Chapter 1

Prologue

Many mass–produced products of modern technology would have appeared completely magical two hundred years ago. Mobile phones and computers are obvious examples, but something as commonplace to us as electric light would perhaps be just as astonishing to an age of candles and oil lamps. It seems reasonable to assume that we are no more prescient than the children of the Enlightenment and that, as science and technology develop further, some things that appear impossible today will become ubiquitous in the future. As Arthur C. Clarke famously wrote, "Any sufficiently advanced technology is indistinguishable from magic". In this book we focus on optics and electromagnetism, an ancient subject so suffused with notions of magic that the word illusion is still used by its modern practitioners in their learned journals. We explain the science of the ultimate optical illusion, invisibility. The ingredients of invisibility can be used for other surprising optical effects, such as perfect imaging and laboratory analogues of black holes. Just as important as the particular applications discussed are the powerful ideas that underlie them, ideas that have a fascinating pedigree and that are far from exhausted. We hope to equip the reader with these versatile and fruitful tools of physics and mathematics.

Although invisibility may seem like magic, its roots are familiar to everyone with (literal) vision. Almost all we need to do is to wonder and ask questions. Take a simple observation from daily life and ask some questions: if a straw is placed in a glass of water it appears to be broken at the water's surface (Fig. 1.1). We know the straw is not really broken (and miraculously repaired when removed from the water), so what does the water change? It can only change our perception of the straw, its image carried by light. The water in the glass distorts our perception of space, and this perception is conveyed by light. We conclude that the water changes the measure of space for light, the way light "sees" distances—the geometry of space. Other transparent substances like glass or air, called optical materials or optical media, should not be qualitatively different from water in the way they distort geometry for light. So we are led to the hypothesis that media appear to light as geometries. In this book we take this geometrical perspective on light in media seriously and develop it to extremes. We also discuss its limitations and find the conditions when the geometry established by media is exact.

Taking some basic facts seriously, scrutinizing them and developing them to extremes is the way science generally develops. The tools for this development are sophisticated instrumentation for finding experimental facts and mathematical theory for refining the ideas; what seems like magic is a brew of applied mathematics.

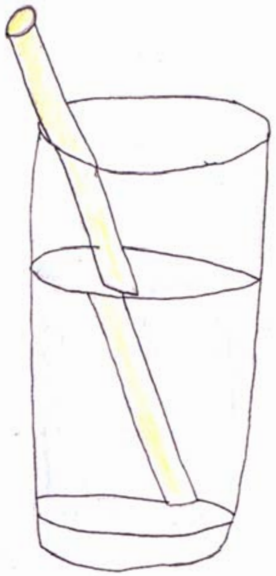

Figure 1.1: Refraction. The image of a straw in a glass of water appears refracted at the water surface. (Credit: Maria Leonhardt.)

But before going into mathematical detail, we can already deduce some aspects of the geometry of light by thinking about things we already know, encouraged by the saying that "research is to see what everybody has seen and to think what nobody has thought" (Jammer [1989]). We know, for example, that a convex lens focuses light (Fig. 1.2); parallel bundles of light rays are focused at one point, which suggests that in the geometry of light established by the lens parallel lines meet. The Greek mathematician Euclid, who developed geometry from five axioms, postulated that parallels never meet, but Euclid's geometry is the geometry of flat space. Euclid's parallel axiom is in fact the defining characteristic of flat space. The light rays focused by the lens do not seem to conform to Euclid's postulate; the geometry of light is non–Euclidean, light may perceive a medium as a curved space. Only in exceptional cases is the geometry established by an optical material that of flat space. One of the exceptional cases is obvious: imagine being completely immersed in a transparent substance, like a diver in water. In this situation space does not appear to be distorted at all, except when the diver looks from below at the water's surface where the flat space established by the water ends. We will prove that having two different media, say water and air, with an interface between them, is already sufficient to establish a curved geometry for light. The straw in the glass of water appears broken because the geometry of light is curved. We will deduce the conditions when the geometry made by media is flat and show that such media can make things disappear from view.

James Clerk Maxwell discovered that light is an electromagnetic wave. With his theory of electromagnetism he also laid the foundation for most of modern technology. The geometry of curved space, on the other hand, is normally encountered by physicists only in Albert Einstein's general relativity. To understand the geometry of light we need to combine aspects of both theories. Yet for most physics and engineering students, ordinary electromagnetism with its vector calculus is already a challenge. In this textbook we build up the required mathematics, differential geometry, step by step with many exercises designed to help the reader gain expertise and confidence in the mathematical machinery we set forth. We strongly recommend doing as many of the exercises as possible, because there is no easier path to

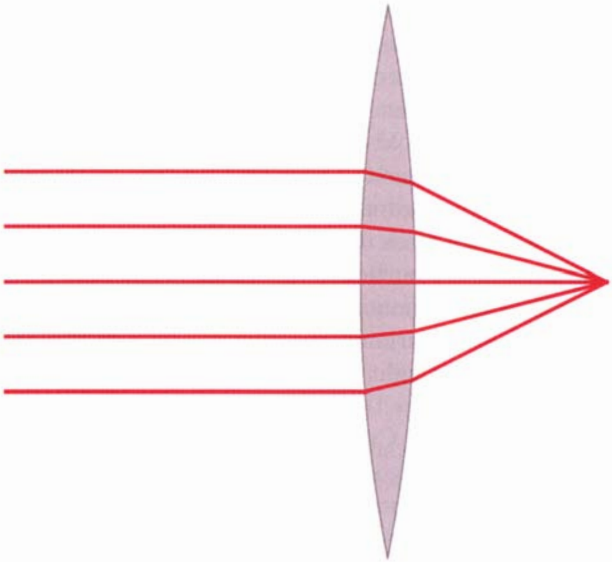

Figure 1.2: Parallel light rays (red) meet in the focus of a lens (grey).

the necessary geometry, no "royal road". We assume the reader knows basic calculus and analytic geometry, and has some acquaintance with Maxwell's equations. Differential geometry applied to electromagnetism gives insights into the nature of light and establishes the scientific foundations for the applications that follow.

We hope the applications and insights provide a strong enough incentive to work through this book. The profit for the reader is a working knowledge of differential geometry and other versatile tools, with a sense for the way in which physicists and engineers apply mathematics. Potential applications are not confined to optics and electromagnetism, but include waves in fluid mechanics and acoustics and the strange waves of quantum physics. The most difficult part of the book is probably the beginning, the Chapter on Fermat's principle, because there we introduce the main concepts with limited algebra, assisted by tailor–made arguments and visualizations. Concepts are the hardest part of science—one should always remember that their originators also struggled to master them.

One of the joys of this area of optics is that it makes use of a surprisingly wide range of classic physics and mathematics. The appearance of names such as Fermat, Newton, Hamilton, Maxwell, Riemann and Einstein shows that this book is built on old foundations. Indeed, one could describe the recent developments presented here as "new things in old things", to quote a phrase by Michael Berry. This illustrates the continuing importance of the old things, but also the gradual, hard–won shift in perspective that is required to see the new things. How else could it have taken so long before ideas for invisibility and perfect imaging appeared? As the reader will see, they are obvious with hindsight.

The materials required for cloaking and perfect imaging, metamaterials, are not new either; they date back to Ancient Rome. The Romans invented the first optical metamaterial—ruby glass. They probably did not know it, but their recipe for ruby glass contained a crucial ingredient: tiny gold droplets, typically 5–60 nm in size (Wagner et al. [2000]). These gold particles colour the glass in an extraordinary way, as demonstrated by the exquisite Lycurgus Cup (Fig. 1.3). In daylight the cup appears green, but when illuminated from the inside it glows with a ruby colour. The gold nanoparticles in the glass do not colour it golden, but red. One can also make other colours with metal particles; the brilliant colours of medieval stained–glass windows come from metal nanoparticles immersed in the glass. The sizes and shapes of the nanoparticles determine the colour. In a metamaterial, structures smaller than the wavelength of light control the optical properties of the material, their shapes and sizes matter more than their chemistry—metal nanostructures like the gold droplets in the Lycurgus Cup do not appear metallic. Thanks to advances in modern nanotechnology and the science behind it, engineers can now make carefully controlled subwavelength structures with designs based on accurate theoretical predictions, whereas Roman technology mostly relied on trial and error. Rome pioneered the technology of metamaterials and Greece, through geometry, the ideas to make use of them.

Figure 1.3: Lycurgus Cup. This Roman cup is made of ruby glass, the first optical metamaterial. When viewed in reflected light, for example in daylight, it appears green. However, when a light is shone into the cup and transmitted through the glass, it appears red. The cup illustrates the myth of King Lycurgus. He is seen being dragged into the underworld by the Greek nymph Ambrosia, who is disguised as a vine. (Credit: the Trustees of the British Museum.)

There are several excellent monographs on the science and technology of metamaterials (see the list in *Further Reading*), but this is the first textbook on the geometrical ideas behind some of their most exciting applications. We thus explore the Greek path rather than the Roman. Connections between general relativity and optics have been reviewed before (Schleich and Scully [1984]), but with different applications in mind and not in a textbook. The only other textbook that combines general relativity with electromagnetism in media is Post's "Formal Structure of Electromagnetics" (Post [1962]), but the book is, as the title says, formal. Here we hope to breathe life into formalism, to explain some "new things in old things", and to inspire the reader to discover others that, for now, are still magic.

FURTHER READING

This book grew out of the review article Leonhardt and Philbin [2009]. We recommend Post [1962] and Schleich and Scully [1984] for getting a perspective on the geometry of light that complements our book.

On the practicalities and the underlying physics of metamaterials we recommend Milton [2002], Sarychev and Shalaev [2007], Cai and Shalaev [2009] and the monumental Metamaterials Handbook (Capolino [2009]). On numerical aspects we suggest to consult Hao and Mittra [2008]. Wave propagation in metamaterials is discussed in Solymar and Shamonina [2009].

The practical use of general relativity in electrical and optical engineering may seem surprisingly unorthodox: traditionally, relativity has been associated with the physics of gravitation (Misner, Thorne and Wheeler [1973]) and cosmology (Peacock [1999]) or, in engineering (Van Bladel [1984]) has been considered a complication, not a simplification. This situation changed with the advent of transformation optics (Chen, Chan and Sheng [2010]). Geometrical ideas have been applied to construct conductivities that are undetectable by static electric fields (Greenleaf, Lassas and Uhlmann [2003a,b]) which was the precursor of invisibility devices (Gbur [2003], Alu and Engheta [2005], Leonhardt [2006a,b], Milton and Nicorovici [2006], Pendry, Schurig and Smith [2006], Schurig, Pendry and Smith [2006]) based on optical implementations of coordinate transformations. From these developments grew the subject of transformation optics (Chen, Chan and Sheng [2010]).

In Chapter 2 we mention the fascinating history of ideas behind the geometrical perspective on optics and electromagnetism, a history that spans more than three centuries. More recently, in 1923 Gordon noticed that moving isotropic media appear to electromagnetic fields as certain effective space–time geometries. Bortolotti [1926] and Rytov [1938] pointed out that ordinary isotropic media establish non–Euclidean geometries for light. Tamm [1924, 1925] generalized the geometrical approach to anisotropic media and briefly applied this theory (Tamm [1925]) to the propagation of light in curved geometries. Plebanski [1960] formulated the electromagnetic effect of curved space–time or curved coordinates in concise constitutive equations. Dolin [1961] published an early precursor of transformation optics that, however, rather focuses on the construction of new solutions of Maxwell's equations than on the invention of new devices.

SCIENCE/PHYSICS

GEOMETRY AND LIGHT

The Science of Invisibility

Ulf Leonhardt and Thomas Philbin

> "A startlingly original, historically erudite and elegantly written reinterpretation of classical ray and wave optics in terms of geometrical transformations of space, yielding new insights into familiar phenomena and designs for perfect imaging and concealment devices."—Sir Michael Berry, University of Bristol, United Kingdom

The science of invisibility combines two of physics' greatest concepts: Einstein's general relativity and Maxwell's principles of electromagnetism. Recent years have witnessed major breakthroughs in the area, and the authors of this volume—Ulf Leonhardt and Thomas Philbin of Scotland's University of St. Andrews—have been active in the transformation of invisibility from fiction into science. Their work on designing invisibility devices is based on modern metamaterials, inspired by Fermat's principle, analogies between mechanics and optics, and the geometry of curved space.

Suitable for graduate students and advanced undergraduates of engineering, physics, or mathematics, and scientific researchers of all types, this is the first authoritative textbook on invisibility and the science behind it. The book is two books in one: it introduces the mathematical foundations—differential geometry—for physicists and engineers, and it shows how concepts from general relativity become practically useful in electrical and optical engineering, not only for invisibility but also for perfect imaging and other fascinating topics. More than one hundred full-color illustrations and exercises with solutions complement the text.

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