79

CIRCUMNAVIGATION

Imagine you are on a remote island with n identical airplanes and an unlimited supply of fuel and pilots. Each airplane can go a unit distance on a full tank. Aircraft all fly at the same constant speed, can refuel each other instantly and consume fuel at the same constant rate. Suppose you are on a spherical planet that is completely ocean except for the island. You wish to allow one airplane to safely circumnavigate the planet, flying over both the north and south poles before returning safely to the island. All other airplanes must safely return to the island as well. For one aircraft the circumference can be no more than 1 unit. For two aircraft the circumference can be no more than $5/3$ units. What is the largest circumference the planet can have and still allow

(a) three aircraft to refuel each other and get one to safely circumnavigate the planet?

(b) four aircraft to refuel each other and get one to safely circumnavigate the planet?

90

TWO TRIANGLES

Find positive integers x, y, u, v such that $x^2+y^2 = u^2$ and $x^2-xy+y^2 = v^2$.

91

THREE INTEGER TRIANGLES

In the figure the lengths a through i are integers and the angles are multiples of θ as shown. Find the smallest possible value of i.
Case (a): if α is unrestricted.
Case (b): if $\alpha < 90°$.