the reference model, that is, y_m.

The simple analysis that goes into the scheme is as follows: consider $\hat{P}(s)$ to be the transfer function of the linear, time invariant plant and k the constant gain of the servo. The transfer function from y_m to y_p is $k\hat{P}(s)/1 + k\hat{P}(s)$. When the gain k is sufficiently large, the transfer function is approximately 1 over the frequencies of interest, so that $y_m \sim y_p$.

The aim of the scheme is to let the gain k be as high as possible, so that the closed-loop transfer function becomes close to 1, until the onset of instability (a limit cycle) is detected. If the limit cycle oscillations exceed some level, the gain is decreased. Below this level, the gain is increased. The limit cycle detector is typically just a rectifier and low-pass filter.

The series high-gain scheme is intuitive and simple: only one parameter is updated. However, it has the following problems

a) Oscillations are constantly present in the system.

b) Noise in the frequency band of the limit cycle detector causes the gain to decrease well below the critical value.

c) Reference inputs may cause saturation due to the high-gain.

d) Saturation may mask limit cycle oscillations, allowing the gain to increase above the critical value, and leading to instability.

Indeed, tragically, an experimental X-15 aircraft flying this control system crashed in 1966 (cf. Staff of the Flight Research Center [1971]), owing partially to the saturation problems occurring in the high-gain scheme. The roll and pitch axes were controlled by the right and left rear ailerons, using differential and identical commands respectively. The two axes were assumed decoupled for the purpose of control design. However, saturation of the actuators in the pitch axis caused the aircraft to lose controllability in the roll axis (since the ailerons were at maximum deflection). Due to the saturation, the instability remained undetected, and created aerodynamic forces too great for the aircraft to withstand.

Parallel Scheme

As in the series scheme, the desired performance of the closed-loop system is specified through a reference model, and the adaptive system attempts to make the plant output match the reference model output asymptotically. An early reference to this scheme is Osburn, Whitaker, & Kezer [1961]. A block diagram is shown in Figure 0.3. The controller

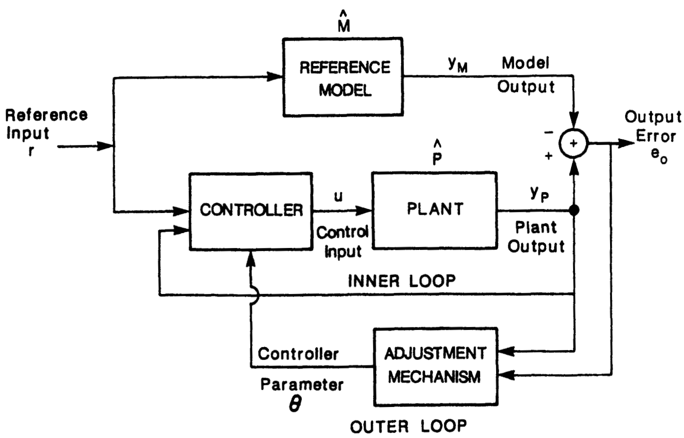

Figure 0.3: Model Reference Adaptive Control—Parallel Scheme

can be thought of as having two loops: an inner or regulator loop that is an ordinary control loop consisting of the plant and regulator, and an outer or adaptation loop that adjusts the parameters of the regulator in such a way as to drive the error between the model output and plant output to zero.

The key problem in the scheme is to obtain an adjustment mechanism that drives the output error $e_0 = y_p - y_m$ to zero. In the earliest applications of this scheme, the following update, called the *gradient update*, was used. Let the vector θ contain the adjustable parameters of the controller. The idea behind the gradient update is to reduce $e_0^2(\theta)$ by adjusting θ along the direction of steepest descent, that is

$$\frac{d\theta}{dt} = -g\frac{\partial}{\partial\theta}(e_0^2(\theta)) \tag{0.2.1}$$

$$= -2g\,e_0(\theta)\frac{\partial}{\partial\theta}(e_0(\theta)) = -2g\,e_0(\theta)\frac{\partial}{\partial\theta}(y_p(\theta)) \tag{0.2.2}$$

where g is a positive constant called the *adaptation gain.*

The interpretation of $e_0(\theta)$ is as follows: it is the output error (also a function of time) obtained by freezing the controller parameter at θ. The gradient of $e_0(\theta)$ with respect to θ is equal to the gradient of y_p with respect to θ, since y_m is independent of θ, and represents the *sensitivity* of the output error to variations in the controller parameter θ.

Several problems were encountered in the usage of the gradient update. The sensitivity function $\partial y_p(\theta) / \partial \theta$ usually depends on the unknown plant parameters, and is consequently unavailable. At this point the so-called *M.I.T. rule*, which replaced the unknown parameters by their estimates at time t, was proposed. Unfortunately, for schemes based on the M.I.T. rule, it is not possible in general to prove closed-loop stability, or convergence of the output error to zero. Empirically, it was observed that the M.I.T. rule performed well when the adaptation gain g and the magnitude of the reference input were small (a conclusion later confirmed analytically by Mareels *et al* [1986]). However, examples of instability could be obtained otherwise (cf. James [1971]).

Parks [1966] found a way of redesigning adaptive systems using Lyapunov theory, so that stable and provably convergent model reference schemes were obtained. The update laws were similar to (0.2.2), with the sensitivity $\partial y_p(\theta) / \partial \theta$ replaced by other functions. The stability and convergence properties of model reference adaptive systems make them particularly attractive and will occupy a lot of our interest in this book.

0.2.3 Self Tuning Regulators

In this technique of adaptive control, one starts from a control design method for known plants. This design method is summarized by a controller structure, and a relationship between plant parameters and controller parameters. Since the plant parameters are in fact unknown, they are obtained using a recursive parameter identification algorithm. The controller parameters are then obtained from the estimates of the plant parameters, in the same way *as if these were the true parameters.* This is usually called a *certainty equivalence principle.*

The resulting scheme is represented on Figure 0.4. An explicit separation between identification and control is assumed, in contrast to the model reference schemes above, where the parameters of the controller are updated directly to achieve the goal of model following. The self tuning approach was originally proposed by Kalman [1958] and clarified by Astrom & Wittenmark [1973]. The controller is called *self tuning*, since it has the ability to tune its own parameters. Again, it can be thought of as having two loops: an inner loop consisting of a conventional controller, but with varying parameters, and an outer loop consisting of an identifier and a design box (representing an on-line solution to a design problem for a system with known parameters) which adjust these controller parameters.

The self tuning regulator is very flexible with respect to its choice of controller design methodology (linear quadratic, minimum variance, gain-phase margin design, ...), and to the choice of identification scheme

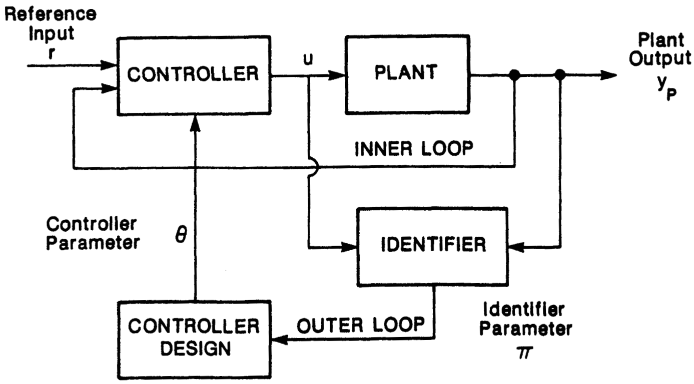

Figure 0.4: Self-tuning Controller

(least squares, maximum likelihood, extended Kalman filtering, ...). The analysis of self tuning adaptive systems is however more complex than the analysis of model reference schemes, due primarily to the (usually nonlinear) transformation from identifier parameters to controller parameters.

Direct and Indirect Adaptive Control

While model reference adaptive controllers and self tuning regulators were introduced as different approaches, the only real difference between them is that model reference schemes are *direct* adaptive control schemes, whereas self tuning regulators are *indirect*. The self tuning regulator first identifies the plant parameters recursively, and then uses these estimates to update the controller parameters through some fixed transformation. The model reference adaptive schemes update the controller parameters directly (no explicit estimate or identification of the plant parameters is made). It is easy to see that the inner or control loop of a self tuning regulator could be the same as the inner loop of a model reference design. Or, in other words, the model reference adaptive schemes can be seen as a special case of the self tuning regulators, with an identity transformation between updated parameters and controller parameters. Through this book, we will distinguish between direct and indirect schemes rather than between model reference and self tuning algorithms.

0.2.4 Stochastic Control Approach

Adaptive controller structures based on model reference or self tuning approaches are based on heuristic arguments. Yet, it would be appealing to obtain such structures from a unified theoretical framework. This can be done (in principle, at least) using stochastic control. The system and its environment are described by a stochastic model, and a criterion is formulated to minimize the expected value of a loss function, which is a scalar function of states and controls. It is usually very difficult to solve stochastic optimal control problems (a notable exception is the linear quadratic gaussian problem). When indeed they can be solved, the optimal controllers have the structure shown in Figure 0.5: an identifier (estimator) followed by a nonlinear feedback regulator.

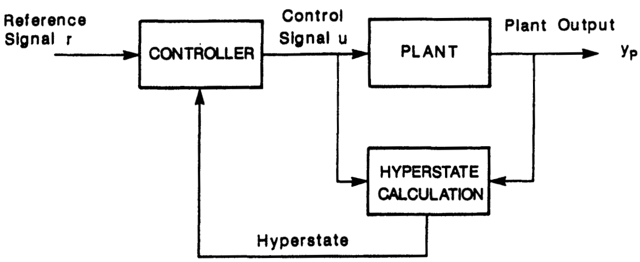

Figure 0.5: "Generic" Stochastic Controller

The estimator generates the conditional probability distribution of the state from the measurements: this distribution is called the *hyperstate* (usually belonging to an infinite dimensional vector space). The self tuner may be thought of as an approximation of this controller, with the hyperstate approximated by the process state and process parameters estimate.

From some limited experience with stochastic control, the following interesting observations can be made of the optimal control law: in addition to driving the plant output to its desired value, the controller introduces *probing* signals which improve the identification and, therefore future control. This, however, represents some cost in terms of control activity. The optimal regulator maintains a balance between the control activity for *learning* about the plant it is controlling and the activity for *controlling* the plant output to its desired value. This property is referred to as *dual control.* While we will not explicitly study stochastic control in this book, the foregoing trade-off will be seen repeatedly: good adaptive control requires correct identification, and for the identification to be complete, the controller signal has to be sufficiently rich to allow for the excitation of the plant dynamics. The presence of this rich enough excitation may result in poor transient performance of the scheme, displaying the trade-off between learning and control performance.

0.3 A SIMPLE EXAMPLE

Adaptive control systems are difficult to analyze because they are nonlinear, time varying systems, even if the plant that they are controlling is linear, time invariant. This leads to interesting and delicate technical problems. In this section, we will introduce some of these problems with a simple example. We also discuss some of the adaptive schemes of the previous section in this context.

We consider a first order, time invariant, linear system with transfer function

$$\hat{P}(s) = \frac{k_p}{s+a} \tag{0.3.1}$$

where $a > 0$ is known. The gain k_p of the plant is unknown, but its sign is known (say $k_p > 0$). The control objective is to get the plant output to match a model output, where the reference model transfer function is

$$\hat{M}(s) = \frac{1}{s+a} \tag{0.3.2}$$

Only gain compensation—or feedforward control—is necessary, namely a gain θ at the plant input, as is shown on Figure 0.6.

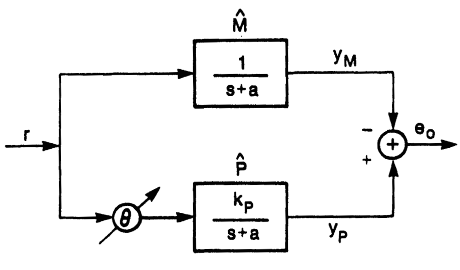

Figure 0.6: Simple Feedforward Controller

Note that, if k_p were known, θ would logically be chosen to be $1/k_p$. We will call

$$\theta^* = \frac{1}{k_p} \tag{0.3.3}$$

the *nominal* value of the parameter θ, that is the value which realizes the output matching objective for all inputs. The design of the various adaptive schemes proceeds as follows.

Gain Scheduling

Let $v(t) \in \mathbb{R}$ be some auxiliary measurement that correlates in known fashion with k_p, say $k_p(t) = f(v(t))$. Then, the gain scheduler chooses at time t

$$\theta(t) = \frac{1}{f(v(t))} \tag{0.3.4}$$

Model Reference Adaptive Control Using the M.I.T. Rule

To apply the M.I.T. rule, we need to obtain $\partial e_0(\theta) / \partial \theta = \partial y_p(\theta) / \partial \theta$, with the understanding that θ is frozen. From Figure 0.6, it is easy to see that

$$\frac{\partial y_p(\theta)}{\partial \theta} = \frac{k_p}{s+a}(\theta r) = k_p y_m \tag{0.3.5}$$

We see immediately that the sensitivity function in (0.3.5) depends on the parameter k_p which is unknown, so that $\partial y_p / \partial \theta$ is not available. However, the sign of k_p is known ($k_p > 0$), so that we may merge the constant k_p with the adaptation gain. The M.I.T rule becomes

$$\dot{\theta} = -g e_0 y_m \qquad g > 0 \tag{0.3.6}$$

Note that (0.3.6) prescribes an update of the parameter θ in the direction opposite to the "correlation" product of e_0 and the model output y_m.

Model Reference Adaptive Control Using the Lyapunov Redesign

The control scheme is exactly as before, but the parameter update law is chosen to make a Lyapunov function decrease along the trajectories of the adaptive system (see Chapter 1 for an introduction to Lyapunov analysis). The plant and reference model are described by

$$\dot{y}_p = -a y_p + k_p \theta r \tag{0.3.7}$$

$$\dot{y}_m = -a y_m + r = -a y_m + k_p \theta^* r \tag{0.3.8}$$

Subtracting (0.3.8) from (0.3.7), we get, with $e_0 = y_p - y_m$

$$\dot{e}_0 = -a e_0 + k_p(\theta - \theta^*) r \tag{0.3.9}$$

Since we would like θ to converge to the nominal value $\theta^* = 1 / k_p$, we define the *parameter error* as

$$\phi = \theta - \theta^* \tag{0.3.10}$$

Note that since θ^* is fixed (though unknown), $\dot{\phi} = \dot{\theta}$.

The Lyapunov redesign approach consists in finding an update law so that the Lyapunov function

$$v(e_0, \phi) = e_0^2 + k_p \phi^2 \tag{0.3.11}$$

is decreasing along trajectories of the error system

$$\begin{aligned} \dot{e}_0 &= -a\, e_0 + k_p \phi r \\ \dot{\phi} &= \text{update law to be defined} \end{aligned} \tag{0.3.12}$$

Note that since $k_p > 0$, the function $v(e_0, \phi)$ is a positive definite function. The derivative of v along the trajectories of the error system (0.3.12) is given by

$$\dot{v}(e_0, \phi)\Big|_{(0.3.12)} = -2a\, e_0^2 + 2k_p e_0 \phi r + 2k_p \phi \dot{\phi} \tag{0.3.13}$$

Choosing the update law

$$\dot{\theta} = \dot{\phi} = -e_0 r \tag{0.3.14}$$

yields

$$\dot{v}(e_0, \phi) = -2a\, e_0^2 \leq 0 \tag{0.3.15}$$

thereby guaranteeing that $e_0^2 + k_p\phi^2$ is decreasing along the trajectories of (0.3.12), (0.3.14) and that e_0, and ϕ are bounded. Note that (0.3.15) is similar in form to (0.3.6), with the difference that e_0 is correlated with r rather that y_m. An adaptation gain g may also be included in (0.3.14).

Since $v(e_0, \phi)$ is decreasing and bounded below, it would appear that $e_0 \to 0$ as $t \to \infty$. This actually follows from further analysis, provided that r is bounded (cf. Barbalat's lemma 1.2.1).

Having concluded that $e_0 \to 0$ as $t \to \infty$, what can we say about θ? Does it indeed converge to $\theta^* = 1/k_p$? The answer is that one can not conclude anything about the convergence of θ to θ^* without extra conditions on the reference input. Indeed, if the reference input was a constant zero signal, there would be no reason to expect θ to converge to θ^*. Conditions for parameter convergence are important in adaptive control and will be studied in great detail. An answer to this question for the simple example will be given for the following indirect adaptive control scheme.

Indirect Adaptive Control (Self Tuning)

To be able to effectively compare the indirect scheme with the direct schemes given before, we will assume that the control objective is still model matching, with the same model as above. Figure 0.7 shows an indirect or self tuning type of model reference adaptive controller.

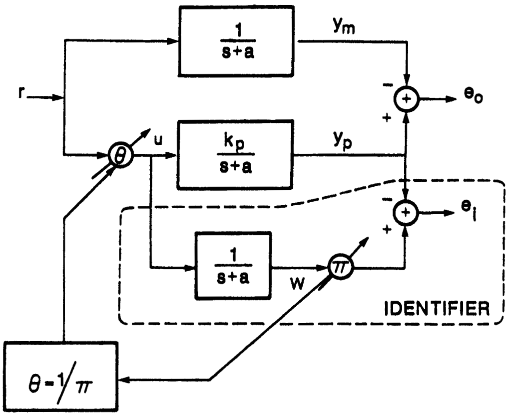

Figure 0.7: A Simple Indirect Controller

The identifier contains an identifier parameter $\pi(t)$ that is an estimate of the unknown plant parameter k_p. Therefore, we define $\pi^* = k_p$. The controller parameter is chosen following the certainty equivalence principle: since $\theta^* = 1 / k_p$ and $\pi^* = k_p$, we let $\theta(t) = 1 / \pi(t)$. The hope is that, as $t \to \infty$, $\pi(t) \to k_p$, so that $\theta(t) \to 1/k_p$.

The update law now is an update law for the identifier parameter $\pi(t)$. There are several possibilities at this point, and we proceed to derive one of them. Define the identifier parameter error

$$\psi(t) \;:=\; \pi(t) - \pi^* \tag{0.3.16}$$

and let

$$w \;=\; \frac{1}{s+a}\,(\theta r) \;=\; \frac{1}{s+a}\,(u) \tag{0.3.17}$$

The signal w may be obtained by stable filtering of the input u, since $a > 0$ is known. The update law is based on the identifier error

$$e_i \;:=\; \pi w - y_p \tag{0.3.18}$$

Equation (0.3.18) is used in the actual implementation of the algorithm. For the analysis, note that

$$y_p = \frac{k_p}{s+a}(\theta r) = \pi^* w \tag{0.3.19}$$

so that

$$e_i = \psi w \tag{0.3.20}$$

Consider the update law

$$\dot{\pi} = \dot{\psi} = -g\, e_i\, w \qquad g > 0 \tag{0.3.21}$$

and let the Lyapunov function

$$v = \psi^2 \tag{0.3.22}$$

This Lyapunov function has the special form of the norm square of the identifier parameter error. Its derivative along the trajectories of the adaptive system is

$$\dot{v} = -g\,\psi^2 w^2 \tag{0.3.23}$$

Therefore, the update law causes a decreasing parameter error and all signals remain bounded.

The question of parameter convergence can be answered quite simply in this case. Note that (0.3.20), (0.3.21) represent the first order linear time varying system

$$\dot{\psi} = -g\, w^2 \psi \tag{0.3.24}$$

which may be explicitly integrated to get

$$\psi(t) = \psi(0) \exp\left(-g \int_0^t w^2(\tau)\, d\tau\right) \tag{0.3.25}$$

It is now easy to see that if

$$\int_0^t w^2(\tau)\, d\tau \to \infty \qquad \text{as } t \to \infty \tag{0.3.26}$$

then $\psi(t) \to 0$, so that $\pi(t) \to \pi^*$ and $\theta(t) \to 1/k_p$, yielding the desired controller. The condition (0.3.26) is referred to as an *identifiability* condition and is much related to the so-called *persistency of excitation* that will be discussed in Chapter 2. It is easily seen that, in particular, it excludes signals which tend to zero as $t \to \infty$.

The difficulty with (0.3.26) is that it depends on w, which in turn depends on u and therefore on both θ and r. Converting it into a condition on the exogenous reference input $r(t)$ only is another of the problems which we will discuss in the following chapters.

The foregoing simple example showed that even when simple feed-forward control of a linear, time invariant, first order plant was involved, the analysis of the resulting closed-loop dynamics could be involved: the equations were *time-varying, linear* equations. Once feedback control is involved, the equations become nonlinear and time varying.