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### GRID POINT POLYGON

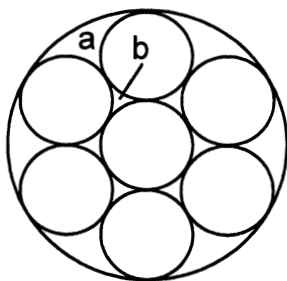
A polygon has sides in order around its perimeter of lengths  $1, 2, 3, \dots, n$ . Its vertices all lie on grid points of a unit grid (that is all vertices have integer coordinates). The polygon does not cross itself, overlap itself or have any sides meeting at  $0^\circ$  or  $180^\circ$  angles.

- (a) Produce such a polygon with the least number of sides.
- (b) Produce such a polygon with the least odd number of sides.

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### SEVEN COOKIES

Seven circular cookies of equal size are cut from a larger circle of dough as shown. What fraction of a cookie could be made with dough scraps  $a+b$ ?



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### GRID POINT PENTAGON

A regular pentagon is drawn on ordinary graph paper. Can more than two of its vertices lie on grid points?

**LOGICAL HATS 1**

There are five slips of paper with the number 7 written on three of them and the number 11 on the other two. Three of the slips go on the hats of logicians A, B and C in some order; the other two slips remain hidden. Each logician can see the numbers on the others' hats but not his own. The logicians are error-free in their reasoning and have all the information given so far. They are asked in turn to identify their number.

A: "I don't know my number."

B: "I don't know my number."

What number is on C?

**ELVES AND TROLL**

You are one of 50 elves trapped by a troll. He will bring all of you into a dark room, put one of 50 different cards from a standard deck of 52 playing cards on each of your foreheads, then turn on the light and ask you each in turn, "What card is on your forehead?" You see the cards on all foreheads but your own and hear the answers of those before you but when you are questioned you may only respond with the name of a card. Those who say the name of their card go free. If you are allowed to confer with the other elves ahead of time what strategy should you devise that allows the maximum number of elves to be released?