

1. Introduction

The radiation of the sun in which the planet is incessantly plunged, penetrates the air, the earth, and the waters; its elements are divided, change direction in every way, and, penetrating the mass of the globe, would raise its temperature more and more, if the heat acquired were not exactly balanced by that which escapes in rays from all points of the surface and expands through the sky. **The Analytical Theory of Heat, J. Fourier**

1.1 Heat transfer

People have always understood that something flows from hot objects to cold ones. We call that flow *heat*. In the eighteenth and early nineteenth centuries, scientists imagined that all bodies contained an invisible fluid which they called *caloric*. Caloric was assigned a variety of properties, some of which proved to be inconsistent with nature (e.g., it had weight and it could not be created nor destroyed). But its most important feature was that it flowed from hot bodies into cold ones. It was a very useful way to think about heat. Later we shall explain the flow of heat in terms more satisfactory to the modern ear; however, it will seldom be wrong to imagine caloric flowing from a hot body to a cold one.

The flow of heat is all-pervasive. It is active to some degree or another in everything. Heat flows constantly from your bloodstream to the air around you. The warmed air buoys off your body to warm the room you are in. If you leave the room, some small buoyancy-driven (or *convective*) motion of the air will continue because the walls can never be perfectly isothermal. Such processes go on in all plant and animal life and in the air around us. They occur throughout the earth, which is hot at its core and cooled around its surface. The only conceivable domain free from heat flow would have to be isothermal and totally isolated from any other region. It would be “dead” in the fullest sense of the word — devoid of any process of any kind.

The overall driving force for these heat flow processes is the cooling (or leveling) of the thermal gradients within our universe. The heat flows that result from the cooling of the sun are the primary processes that we experience naturally. The conductive cooling of Earth's center and the radiative cooling of the other stars are processes of secondary importance in our lives.

The life forms on our planet have necessarily evolved to match the magnitude of these energy flows. But while most animals are in balance with these heat flows, we humans have used our minds, our backs, and our wills to harness to harness and control energy flows that are far more intense than those we experience naturally¹. To emphasize this point we suggest that the reader make an experiment.

Experiment 1.1

Generate as much power as you can, in some way that permits you to measure your own work output. You might lift a weight, or run your own weight up a stairwell, against a stopwatch. Express the result in watts (W). Perhaps you might collect the results in your class. They should generally be less than 1 kW or even 1 horsepower (746 W). How much less might be surprising.

Thus, when we do so small a thing as turning on a 150 W light bulb, we are manipulating a quantity of energy substantially greater than a human being could produce in sustained effort. The power consumed by an oven, toaster, or hot water heater is an order of magnitude beyond our capacity. The power consumed by an automobile can easily be three orders of magnitude greater. If all the people in the United States worked continuously like galley slaves, they could barely equal the output of even a single city power plant.

Our voracious appetite for energy has steadily driven the intensity of actual heat transfer processes upward until they are far greater than those normally involved with life forms on earth. Until the middle of the thirteenth century, the energy we use was drawn indirectly from the sun

¹Some anthropologists think that the term *Homo technologicus* (those who use technology) serves to define human beings, as apart from animals, better than the older term *Homo sapiens* (those who are wise). We may not be as much wiser than the animals as we think we are, but only we do serious sustained tool making.

using comparatively gentle processes — animal power, wind and water power, and the combustion of wood. Then population growth and deforestation drove the English to using coal. By the end of the seventeenth century, England had almost completely converted to coal in place of wood. At the turn of the eighteenth century, the first commercial steam engines were developed, and that set the stage for enormously increased consumption of coal. Europe and America followed England in these developments.

The development of fossil energy sources has been a bit like Jules Verne's description in *Around the World in Eighty Days* in which, to win a race, a crew burns the inside of a ship to power the steam engine. The combustion of nonrenewable fossil energy sources (and, more recently, the fission of uranium) has led to remarkably intense energy releases in power-generating equipment. The energy transferred as heat in a nuclear reactor is on the order of *one million watts per square meter*.

A complex system of heat and work transfer processes is invariably needed to bring these concentrations of energy back down to human proportions. We must understand and control the processes that divide and diffuse intense heat flows down to the level on which we can interact with them. To see how this works, consider a specific situation. Suppose we live in a town where coal is processed into fuel-gas and coke. Such power supplies used to be common, and they may return if natural gas supplies ever dwindle. Let us list a few of the process heat transfer problems that must be solved before we can drink a glass of iced tea.

- A variety of high-intensity heat transfer processes are involved with combustion and chemical reaction in the gasifier unit itself.
- The gas goes through various cleanup and pipe-delivery processes to get to our stoves. The heat transfer processes involved in these stages are generally less intense.
- The gas is burned in the stove. Heat is transferred from the flame to the bottom of the teakettle. While this process is small, it is intense because boiling is a very efficient way to remove heat.
- The coke is burned in a steam power plant. The heat transfer rates from the combustion chamber to the boiler, and from the wall of the boiler to the water inside, are very intense.

- The steam passes through a turbine where it is involved with many heat transfer processes, including some condensation in the last stages. The spent steam is then condensed in any of a variety of heat transfer devices.
- Cooling must be provided in each stage of the electrical supply system: the winding and bearings of the generator, the transformers, the switches, the power lines, and the wiring in our houses.
- The ice cubes for our tea are made in an electrical refrigerator. It involves three major heat exchange processes and several lesser ones. The major ones are the condensation of refrigerant at room temperature to reject heat, the absorption of heat from within the refrigerator by evaporating the refrigerant, and the balancing heat leakage from the room to the inside.
- Let's drink our iced tea quickly because heat transfer from the room to the water and from the water to the ice will first dilute, and then warm, our tea if we linger.

A society based on power technology teems with heat transfer problems. Our aim is to learn the principles of heat transfer so we can solve these problems and design the equipment needed to transfer thermal energy from one substance to another. In a broad sense, all these problems resolve themselves into collecting and focusing large quantities of energy for the use of people, and then distributing and interfacing this energy with people in such a way that they can use it on their own puny level.

We begin our study by recollecting how heat transfer was treated in the study of thermodynamics and by seeing why thermodynamics is not adequate to the task of solving heat transfer problems.

1.2 Relation of heat transfer to thermodynamics

The First Law with work equal to zero

The subject of thermodynamics, as taught in engineering programs, makes constant reference to the heat transfer between systems. The First Law of Thermodynamics for a closed system takes the following form on a

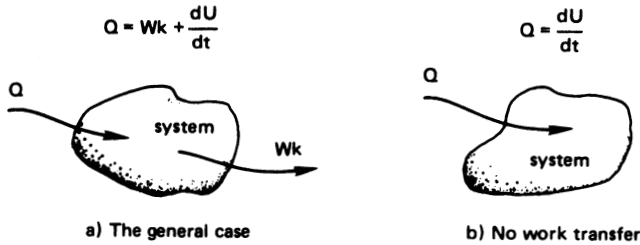


Figure 1.1 The First Law of Thermodynamics for a closed system.

rate basis:

$$\underbrace{Q}_{\substack{\text{positive toward} \\ \text{the system}}} = \underbrace{Wk}_{\substack{\text{positive away} \\ \text{from the system}}} + \underbrace{\frac{dU}{dt}}_{\substack{\text{positive when} \\ \text{the system's} \\ \text{energy increases}}} \quad (1.1)$$

where Q is the heat transfer rate and Wk is the work transfer rate. They may be expressed in joules per second (J/s) or watts (W). The derivative dU/dt is the rate of change of internal thermal energy, U , with time, t . This interaction is sketched schematically in Fig. 1.1a.

The analysis of heat transfer processes can generally be done without reference to any work processes, although heat transfer might subsequently be combined with work in the analysis of real systems. If $p dV$ work is the only work that occurs, then eqn. (1.1) is

$$Q = p \frac{dV}{dt} + \frac{dU}{dt} \quad (1.2a)$$

This equation has two well-known special cases:

$$\text{Constant volume process:} \quad Q = \frac{dU}{dt} = mc_v \frac{dT}{dt} \quad (1.2b)$$

$$\text{Constant pressure process:} \quad Q = \frac{dH}{dt} = mc_p \frac{dT}{dt} \quad (1.2c)$$

where $H \equiv U + pV$ is the enthalpy, and c_v and c_p are the specific heat capacities at constant volume and constant pressure, respectively.

When the substance undergoing the process is incompressible (so that V is constant for any pressure variation), the two specific heats are equal:

$c_v = c_p \equiv c$. The proper form of eqn. (1.2a) is then

$$Q = \frac{dU}{dt} = mc \frac{dT}{dt} \quad (1.3)$$

Since solids and liquids can frequently be approximated as being incompressible, we shall often make use of eqn. (1.3).

If the heat transfer were reversible, then eqn. (1.2a) would become²

$$\underbrace{T \frac{dS}{dt}}_{Q_{\text{rev}}} = \underbrace{p \frac{dV}{dt}}_{W_{\text{rev}}} + \frac{dU}{dt} \quad (1.4)$$

That might seem to suggest that Q can be evaluated independently for inclusion in either eqn. (1.1) or (1.3). However, it cannot be evaluated using $T dS$, because real heat transfer processes are all irreversible and S is not defined as a function of T in an irreversible process. The reader will recall that engineering thermodynamics might better be named *thermostatics*, because it only describes the equilibrium states on either side of irreversible processes.

Since the rate of heat transfer cannot be predicted using $T dS$, how can it be determined? If $U(t)$ were known, then (when $Wk = 0$) eqn. (1.3) would give Q , but $U(t)$ is seldom known *a priori*.

The answer is that a new set of physical principles must be introduced to predict Q . The principles are *transport laws*, which are not a part of the subject of thermodynamics. They include Fourier's law, Newton's law of cooling, and the Stefan-Boltzmann law. We introduce these laws later in the chapter. The important thing to remember is that a description of heat transfer requires that additional principles be combined with the First Law of Thermodynamics.

Reversible heat transfer as the temperature gradient vanishes

Consider a wall connecting two thermal reservoirs as shown in Fig. 1.2. As long as $T_1 > T_2$, heat will flow *spontaneously and irreversibly* from 1 to 2. In accordance with our understanding of the Second Law of Thermodynamics, we expect the entropy of the universe to increase as a consequence of this process. If $T_2 \rightarrow T_1$, the process will approach being quasistatic and reversible. But the rate of heat transfer will also approach

² T = absolute temperature, S = entropy, V = volume, p = pressure, and "rev" denotes a reversible process.

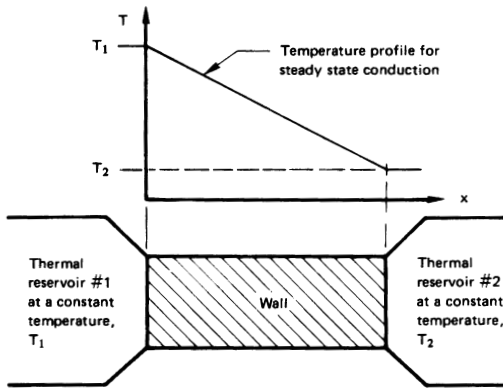


Figure 1.2 Irreversible heat flow between two thermal reservoirs through an intervening wall.

zero if there is no temperature difference to drive it. Thus all real heat transfer processes generate entropy.

Now we come to a dilemma: If the irreversible process occurs at steady state, the properties of the wall do not vary with time. We know that the entropy of the wall depends on its state and must therefore be constant. How, then, does the entropy of the universe increase? We turn to this question next.

Entropy production

The entropy increase of the universe as the result of a process is the sum of the entropy changes of *all* elements that are involved in that process. The *rate of entropy production* of the universe, \dot{S}_{Un} , resulting from the preceding heat transfer process through a wall is

$$\dot{S}_{\text{Un}} = \dot{S}_{\text{res 1}} + \underbrace{\dot{S}_{\text{wall}}}_{= 0, \text{ since } S_{\text{wall}} \text{ must be constant}} + \dot{S}_{\text{res 2}} \quad (1.5)$$

where the dots denote time derivatives (i.e., $\dot{x} \equiv dx/dt$). Since the reservoir temperatures are constant,

$$\dot{S}_{\text{res}} = \frac{Q}{T_{\text{res}}}. \quad (1.6)$$

Now $Q_{\text{res 1}}$ is negative and equal in magnitude to $Q_{\text{res 2}}$, so eqn. (1.5) becomes

$$\dot{S}_{\text{Un}} = \left| Q_{\text{res 1}} \right| \left(\frac{1}{T_2} - \frac{1}{T_1} \right). \quad (1.7)$$

The term in parentheses is positive, so $\dot{S}_{\text{Un}} > 0$. This agrees with Clausius's statement of the Second Law of Thermodynamics.

Notice an odd fact here: The rate of heat transfer, Q , and hence \dot{S}_{Un} , is determined by the wall's resistance to heat flow. Although the wall is the agent that causes the entropy of the universe to increase, its own entropy does not change. Only the entropies of the reservoirs change.

1.3 Modes of heat transfer

Figure 1.3 shows an analogy that might be useful in fixing the concepts of heat conduction, convection, and radiation as we proceed to look at each in some detail.

Heat conduction

Fourier's law. Joseph Fourier (see Fig. 1.4) published his remarkable book *Théorie Analytique de la Chaleur* in 1822. In it he formulated a very complete exposition of the theory of heat conduction.

He began his treatise by stating the empirical law that bears his name: *the heat flux,³ q (W/m²), resulting from thermal conduction is proportional to the magnitude of the temperature gradient and opposite to it in sign.* If we call the constant of proportionality, k , then

$$q = -k \frac{dT}{dx} \quad (1.8)$$

The constant, k , is called the *thermal conductivity*. It obviously must have the dimensions W/m·K, or J/m·s·K, or Btu/h·ft·°F if eqn. (1.8) is to be dimensionally correct.

The heat flux is a vector quantity. Equation (1.8) tells us that if temperature decreases with x , q will be positive—it will flow in the x -direction. If T increases with x , q will be negative—it will flow opposite the x -direction. In either case, q will flow from higher temperatures to lower temperatures. Equation (1.8) is the one-dimensional form of Fourier's law. We develop its three-dimensional form in Chapter 2, namely:

$$\vec{q} = -k \nabla T$$

³The heat flux, q , is a heat rate per unit area and can be expressed as Q/A , where A is an appropriate area.

5.6 Transient heat conduction to a semi-infinite region

Introduction

Bronowski's classic television series, *The Ascent of Man* [5.9], included a brilliant reenactment of the ancient ceremonial procedure by which the Japanese forged Samurai swords (see Fig. 5.13). The metal is heated, folded, beaten, and formed, over and over, to create a blade of remarkable toughness and flexibility. When the blade is formed to its final configuration, a tapered sheath of clay is baked on the outside of it, so the cross section is as shown in Fig. 5.13. The red-hot blade with the clay sheath is then subjected to a rapid quenching, which cools the uninsulated cutting edge quickly and the back part of the blade very slowly. The result is a layer of case-hardening that is hardest at the edge and less hard at points farther from the edge.

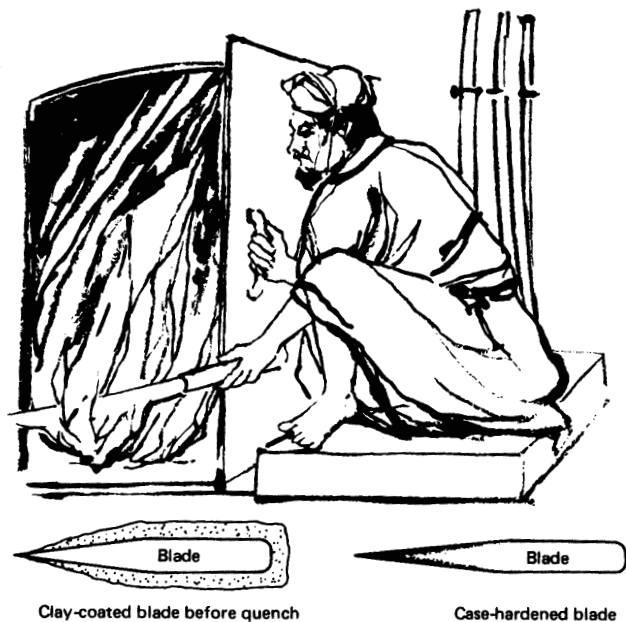


Figure 5.13 The ceremonial case-hardening of a Samurai sword.

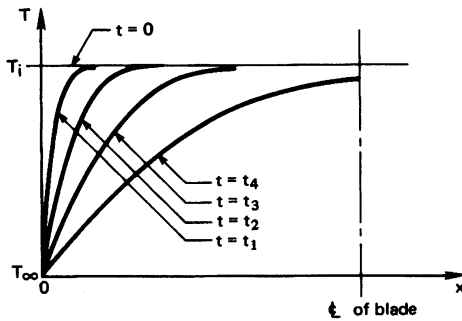


Figure 5.14 The initial cooling of a thin sword blade. Prior to $t = t_4$, the blade might as well be infinitely thick insofar as cooling is concerned.

The blade is then tough and ductile, so it will not break, but has a fine hard outer shell that can be honed to sharpness. We need only look a little way up the side of the clay sheath to find a cross section that was thick enough to prevent the blade from experiencing the sudden effects of the cooling quench. The success of the process actually relies on the *failure* of the cooling to penetrate the clay very deeply in a short time.

Now we ask how we can say whether or not the influence of a heating or cooling process is restricted to the surface of a body. Or if we turn the question around: “Under what conditions can we view the depth of a body as *infinite* with respect to the thickness of the region that has felt the heat transfer process?”

Consider next the cooling process within the blade in the absence of the clay retardant and when \bar{h} is very large. Actually, our considerations will apply initially to any finite body whose boundary suddenly changes temperature. The temperature distribution, in this case, is sketched in Fig. 5.14 for four sequential times. Only the fourth curve—that for which $t = t_4$ —is noticeably influenced by the opposite wall. Up to that time, the wall might as well have infinite depth.

Since any body subjected to a sudden change of temperature is infinitely large in comparison with the initial region of temperature change, we must learn how to treat heat transfer in this period.

Solution aided by dimensional analysis

The calculation of the temperature distribution in a semi-infinite region poses a difficulty: we can impose a definite b.c. at only one position—the exposed boundary. We get around that difficulty in a nice way with the help of dimensional analysis.

When the one boundary of a semi-infinite region, initially at $T = T_i$, is suddenly cooled (or heated) to a new temperature, T_∞ , as in Fig. 5.14, the dimensional function equation is

$$T - T_\infty = \text{fn}[t, x, \alpha, (T_i - T_\infty)]$$

where there is *no characteristic length or time*. Since there are five variables in °C, s, and m, we should look for two dimensional groups.

$$\underbrace{\frac{T - T_\infty}{T_i - T_\infty}}_{\Theta} = \text{fn}\left(\underbrace{\frac{x}{\sqrt{\alpha t}}}_{\zeta}\right) \quad (5.44)$$

The very important thing that we learn from this exercise in dimensional analysis is that position and time collapse into one independent variable. This means that the heat conduction equation and its b.c.s must transform from a partial differential equation into a simpler ordinary differential equation in the single variable, $\zeta = x/\sqrt{\alpha t}$. Thus, we transform each side of

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

as follows, where we call $T_i - T_\infty \equiv \Delta T$:

$$\frac{\partial T}{\partial t} = (T_i - T_\infty) \frac{\partial \Theta}{\partial t} = \Delta T \frac{\partial \Theta}{\partial \zeta} \frac{\partial \zeta}{\partial t} = \Delta T \left(-\frac{x}{2t\sqrt{\alpha t}}\right) \frac{\partial \Theta}{\partial \zeta};$$

$$\frac{\partial T}{\partial x} = \Delta T \frac{\partial \Theta}{\partial \zeta} \frac{\partial \zeta}{\partial x} = \frac{\Delta T}{\sqrt{\alpha t}} \frac{\partial \Theta}{\partial \zeta};$$

$$\text{and } \frac{\partial^2 T}{\partial x^2} = \frac{\Delta T}{\sqrt{\alpha t}} \frac{\partial^2 \Theta}{\partial \zeta^2} \frac{\partial \zeta}{\partial x} = \frac{\Delta T}{\alpha t} \frac{\partial^2 \Theta}{\partial \zeta^2}.$$

Substituting the first and last of these derivatives in the heat conduction equation, we get the ordinary differential equation

$$\frac{d^2 \Theta}{d\zeta^2} = -\frac{\zeta}{2} \frac{d\Theta}{d\zeta} \quad (5.45)$$

Notice that we changed from partial to total derivative notation since Θ now depends solely on ζ . The i.c. for eqn. (5.45) is

$$T(t = 0) = T_i \quad \text{or} \quad \Theta(\zeta \rightarrow \infty) = 1 \quad (5.46)$$

and the one known b.c. is

$$T(x=0) = T_\infty \quad \text{or} \quad \Theta(\zeta=0) = 0 \quad (5.47)$$

If we call $d\Theta/d\zeta \equiv \chi$, then eqn. (5.45) becomes the first-order equation

$$\frac{d\chi}{d\zeta} = -\frac{\zeta}{2}\chi$$

which can be integrated once to get

$$\chi \equiv \frac{d\Theta}{d\zeta} = C_1 e^{-\zeta^2/4} \quad (5.48)$$

and we integrate this a second time to get

$$\Theta = C_1 \int_0^\zeta e^{-\zeta'^2/4} d\zeta' + \underbrace{\Theta(0)}_{= 0 \text{ according to the b.c.}} \quad (5.49)$$

The b.c. is now satisfied, and we need only substitute eqn. (5.49) in the i.c., eqn. (5.46), to solve for C_1 :

$$1 = C_1 \int_0^\infty e^{-\zeta'^2/4} d\zeta'$$

This particular definite integral is given by integral tables as $\sqrt{\pi}$, so

$$C_1 = \frac{1}{\sqrt{\pi}}$$

Thus the solution to the problem of conduction in a semi-infinite region, subject to a b.c. of the first kind is

$$\Theta = \frac{1}{\sqrt{\pi}} \int_0^\zeta e^{-\zeta'^2/4} d\zeta' = \frac{2}{\sqrt{\pi}} \int_0^{\zeta/2} e^{-s^2} ds \equiv \text{erf}(\zeta/2) \quad (5.50)$$

The second integral in eqn. (5.50), obtained by a change of variables, is called the *error function* (erf). Its name arises from its relationship to certain statistical problems related to the Gaussian distribution, which describes random errors. In Table 5.3, we list values of the error function and the complementary error function, $\text{erfc}(x) \equiv 1 - \text{erf}(x)$. Equation (5.50) is also plotted in Fig. 5.15.

Table 5.3 Error function and complementary error function.

$\zeta/2$	$\text{erf}(\zeta/2)$	$\text{erfc}(\zeta/2)$	$\zeta/2$	$\text{erf}(\zeta/2)$	$\text{erfc}(\zeta/2)$
0.00	0.00000	1.00000	1.10	0.88021	0.11980
0.05	0.05637	0.94363	1.20	0.91031	0.08969
0.10	0.11246	0.88754	1.30	0.93401	0.06599
0.15	0.16800	0.83200	1.40	0.95229	0.04771
0.20	0.22270	0.77730	1.50	0.96611	0.03389
0.30	0.32863	0.67137	1.60	0.97635	0.02365
0.40	0.42839	0.57161	1.70	0.98379	0.01621
0.50	0.52050	0.47950	1.80	0.98909	0.01091
0.60	0.60386	0.39614	1.8214	0.99000	0.01000
0.70	0.67780	0.32220	1.90	0.99279	0.00721
0.80	0.74210	0.25790	2.00	0.99532	0.00468
0.90	0.79691	0.20309	2.50	0.99959	0.00041
1.00	0.84270	0.15730	3.00	0.99998	0.00002

In Fig. 5.15 we see that the early-time curves shown in Fig. 5.14 have collapsed into a single curve. This was accomplished by the *similarity transformation*, as we call it⁵: $\zeta/2 = x/2\sqrt{\alpha t}$. From the figure or from Table 5.3, we see that $\Theta \geq 0.99$ when

$$\frac{\zeta}{2} = \frac{x}{2\sqrt{\alpha t}} \geq 1.8214 \quad \text{or} \quad x \geq \delta_{99} \equiv 3.64\sqrt{\alpha t} \quad (5.51)$$

In other words, the local value of $(T - T_\infty)$ is more than 99% of $(T_i - T_\infty)$ for positions in the slab beyond farther from the surface than $\delta_{99} = 3.64\sqrt{\alpha t}$.

Example 5.4

For what maximum time can a samurai sword be analyzed as a semi-infinite region after it is quenched, if it has no clay coating and $\bar{h}_{\text{external}} \cong \infty$?

SOLUTION. First, we must guess the half-thickness of the sword (say, 3 mm) and its material (probably wrought iron with an average α

⁵The transformation is based upon the “similarity” of spatial and temporal changes in this problem.

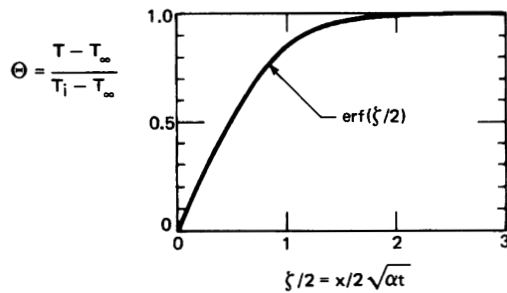


Figure 5.15 Temperature distribution in a semi-infinite region.

around $1.5 \times 10^{-5} \text{ m}^2/\text{s}$). The sword will be semi-infinite until δ_{99} equals the half-thickness. Inverting eqn. (5.51), we find

$$t \leq \frac{\delta_{99}^2}{3.64^2 \alpha} = \frac{(0.003 \text{ m})^2}{13.3(1.5)(10)^{-5} \text{ m}^2/\text{s}} = 0.045 \text{ s}$$

Thus the quench would be felt at the centerline of the sword within only 1/20 s. The thermal diffusivity of clay is smaller than that of steel by a factor of about 30, so the quench time of the coated steel must continue for over 1 s before the temperature of the steel is affected at all, if the clay and the sword thicknesses are comparable. ■

Equation (5.51) provides an interesting foretaste of the notion of a fluid boundary layer. In the context of Fig. 1.9 and Fig. 1.10, we observe that free stream flow around an object is disturbed in a thick layer near the object because the fluid adheres to it. It turns out that the thickness of this boundary layer of altered flow velocity increases in the downstream direction. For flow over a flat plate, this thickness is approximately $4.92\sqrt{\nu t}$, where t is the time required for an element of the stream fluid to move from the leading edge of the plate to a point of interest. This is quite similar to eqn. (5.51), except that the thermal diffusivity, α , has been replaced by its counterpart, the kinematic viscosity, ν , and the constant is a bit larger. The velocity profile will resemble Fig. 5.15.

If we repeated the problem with a boundary condition of the third kind, we would expect to get $\Theta = \Theta(\text{Bi}, \zeta)$, except that there is no length, L , upon which to build a Biot number. Therefore, we must replace L with $\sqrt{\alpha t}$, which has the dimension of length, so

$$\Theta = \Theta\left(\zeta, \frac{\bar{h}\sqrt{\alpha t}}{k}\right) \equiv \Theta(\zeta, \beta) \tag{5.52}$$

The term $\beta \equiv \bar{h}\sqrt{\alpha t}/k$ is like the product: $\text{Bi}\sqrt{\text{Fo}}$. The solution of this problem (see, e.g., [5.6], §2.7) can be conveniently written in terms of the complementary error function, $\text{erfc}(x) \equiv 1 - \text{erf}(x)$:

$$\Theta = \text{erf} \frac{\zeta}{2} + \exp(\beta\zeta + \beta^2) \left[\text{erfc} \left(\frac{\zeta}{2} + \beta \right) \right] \quad (5.53)$$

We offer our own original graph of this result in Fig. 5.16.

Example 5.5

Most of us have passed our finger through an 800°C candle flame and know that if we limit exposure to about 1/4 s we will not be burned. Why not?

SOLUTION. The short exposure to the flame causes only a *very* superficial heating, so we consider the finger to be a semi-infinite region and go to eqn. (5.53) to calculate $(T_{\text{burn}} - T_{\text{flame}})/(T_i - T_{\text{flame}})$. It turns out that the burn threshold of human skin, T_{burn} , is about 65°C. (That is why 140°F or 60°C tap water is considered to be “scalding.”) Therefore, we shall calculate how long it will take for the surface temperature of the finger to rise from body temperature (37°C) to 65°C, when it is protected by an assumed $\bar{h} \cong 100 \text{ W/m}^2\text{K}$. We shall assume that the thermal conductivity of human flesh equals that of its major component—water—and that the thermal diffusivity is equal to the known value for beef. Then

$$\begin{aligned} \Theta &= \frac{65 - 800}{37 - 800} = 0.963 \\ \beta\zeta &= \frac{\bar{h}x}{k} = 0 \quad \text{since } x = 0 \text{ at the surface} \\ \beta^2 &= \frac{\bar{h}^2 \alpha t}{k^2} = \frac{100^2(0.135 \times 10^{-6})t}{0.63^2} = 0.0034(t \text{ s}) \end{aligned}$$

The situation is quite far into the corner of Fig. 5.16. We read $\beta^2 \cong 0.001$, which corresponds with $t \cong 0.3 \text{ s}$. For greater accuracy, we must go to eqn. (5.53):

$$0.963 = \underbrace{\text{erf } 0}_{=0} + e^{0.0034t} \left[\text{erfc} \left(0 + \sqrt{0.0034t} \right) \right]$$

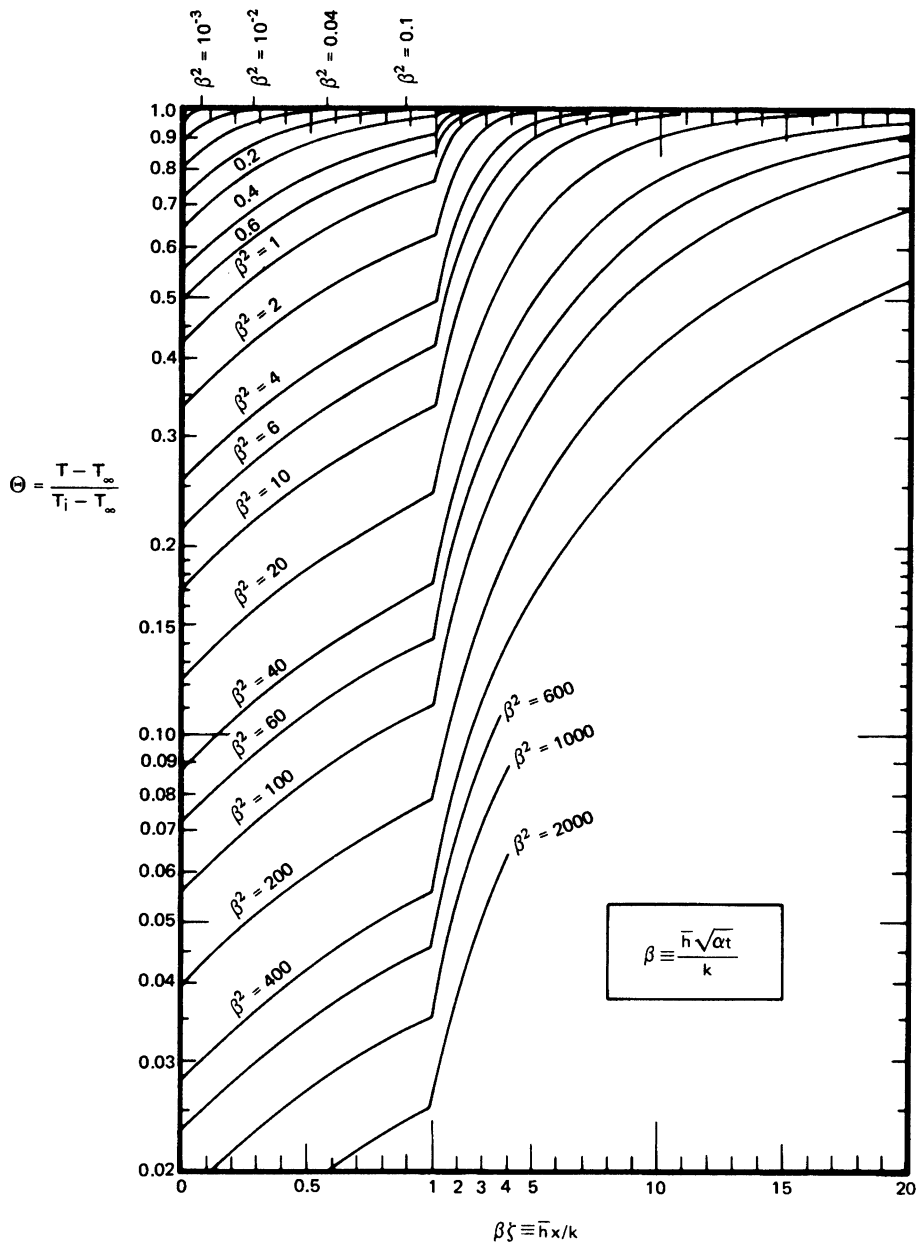


Figure 5.16 The cooling of a semi-infinite region by an environment at T_∞ , through a heat transfer coefficient, \bar{h} .

By trial and error, we get $t \cong 0.33$ s. In fact, it can be shown that

$$\Theta(\zeta = 0, \beta) \cong 1 - \frac{2\beta}{\sqrt{\pi}} \quad \text{for } \beta \ll 1 \quad \blacksquare$$

which can be solved directly for $\beta = (1 - 0.963)\sqrt{\pi}/2 = 0.03279$, leading to the same answer.

Thus, it would require about 1/3 s to bring the skin to the burn point if we have chosen a correct value of the heat transfer coefficient.

Experiment 5.1

Immerse your hand in the subfreezing air in the freezer compartment of your refrigerator. Next immerse your finger in a mixture of ice cubes and water, but do not move it. Then, immerse your finger in a mixture of ice cubes and water, swirling it around as you do so. Describe your initial sensation in each case, and explain the differences in terms of Fig. 5.16. What variable has changed from one case to another?

Heat transfer

Heat will be removed from the exposed surface of a semi-infinite region, with a b.c. of either the first or the third kind, in accordance with Fourier's law:

$$q = -k \left. \frac{\partial T}{\partial x} \right|_{x=0} = \frac{k(T_\infty - T_i)}{\sqrt{\alpha t}} \left. \frac{d\Theta}{d\zeta} \right|_{\zeta=0}$$

Differentiating Θ as given by eqn. (5.50), we obtain, for the b.c. of the first kind,

$$q = \frac{k(T_\infty - T_i)}{\sqrt{\alpha t}} \left(\frac{1}{\sqrt{\pi}} e^{-\zeta^2/4} \right)_{\zeta=0} = \frac{k(T_\infty - T_i)}{\sqrt{\pi \alpha t}} \quad (5.54)$$

Thus, q decreases with increasing time, as $t^{-1/2}$. When the temperature of the surface is first changed, the heat removal rate is enormous. Then it drops off rapidly.

It often occurs that we suddenly apply a specified input heat flux, q_w , at the boundary of a semi-infinite region. In such a case, we can

differentiate the heat diffusion equation with respect to x , so

$$\alpha \frac{\partial^3 T}{\partial x^3} = \frac{\partial^2 T}{\partial t \partial x}$$

When we substitute $q = -k \partial T / \partial x$ in this, we obtain

$$\alpha \frac{\partial^2 q}{\partial x^2} = \frac{\partial q}{\partial t}$$

with the b.c.'s:

$$q(x = 0, t > 0) = q_w \quad \text{or} \quad \left. \frac{q_w - q}{q_w} \right|_{x=0} = 0$$

$$q(x \geq 0, t = 0) = 0 \quad \text{or} \quad \left. \frac{q_w - q}{q_w} \right|_{t=0} = 1$$

What we have done here is quite elegant. We have made the problem of predicting the local heat flux q into exactly the same form as that of predicting the local temperature in a semi-infinite region subjected to a step change of wall temperature. Therefore, the solution must be the same:

$$\frac{q_w - q}{q_w} = \operatorname{erf} \left(\frac{x}{2\sqrt{\alpha t}} \right). \quad (5.55)$$

The temperature distribution is obtained by integrating Fourier's law. At the wall, for example:

$$\int_{T_i}^{T_w} dT = - \int_{\infty}^0 \frac{q}{k} dx$$

where $T_i = T(x \rightarrow \infty)$ and $T_w = T(x = 0)$. Then

$$T_w = T_i + \frac{q_w}{k} \int_0^{\infty} \operatorname{erfc}(x/2\sqrt{\alpha t}) dx$$

This becomes

$$T_w = T_i + \frac{q_w}{k} \sqrt{\alpha t} \underbrace{\int_0^{\infty} \operatorname{erfc}(\zeta/2) d\zeta}_{=2/\sqrt{\pi}}$$

so

$$T_w(t) = T_i + 2 \frac{q_w}{k} \sqrt{\frac{\alpha t}{\pi}} \quad (5.56)$$

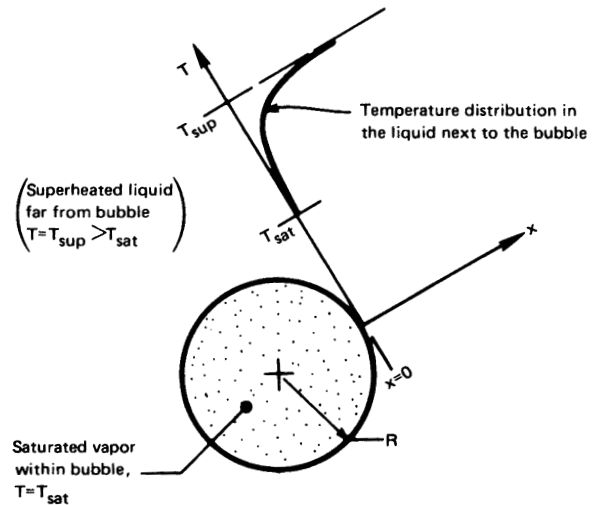


Figure 5.17 A bubble growing in a superheated liquid.

Example 5.6 Predicting the Growth Rate of a Vapor Bubble in an Infinite Superheated Liquid

This prediction is relevant to a large variety of processes, ranging from nuclear thermohydraulics to direct-contact heat exchange. It was originally presented by Max Jakob and others in the early 1930s (see, e.g., [5.10, Chap. I]). Jakob (pronounced Yah'-kob) was an important figure in heat transfer during the 1920s and 1930s. He left Nazi Germany in 1936 to come to the United States. We encounter his name again later.

Figure 5.17 shows how growth occurs. When a liquid is superheated to a temperature somewhat above its boiling point, a small gas or vapor cavity in that liquid will grow. (That is what happens in the superheated water at the bottom of a teakettle.)

This bubble grows into the surrounding liquid because its boundary is kept at the saturation temperature, T_{sat} , by the near-equilibrium coexistence of liquid and vapor. Therefore, heat must flow from the superheated surroundings to the interface, where evaporation occurs. So long as the layer of cooled liquid is thin, we should not suffer too much error by using the one-dimensional semi-infinite region solution to predict the heat flow.

Thus, we can write the energy balance at the bubble interface:

$$\underbrace{\left(-q \frac{\text{W}}{\text{m}^2}\right) (4\pi R^2 \text{ m}^2)}_{Q \text{ into bubble}} = \underbrace{\left(\rho_g h_{fg} \frac{\text{J}}{\text{m}^3}\right) \left(\frac{dV}{dt} \frac{\text{m}^3}{\text{s}}\right)}_{\text{rate of energy increase of the bubble}}$$

and then substitute eqn. (5.54) for q and $4\pi R^3/3$ for the volume, V . This gives

$$\frac{k(T_{\text{sup}} - T_{\text{sat}})}{\sqrt{\alpha\pi t}} = \rho_g h_{fg} \frac{dR}{dt} \quad (5.57)$$

Integrating eqn. (5.57) from $R = 0$ at $t = 0$ up to R at t , we obtain Jakob's prediction:

$$R = \frac{2}{\sqrt{\pi}} \frac{k\Delta T}{\rho_g h_{fg} \sqrt{\alpha}} \sqrt{t} \quad (5.58) \quad \blacksquare$$

This analysis was done without assuming the curved bubble interface to be plane, 24 years after Jakob's work, by Plesset and Zwick [5.11]. It was verified in a more exact way after another 5 years by Scriven [5.12]. These calculations are more complicated, but they lead to a very similar result:

$$R = \frac{2\sqrt{3}}{\sqrt{\pi}} \frac{k\Delta T}{\rho_g h_{fg} \sqrt{\alpha}} \sqrt{t} = \sqrt{3} R_{\text{Jakob}}. \quad (5.59)$$

Both predictions are compared with some of the data of Dergarabedian [5.13] in Fig. 5.18. The data and the exact theory match almost perfectly. The simple theory of Jakob et al. shows the correct dependence on R on all its variables, but it shows growth rates that are low by a factor of $\sqrt{3}$. This is because the expansion of the spherical bubble causes a relative motion of liquid toward the bubble surface, which helps to thin the region of thermal influence in the radial direction. Consequently, the temperature gradient and heat transfer rate are higher than in Jakob's model, which neglected the liquid motion. Therefore, the temperature profile flattens out more slowly than Jakob predicts, and the bubble grows more rapidly.

Experiment 5.2

Touch various objects in the room around you: glass, wood, corkboard, paper, steel, and gold or diamond, if available. Rank them in order of which feels coldest at the first instant of contact (see Problem 5.29).

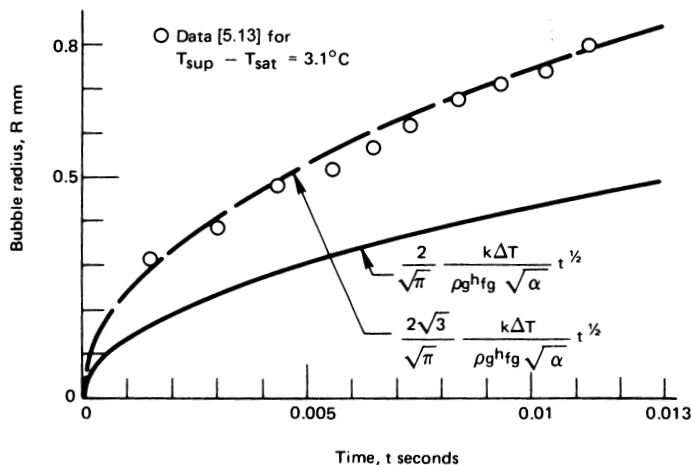


Figure 5.18 The growth of a vapor bubble—predictions and measurements.

The more advanced theory of heat conduction (see, e.g., [5.6]) shows that if two semi-infinite regions at uniform temperatures T_1 and T_2 are placed together suddenly, their interface temperature, T_s , is given by⁶

$$\frac{T_s - T_2}{T_1 - T_2} = \frac{\sqrt{(k\rho c_p)_1}}{\sqrt{(k\rho c_p)_1} + \sqrt{(k\rho c_p)_2}}$$

If we identify one region with your body ($T_1 \approx 37^\circ\text{C}$) and the other with the object being touched ($T_2 \approx 20^\circ\text{C}$), we can determine the temperature, T_s , that the surface of your finger will reach upon contact. Compare the ranking you obtain experimentally with the ranking given by this equation.

Notice that your bloodstream and capillary system provide a heat source in your finger, so the equation is valid only for a moment. Then you start replacing heat lost to the objects. If you included a diamond

⁶For semi-infinite regions, initially at uniform temperatures, T_s does not vary with time. For finite bodies, T_s will eventually change. A constant value of T_s means that each of the two bodies independently behaves as a semi-infinite body whose surface temperature has been changed to T_s at time zero. Consequently, our previous results—eqns. (5.50), (5.51), and (5.54)—apply to each of these bodies while they may be treated as semi-infinite. We need only replace T_∞ by T_s in those equations.

among the objects that you touched, you will notice that it warmed up almost instantly. Most diamonds are quite small but are possessed of the highest known value of α . Therefore, they can behave as a semi-infinite region only for an instant. Immediately after, they feel warm to the touch.

Conduction to a semi-infinite region with a harmonically oscillating temperature at the boundary

Suppose that we approximate the annual variation of the ambient temperature as sinusoidal; then we may ask what the influence of this variation will be beneath the ground. We want to calculate $T - \bar{T}$ (where \bar{T} is the time-average surface temperature) as a function of: depth, x ; thermal diffusivity, α ; frequency of oscillation, ω ; amplitude of oscillation, ΔT ; and time, t . There are six variables in K, m, and s, so the problem can be represented in three dimensionless variables:

$$\Theta \equiv \frac{T - \bar{T}}{\Delta T}; \quad \Omega \equiv \omega t; \quad \xi \equiv x \sqrt{\frac{\omega}{2\alpha}}.$$

We pose the problem as follows in these variables. The heat conduction equation is

$$\frac{1}{2} \frac{\partial^2 \Theta}{\partial \xi^2} = \frac{\partial \Theta}{\partial \Omega} \quad (5.60)$$

and the b.c.'s are

$$\Theta \Big|_{\xi=0} = \cos \omega t \quad \text{and} \quad \Theta \Big|_{\xi>0} = \text{finite} \quad (5.61)$$

No i.c. is needed because, after the initial transient decays, the remaining steady oscillation must be periodic.

The solution is given by Carslaw and Jaeger (see [5.6, §2.6] or work Problem 5.16). It is

$$\Theta(\xi, \Omega) = e^{-\xi} \cos(\Omega - \xi) \quad (5.62)$$

This result is plotted in Fig. 5.19. It shows that the surface temperature variation decays exponentially into the region and suffers a phase shift as it does so.

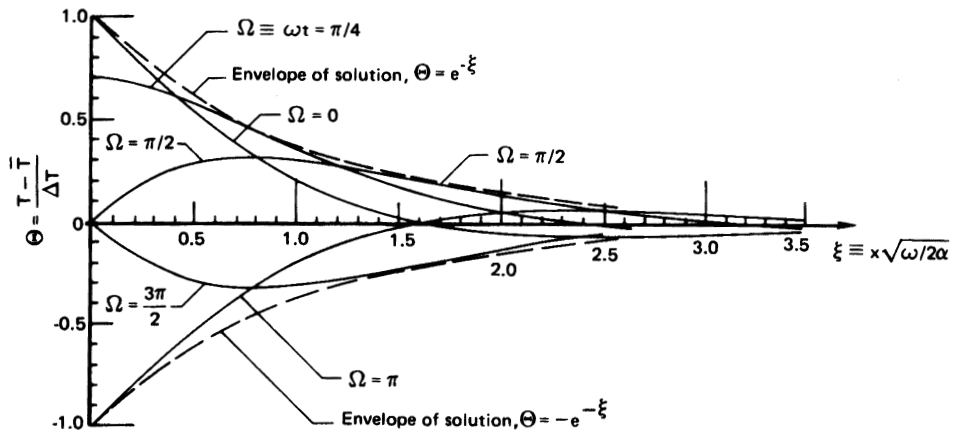


Figure 5.19 The temperature variation within a semi-infinite region whose temperature varies harmonically at the boundary.

Example 5.7

How deep in the earth must we dig to find the temperature wave that was launched by the coldest part of the last winter if it is now high summer?

SOLUTION. $\omega = 2\pi$ rad/yr, and $\Omega = \omega t = 0$ at the present. First, we must find the depths at which the $\Omega = 0$ curve reaches its local extrema. (We pick the $\Omega = 0$ curve because it gives the highest temperature at $t = 0$.)

$$\left. \frac{d\Theta}{d\xi} \right|_{\Omega=0} = -e^{-\xi} \cos(0 - \xi) + e^{-\xi} \sin(0 - \xi) = 0$$

This gives

$$\tan(0 - \xi) = 1 \quad \text{so} \quad \xi = \frac{3\pi}{4}, \frac{7\pi}{4}, \dots$$

and the first minimum occurs where $\xi = 3\pi/4 = 2.356$, as we can see in Fig. 5.19. Thus,

$$\xi = x\sqrt{\omega/2\alpha} = 2.356$$

or, if we take $\alpha = 0.139 \times 10^{-6}$ m²/s (given in [5.14] for coarse, gravelly

earth),

$$x = 2.356 \sqrt{\frac{2\pi}{2(0.139 \times 10^{-6})} \frac{1}{365(24)(3600)}} = 2.783 \text{ m}$$

If we dug in the earth, we would find it growing colder and colder until it reached a maximum coldness at a depth of about 2.8 m. Farther down, it would begin to warm up again, but not much. In midwinter ($\Omega = \pi$), the reverse would be true. ■

5.7 Steady multidimensional heat conduction

Introduction

The general equation for $T(\vec{r})$ during steady conduction in a region of constant thermal conductivity, without heat sources, is called Laplace's equation:

$$\nabla^2 T = 0 \quad (5.63)$$

It looks easier to solve than it is, since [recall eqn. (2.12) and eqn. (2.14)] the Laplacian, $\nabla^2 T$, is a sum of several second partial derivatives. We solved one two-dimensional heat conduction problem in Example 4.1, but this was not difficult because the boundary conditions were made to order. Depending upon one's mathematical background and the specific problem, the analytical solution of multidimensional problems can be anything from straightforward calculation to a considerable challenge. The reader who wishes to study such analyses in depth should refer to [5.6] or [5.15], where such calculations are discussed in detail.

Faced with a steady multidimensional problem, three routes are open to us:

- Find out whether or not the analytical solution is already available in a heat conduction text or in other published literature.
- Solve the problem.
 - (a) Analytically.
 - (b) Numerically.
- Obtain the solution graphically if the problem is two-dimensional.

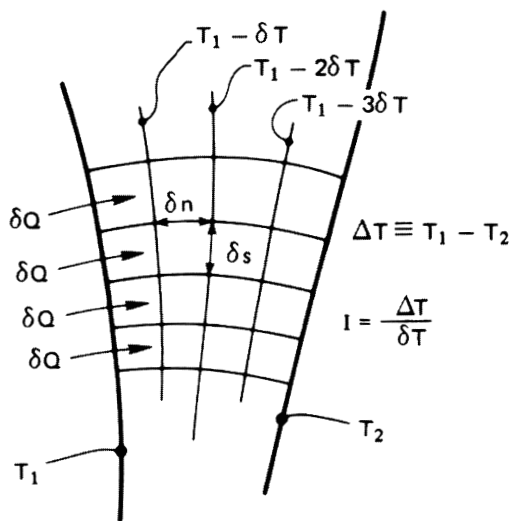


Figure 5.20 The two-dimensional flow of heat between two isothermal walls.

The last of these options is out of style as a solution method, yet it is remarkably simple and effective. We turn to it next since anyone who takes the trouble to master it will develop an uncommonly strong intuitive understanding of multidimensional heat transfer along the way.

The flux plot

The method of *flux plotting* will solve all steady planar problems in which all boundaries are held at either of two temperatures or are insulated. With a little skill, it provides accuracies of a few percent—almost always greater than the accuracy with which the b.c.'s and k can be specified. And it reveals the physics of the problem very clearly.

Figure 5.20 shows heat flowing from one isothermal wall to another in a regime that does not conform to any convenient coordinate scheme. We identify a series of channels, each which carries the same heat flow, δQ W/m. We also include a set of equally spaced isotherms, δT apart, between the walls. Since the heat fluxes in all channels are the same,

$$|\delta Q| = k \frac{\delta T}{\delta n} \delta s \quad (5.64)$$

Notice that if we arrange things so that δQ , δT , and k are the same for flow through each rectangle in the flow field, then $\delta s / \delta n$ must be the

Total emittance charts and the mean beam length provide a simple, but crude, tool for dealing with gas radiation. Since the introduction of these ideas in the mid-twentieth century, major advances have been made in our knowledge of the radiative properties of gases and in the tools available for solving gas radiation problems. In particular, band models of gas radiation, and better measurements, have led to better procedures for dealing with the total radiative properties of gases (see, in particular, References [10.11] and [10.13]). Tools for dealing with radiation in complex enclosures have also improved. The most versatile of these is the previously-mentioned Monte Carlo method [10.4, 10.7], which can deal with nongray, nondiffuse, and nonisothermal walls with nongray, scattering, and nonisothermal gases. An extensive literature also deals with approximate analytical techniques, many of which are based on the idea of a “gray gas” — one for which ϵ_λ and α_λ are independent of wavelength. However, as we have pointed out, the gray gas model is not even a *qualitative* approximation to the properties of real gases.⁷

Finally, it is worth noting that gaseous radiation is frequently less important than one might imagine. Consider, for example, two flames: a bright orange candle flame and a “cold-blue” hydrogen flame. Both have a great deal of water vapor in them, as a result of oxidizing H_2 . But the candle will warm your hands if you place them near it and the hydrogen flame will not. Yet the temperature in the hydrogen flame is *higher*. It turns out that what is radiating both heat and light from the candle is soot — small solid particles of almost thermally black carbon. The CO_2 and H_2O in the candle flame actually contribute relatively little to radiation.

10.6 Solar energy

The sun

The sun continually irradiates the earth at a rate of about 1.74×10^{14} kW. If we imagine this energy to be distributed over a circular disk with the earth's diameter, the solar irradiation is about 1367 W/m^2 , as measured

⁷Edwards [10.11] describes the gray gas as a “myth.” He notes, however, that spectral variations may be overlooked for a gas containing spray droplets or particles [in a range of sizes] or for some gases that have wide, weak absorption bands within the spectral range of interest [10.2]. Some accommodation of molecular properties can be achieved using the *weighted sum of gray gases* concept [10.12], which treats a real gas as superposition of gray gases having different properties.

by satellites above the atmosphere. Much of this energy reaches the ground, where it sustains the processes of life.

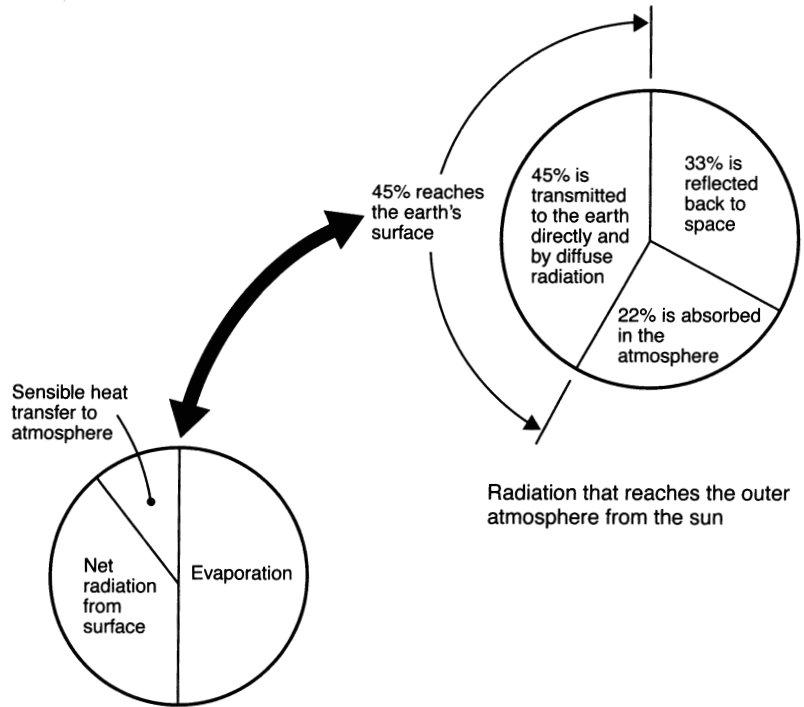
The temperature of the sun varies from tens of millions of kelvin in its core to between 4000 and 6000 K at its surface, where most of the sun's thermal radiation originates. The wavelength distribution of the sun's energy is not quite that of a black body, but it may be approximated as such. A straightforward calculation (see Problem 10.49) shows that a black body of the sun's size and distance from the earth would produce the same irradiation as the sun if its temperature were 5777 K.

The solar radiation reaching the earth's surface is always less than that above the atmosphere owing to atmospheric absorption and the earth's curvature and rotation. Solar radiation usually arrives at an angle of less than 90° to the surface because the sun is rarely directly overhead. We have seen that a radiant heat flux arriving at an angle less than 90° is reduced by the cosine of that angle (Fig. 10.4). The sun's angle varies with latitude, time of day, and day of year. Trigonometry and data for the earth's rotation can be used to find the appropriate angle.

Figure 10.2 shows the reduction of solar radiation by atmospheric absorption for one particular set of atmospheric conditions. In fact, when the sun passes through the atmosphere at a low angle (near the horizon), the path of radiation through the atmosphere is longer, providing relatively more opportunity for atmospheric absorption and scattering. Additional moisture in the air can increase the absorption by H_2O , and, of course, clouds can dramatically reduce the solar radiation reaching the ground. The consequence of these various effects is that the solar radiation received on the ground is almost never more than 1200 W/m^2 and is often only a few hundred W/m^2 . Extensive data are available for estimating the ground level solar irradiation at a given location, time, and date [10.14, 10.15].

The distribution of the Sun's energy and atmospheric irradiation

Figure 10.24 shows what becomes of the solar energy that impinges on the earth if we average it over the year and the globe, taking account of all kinds of weather. Only 45% of the sun's energy actually reaches the earth's surface. The mean energy received is about 235 W/m^2 if averaged over the surface and the year. The lower left-hand portion of the figure shows how this energy is, in turn, all returned to the atmosphere and to space.



The flow of energy from the earth's surface back to - and through - the earth's atmosphere

Figure 10.24 The approximate distribution of the flow of the sun's energy to and from the earth's surface [10.16].

The solar radiation reaching the earth's surface includes direct radiation that has passed through the atmosphere and diffuse radiation that has been scattered, but not absorbed, by the atmosphere. Atmospheric gases also irradiate the surface. This irradiation is quite important to maintaining the temperature of objects on the surface.

In Section 10.5, saw that the energy radiated by a gas depends upon the depth of the gas, its temperature, and the molecules present in it. The emittance of the atmosphere has been characterized in detail [10.16,

10.17, 10.18]. For practical calculations, however, it is often convenient to treat the sky as a black radiator having some appropriate temperature. This effective *sky temperature* usually lies between 5 and 30 K below the ground level air temperature. The sky temperature decreases as the amount of water vapor in the air goes down. For cloudless skies, the sky temperature may be estimated using the dew-point temperature, T_{dp} , and the hour past midnight, t :

$$T_{sky} = T_{air} \left[0.711 + 0.0056 T_{dp} + 7.3 \times 10^{-5} T_{dp}^2 + 0.013 \cos(2\pi t/24) \right]^{1/4} \quad (10.55)$$

where T_{sky} and T_{air} are in kelvin and T_{dp} is in °C. This equation applies for dew points from -20°C to 30°C [10.19].

It is fortunate that sky temperatures are relatively warm. In the absence of an atmosphere, not only would more of the sun's radiation reach the ground during the day, but at night heat would radiate directly into the bitter cold of outer space. Such conditions prevail on the Moon, where average daytime surface temperatures are about 110°C while average nighttime temperatures plunge to about -150°C .

Selective emitters, absorbers, and transmitters

We have noted that most of the sun's energy lies at wavelengths near the visible region of the electromagnetic spectrum and that most of the radiation from objects at temperatures typical of the earth's surface is on much longer, infrared wavelengths (see pg. 537). One result is that materials may be chosen or designed to be selectively good emitters or reflectors of both solar and infrared radiation.

Table 10.4 shows the infrared emittance and solar absorptance for several materials. Among these, we identify several particularly selective solar absorbers and solar reflectors. The selective absorbers have a high absorptance for solar radiation and a low emittance for infrared radiation. Consequently, they do not strongly reradiate the solar energy that they absorb. The selective solar reflectors, on the other hand, reflect solar energy strongly and also radiate heat efficiently in the infrared. Solar reflectors stay much cooler than solar absorbers in bright sunlight.

A wide range of selective coatings have been developed for solar absorbers operating in various temperature ranges. Coatings with solar absorptance above 90% and infrared emittance below 10% are commercially available. A comprehensive review of selective absorber materials is given in [10.20].

Table 10.4 Solar absorptance and infrared emittance for several surfaces near 300 K [10.4, 10.14].

Surface	α_{solar}	ϵ_{IR}
Aluminum, pure	0.09	0.1
Carbon black in acrylic binder	0.94	0.83
Copper, polished	0.3	0.04
<i>Selective Solar absorbers</i>		
Black Cr on Ni plate	0.95	0.09
CuO on Cu (Ebanol C)	0.90	0.16
Nickel black on steel	0.81	0.17
Sputtered cermet on steel	0.96	0.16
<i>Selective Solar Reflectors</i>		
Magnesium oxide	0.14	0.7
Snow	0.2–0.35	0.82
White paint		
Acrylic	0.26	0.90
Zinc Oxide	0.12–0.18	0.93

Example 10.12

In Section 10.2, we discussed white paint on a roof as a selective solar absorber. Consider now a barn roof under a sunlit sky. The solar radiation on the plane of the roof is 600 W/m^2 , the air temperature is 35°C , and a light breeze produces a convective heat transfer coefficient of $\bar{h} = 8 \text{ W/m}^2\text{K}$. The sky temperature is 18°C . Find the temperature of the roof if it is painted with white acrylic paint, and find it again if painted with a non-selective black paint having $\epsilon = 0.9$.

SOLUTION. Heat loss from the roof to the inside of the barn will lower the roof temperature. Since we don't have enough information to evaluate that loss, we can make an upper bound on the roof temperature by assuming that no heat is transferred to the interior. Then, an energy balance on the roof must account for radiation absorbed from the sun and the sky and for heat lost by convection and reradiation:

$$\alpha_{solar} q_{solar} + \epsilon_{IR} \sigma T_{sky}^4 = \bar{h} (T_{roof} - T_{air}) + \epsilon_{IR} \sigma T_{roof}^4$$

Rearranging and substituting the given numbers,

$$8 [T_{\text{roof}} - (273 + 35)] + \epsilon_{\text{IR}} (5.67 \times 10^{-8}) [T_{\text{roof}}^4 - (273 + 18)^4] = \alpha_{\text{solar}} (600)$$

For the non-selective black paint, $\alpha_{\text{solar}} = \epsilon_{\text{IR}} = 0.90$. Solving by iteration, we find

$$T_{\text{roof}} = 338 \text{ K} = 65^\circ\text{C}$$

For white acrylic paint, from Table 10.4, $\alpha_{\text{solar}} = 0.26$ and $\epsilon_{\text{IR}} = 0.90$. We find

$$T_{\text{roof}} = 312 \text{ K} = 39^\circ\text{C}$$

The white painted roof is only a few degrees warmer than the air. ■

Ordinary window glass is a very selective transmitter of solar radiation. Glass is nearly transparent to wavelengths below $2.7 \mu\text{m}$ or so, passing more than 90% of the incident solar energy. At longer wavelengths, in the infrared, glass is virtually opaque to radiation. A consequence of this fact is that solar energy passing through a window cannot pass back out as infrared reradiation. This is precisely why we make greenhouses out of glass. A *greenhouse* is a structure in which we use glass capture solar energy in the interior of a lower temperature space. The glass allows sunlight to enter the space, it stops air from flowing into the space, and it absorbs infrared reradiation from the interior rather than letting it pass directly back to the sky. All these factors help make the interior warm relative to the outside.

The atmospheric greenhouse effect and global warming

The atmosphere creates a *greenhouse effect* on the earth's surface that is very similar to that caused by a pane of glass. Solar energy passes through the atmosphere, arriving mainly on wavelengths between about 0.3 and $3 \mu\text{m}$. The earth's surface, having a mean temperature of 15°C or so, radiates mainly on infrared wavelengths longer than $5 \mu\text{m}$. Certain atmospheric gases have strong absorption bands at these longer wavelengths. Those gases absorb energy radiated from the surface, and then reemit it toward both the surface and outer space, reducing the net rate of radiative heat loss from the surface to outer space. The result is that the surface remains some 30 K warmer than the atmosphere. In effect,

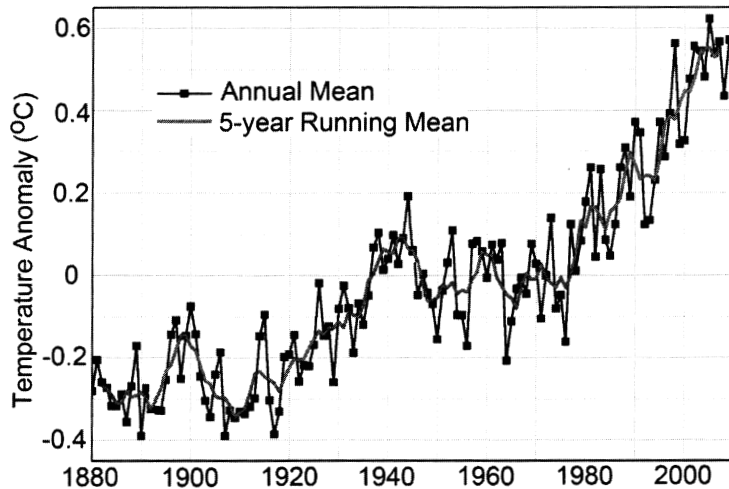


Figure 10.25 Global surface temperature change relative to the mean temperature from 1950–1980 (Courtesy of the NASA Goddard Institute for Space Studies [10.22, 10.23]).

the atmosphere functions as a radiation shield against infrared heat loss to space.

The gases mainly responsible for the the atmospheric greenhouse effect are CO_2 , H_2O , CH_4 , N_2O , O_3 , and some chlorofluorocarbons [10.21]. If the concentration of these gases rises or falls, the strength of the greenhouse effect will change and the surface temperature will also rise or fall. With the exception of the chlorofluorocarbons, each of these gases is created, in part, by natural processes: H_2O by evaporation, CO_2 by animal respiration, CH_4 through plant decay and digestion by livestock, and so on. Human activities, however, have significantly increased the concentrations of all of the gases. Fossil fuel combustion increased the CO_2 concentration by more than 30% during the twentieth century. Methane concentrations have risen through the transportation and leakage of hydrocarbon fuels. Ground level ozone concentrations have risen as a result of photochemical interactions of other pollutants. Chlorofluorocarbons are human-made chemicals.

In parallel to the rising concentrations of these gases, the surface temperature of the earth has risen significantly. Over the course of the

twentieth century, a rise of 0.6–0.7 K occurred, with 0.4–0.5 K of that rise coming after 1950 (see Fig. 10.25). The data showing this rise are extensive, are derived from multiple sources, and have been the subject of detailed scrutiny: there is relatively little doubt that surface temperatures have increased [10.22, 10.23, 10.24]. The question of how much of the rise should be attributed to anthropogenic greenhouse gases, however, remains a subject of intense debate.

Many factors must be considered in examining the causes of global warming. Carbon dioxide, for example, is present in such high concentrations that adding more of it increases absorption less rapidly than might be expected. Other gases that are present in smaller concentrations, such as methane, have far stronger effects per additional kilogram. The concentration of water vapor in the atmosphere rises with increasing surface temperature, amplifying any warming trend. Increased cloud cover has both warming and cooling effects. The melting of polar ice caps as temperatures rise reduces the planet's reflectance, or *albedo*, allowing more solar energy to be absorbed. Small temperature rises that have been observed in the oceans represent enormous amounts of stored energy that must be taken into account. Atmospheric aerosols (two-thirds of which are produced by sulfate and carbon pollution from fossil fuels) also tend to reduce the greenhouse effect. All of these factors must be built into an accurate climate model (see, for example, [10.25]).

The current consensus among mainstream researchers is that the global warming seen during the last half of the twentieth century is mainly attributable to human activity, principally through the combustion of fossil fuels [10.24]. Numerical models have been used to project a continuing temperature rise in the twenty-first century. These are based on various scenarios of future fossil fuel use and future government policies for reducing greenhouse gas emissions. Regrettably, the outlook is not very positive, with best estimates of twenty-first century warming ranging from roughly 1.8–4.0 K.

The potential for solar power

One alternative to the continuing use of fossil fuels is solar energy. With so much solar energy falling upon all parts of the world, and with the apparent safety, reliability, and cleanliness of most schemes for utilizing solar energy, one might ask why we do not generally use solar power already. The reason is that solar power involves many serious heat transfer and thermodynamics design problems and may pose environmental threats of its own. We shall discuss the problems qualitatively and refer