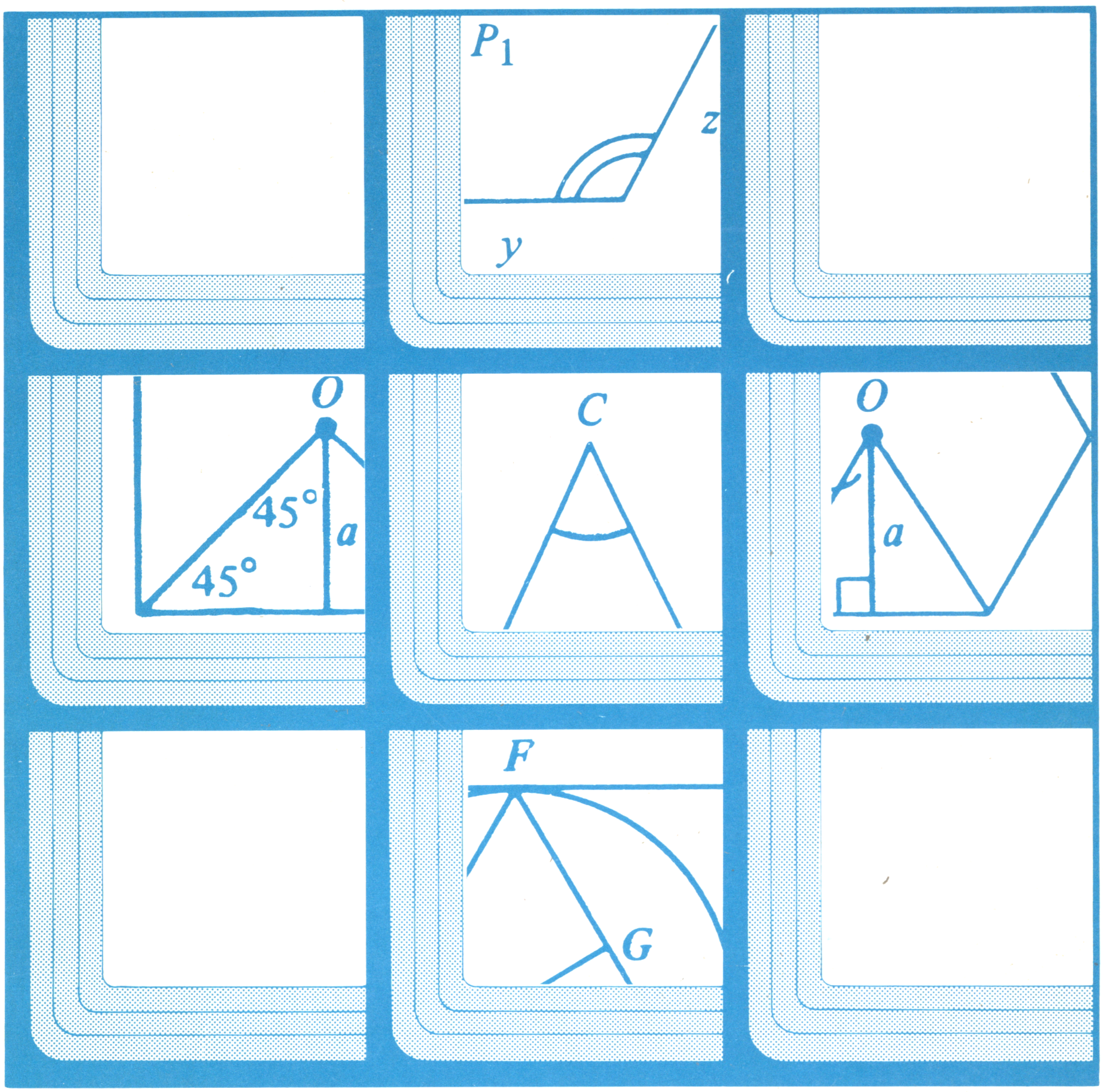
TEACHER'S MANUAL OF SOLUTIONS TO ACCOMPANY

GEOMETRY

Second Edition



POSAMENTIER / BANKS / BANNISTER

TEACHER'S MANUAL OF SOLUTIONS TO ACCOMPANY

GEOMETRY

Its Elements and Structure

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Second Edition

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CONTENTS

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Exercises

1. $\overline{CB}$ 2. $\overline{CD}$ 3. C 4. $\overline{AD}$ 5. $\overline{CB}$

6. $\overline{AD}$ with the interior points of $\overline{CB}$ removed.

7. null set 8. $\overline{AB}$ 9. $\overline{AB}$, $\overline{AC}$, $\overline{CB}$

10. Intersection is the null set.
Union is the set of whole numbers.

11. Intersection is the null set.
Union is the set of nonzero integers.

12. Yes 13. No, not when $A \subseteq B$.

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14. $A \cup B = \{1, 3, 5, 7, 9, 10, 11, 13, 15, 17, 19, 20, 21, 23\}$

15. $A \cap B = \{5, 15\}$

16. $A \cap A = A$

17. $U \cup B = U$ 18. $(A \cup B) \cup U = U$

19. $(A \cap B) \cap U = A \cap B = \{5, 15\}$

20. $U \cap B = B$ 21. $A \cup (B \cup U) = U$

22. $A \cap (B \cap U) = A \cap B = \{5, 15\}$

23. $U \cap (A \cup B) = A \cup B$

24. $U \cap A = A$ 25. $\emptyset \cap U = \emptyset$

26. $(U \cap A) \cup (U \cap B) = A \cup B$

27. $(U \cap A) \cup B = A \cup B$

28. $(A \cup U) \cap \emptyset = \emptyset$

29. Point O 30. Point T 31. $\triangle ABD$ and its interior.

32. Circles O and O'

33. O - O' or complement of O' relative to O.

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Class Exercises

1. Intuition 2. Most people would say no.

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3. Induction 4. 3 5. 4 6. 5

7. 6 8. 7 9. $n + 1$ 10. Inductive.

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Exercises

1. Intuition 2. Induction 3. 2, 5, 9

4. 54 5. sum is 180°

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6. sum is 540° 7. 37

Page 9

8. 15 9. $\frac{1}{11}$ 10. $\frac{20}{21}$

11. $\frac{512}{2187}$ 12. $\frac{243^2}{512}$

13. Sum of the first n even numbers equals $n(n + 1)$

14. Sum of first n multiples of 5 equals $5/2\ n(n + 1)$

15. $(1)(2) + (2)(3) + (3)(4) + \ldots + n(n + 1) = \frac{(n-1)n\ (n + 1)}{3}$

16. $3 + 5 + 7 + 9 + \ldots + (2n + 1) = n(n + 2)$.

17. 70°, 70°, 40°. 18. 25°, 25°, 130°.

19. 40°, 100°, 40°.

20. Two of the angles have equal measure.

21. Exterior angles equal interior angles equal 360.

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Exercises

1. If equal numbers are added to equal numbers the sums are equal.

2. Multiplication commutative.

3. Multiplication and addition properties of equality.

4. Distributive property.

5. Multiplication property of equality.

6. All are, properly interpreted. For example in 1 we have if two sums are equal and one pair of addends are equal, then the other pair are also equal.

7. If two angles are vertical angles, then their sides form two pairs of opposite rays.

8. If an angle forms a linear pair with one of the interior angles of a triangle, then it is an exterior angle of the triangle.

9. If two angles form a linear pair, then their non-common sides are collinear.

10. The set of all points is space.

11. See 7 through 9 above.

12. Perpendicular lines are two lines that form equal adjacent angles.

13. Not if they remain postulates. If a postulate is proved it becomes a theorem.

14. It could be but it would have to be related to other elements in some way.

15. It would not be restrictive enough. There are plenty of sets of points in a plane that are not collinear.

16. This would be circular defining.

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Class Exercises

1. 9 2. 3 3. 17 4. 5

5. Not necessarily 6. -10 and 4

7. Either $x < 3$ or $x = 3$.

8. Yes. 9. Yes.

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Class Exercises continued

10. $x \geq -6$ 11. $x \leq 6$ 12. $-3/2 < x < \frac{1}{2}$

13. $(-17/10) < x < (27/10)$

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Class Exercises

1. Points of the line are a subset of points of the plane.

2. Yes. 3. Yes. 4. No. 5. Yes 6. Yes.

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Exercises

1. *d* 2. *c* 3. *a* 4. *e* 5. *b*

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6. both 7. neither 8. coplanar 9. coplanar

10. $\overline{AF}$ 11. A 12. $\overline{AD}$ 13. ∅ 14. $\overrightarrow{BE}$ 15. ∅

16. one 17. no, no, yes 18. True

19. False 20. False 21. False 22. False

23. True

24. A line in a plane separates the plane into two half planes.

25. Triangle. Three, the interior, the triangle, and the exterior.

26. Four half lines, four rays, four angles, four half planes: any eight of these.

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Class Exercises

1. False 2. False 3. True

4. False, $\overrightarrow{BA}$ is a side, not $\overrightarrow{AB}$

5. ∠ EOD, ∠ DOC, ∠ 1, ∠ 2, ∠ AOC, ∠ AOE, ∠ BOE.

6. Two names for the same angle.

7. ∠ DOB or ∠ BOA, or ∠ BOE.

8. B

9. $\overleftrightarrow{AD}$ ∪ half plane or the B side of $\overleftrightarrow{AD}$.

10. No, but $\overleftrightarrow{OB}$ does.

11. 38. 12. 112. 13. 30.

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Exercises

1. acute 2. right 3. obtuse 4. acute

5. obtuse 6. acute 7. ∠ POT, ∠ ROT

8. ∠ POR 9. ∠ ROP

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Exercises continued

10. If m∠ ROS = ½ m∠ POR, since they are complementary, m∠ ROS = 30, m∠ POR = 60. Since m∠ TOS = ½ m∠ ROS, m∠ TOS = 15
Since m∠ TOP = m∠ POS + m∠ SOT, m∠ TOP = 90 + 15 = 105

11. 2/3 x 90 = 60 12. $1\frac{2}{3}$ x 90 = 150 13. 2 x 90 = 180

14. ½ x 90 = 45 15. 90 16. 180

Page 26

17. 58 18. 131 19. 28 20. 104

27. 35 28. 18 29. 75 30. 125

31. 8 32. 172 33. 90 34. 105

35. 58 36. 38, 58, 84 37. 32, 58, 90

38. 56, 62, 62

39. right 40. acute 41. obtuse

42. each is acute and less than 45 degrees.

43. each is equal to 45 degrees.

44. each is less than 45 degrees.

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45. $x + 2x = 90$; $x = 30$; $2x = 60$

46. $x + x - 36 = 90$; $2x = 126$; $x = 63$; $x - 36 = 27$

47. $5x + 6 + x = 180$; $6x = 174$; $x = 29$; $5x + 6 = 151$

48. $180 - x = 5(90-x) + 10$; $180 - x = 450 - 5x + 10$; $4x = 280$; $x = 70$

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Class Exercises

1. No 2. Yes 3. Yes 4. Yes 5. No

6. △ADO, △DOC, △COB, △BOA, △ADC, △CBA, △DAB, △DCB

7. △ADB is isosceles, △AOD is scalene, △AOB is scalene, △ADB is acute, △ACD is right, △AOB is right.

8. $\overline{AQ}$, $\overline{PC}$, $\overline{RB}$

9. KNCM, KBNM, KBCM, AMNB, AKNC

Page 32

1. Pentagon 2. Triangle 3. Hexagon 4. Pentagon

5. sides 6. 3-sided 7. n-gon 8. vertex, midpoint

9. two 10. hypotenuse 11. equiangular

12. D 13. triangle

14. △AED, △ADC, △ABC, are triangles formed by diagonals from A.

15. △AED is isosceles. △ADC is equilateral. △ABC is scalene.

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Exercises continued

16. △AED is obtuse, △ADC is acute, △ABC is right

17. ∠ A, ∠ D, ∠ C

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18. Pentagon 19. Quadrilateral 20. Decagon

21. Dodecagon 22. 9-gon 23. 7-gon

24. trilateral or 3-gon

25. 6-gon

26. $\overline{AC}$, $\overline{DB}$ 27, $\overline{TQ}$, $\overline{TR}$, $\overline{PS}$, $\overline{PR}$, $\overline{QS}$

28. $\overline{PN}$, $\overline{PM}$, $\overline{PL}$, $\overline{OK}$, $\overline{OL}$, $\overline{OM}$, $\overline{NK}$, $\overline{NL}$, $\overline{MK}$

29. 61, 64, 55 30. 30, 60, 90

31. 30, 60, 90 32. 180

33. m∠ M = 118, m∠ L = 55, m∠ K = 104, m∠ J = 83

34. m∠ P = 135, m∠ O = 45, m∠ N = 135, m∠ Q = 45

35. m∠ U = 50, m∠ T = 72, m∠ S = 108, m∠ R = 130

36. 360

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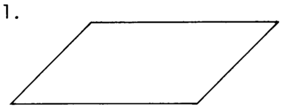
Exercises

1.


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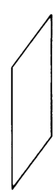

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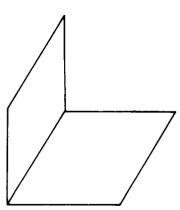

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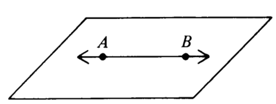

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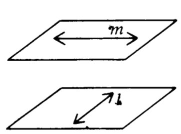

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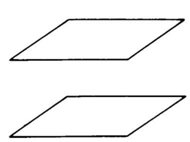

7. False 8. False 9. False 10. True 11. False

12. space 13. a closed half plane 14. plane

15. above 16. half spaces 17. all

18. parallel lines

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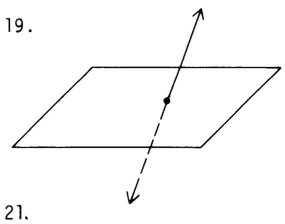
Exercises continued

19.


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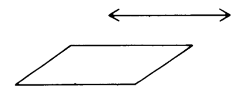

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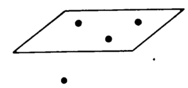

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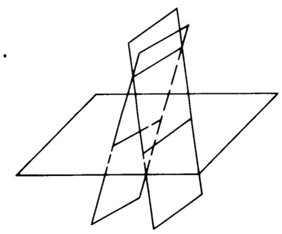

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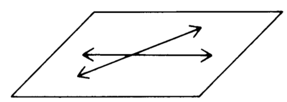

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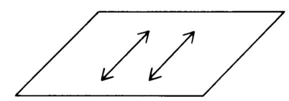

25. False 26. False 27. True 28. False

29. ∠ A-EF-C, ∠ A-EF-G

30. ∠ CFB, ∠ BFG

31. They have a common edge and a common face with disjoint interiors.

32. ∠ A-EF-C, ∠ A-EF-G

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Review Exercises

1. ∅ 2. A 3. B 4. U 5. U 6. U

7. A 8. A

9. { }, {1}, {4}, {6}, {8}, {1, 4}, {1, 6}, {1, 8}, {4, 6}, {4, 8}, {6, 8}, {1, 4, 6}, {1, 4, 8}, {1, 6, 8}, {4, 6, 8} {1, 4, 6, 8}

10. The null set, a single point, the set of all points of the line.

11. Intuition

12. Beliefs held without a conscious logical reason.

13. Conclusions based on many specific cases.

14. $\frac{1}{81}$ 15. 37 16. 38 17. 39

18. $(n/2)(1 + n)$

19. Truthful is dependable is trustworthy is truthful.

20. This with "Space is the set of all points" illustrates circular defining.

21. Theorem

22. When it is impossible for "if" to be true and "then" false.

23. $\overleftrightarrow{AB}$ 24. {A, R, C, D}, {B, C, D}, {A,R,B,C}, {A, B, D}

25. A, R, C

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Review Exercises continued

26. A 27. $\overrightarrow{AB}$ 28. ∅ 29. R 30. $\overline{AC}$

31. Plane P 32. 31 33. 132 34. 20

35. obtuse 36. right 37. acute 38. acute

39. 60 40. 45

45. 64 46. 81 47. 30 48. 45

49. 60 50. 45 51. 90 52. 135

53. heptagon 54. nonagon 55. 5-gon 56. 6-gon

57. $\overline{AC}$, $\overline{DB}$ 58. None

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59. $\overline{GE}$, $\overline{GF}$ legs, $\overline{EF}$ hypotenuse

60. $\overline{PG}$, $\overline{QE}$, $\overline{RF}$ 61. False

62. True (if they intersect)

63. False 64. False 65. A - BC - E, E - BC - D

Chapter Test

1. ∠ EDH 2. △ABF

3. The statement is true, both "if" and "then" are false.

4. The measure of $\overline{AB}$

5. Segment with A and B endpoints

6. Line AB

7. Ray with A endpoint and through B.

8. False 9. True 10. False 11. True 12. True

13. $2x + 3x = 90$; $5x = 90$; $x = 18$

14. $5x = 180$; $x = 36$ 15. 90

16. acute 17. obtuse 18. False

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Exercises

1. True, "if" is false.
2. False, "if" is true and "then" is false.
3. True, "if" is false.
4. False, for $x = 4$ "if" is true and "then is false.
5. True, "if" is false.
6. True, "if" is false (some roses are yellow)
7. False, true "if" and false "then."
8. True, when "if" is true, so is "then". When "if" is false, so is "then."
9. False, "if" is true and "then" is false for $x = -4$, $y = 4$.
10. True, "if" is false.
11. False, "if" is true and "then" false for the number 2.
12. False, "if" is true and "then" is false for the number 9.
13. False, "if" is true and "then" is false for the number 13.
14. False, "if" is true and "then" is false for the number 15.
15. True, "if" and "then" are both true or both false.
16. True, "if" and "then" are both true or both false.
17. False, "if" is true and "then" false for the number 21.
18. False, "if" is true and "then" false for the number 34.
19. True, a postulate.
20. False, true only if B is between A and C.
21. True, a postulate.

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22. True, "if" and "then" both false.
23. True, both "if" and "then" are true.
24. With a false "if" it is impossible to have true "if" and false "then".
25. With a true "then" it is impossible to have a true "if" and a false "then."

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Exercises

1. Definition of a right angle.
2. Definition of complementary angles.
3. Substitution.

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4. Addition postulate (add -15).
5. Definition of supplementary angles and of linear pair.
6. Substitution.
7. Multiplication postulate (multiply by ½).
8. Definition of a right angle.
9. corresponds
10. noncoplanar.
11. 0 and 180.
12. 3
13. line (if they intersect).
14. plane
15. supplementary
16. 45
17. 70
18. Reflexive
19. Multiplication property of equality, symmetric property, transitivity.
20. Angle sum postulate.
21. Angle difference postulate
22. Angle uniqueness postulate
23. Supplementary angle postulate.
24. Points-in-a-plane postulate.
25. True
26. True
27. False
28. False

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29. ∠ AOC and ∠ COB are supplementary (Supplementary angle postulate).
30. m∠ COB = m∠ DOA (definition of supplementary angles and addition property of equality).
31. m∠ 3 (substitution and addition property of equality).

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Exercises

1. The line postulate.
2. The points-in-a-plane postulate.
3. Definition of midpoint.
4. The distance postulate.
5. The point uniqueness postulate.
6. The line postulate and the plane postulate.
7. The space postulate.
8. The plane intersection postulate.
9. The addition property of equality.
10. The substitution postulate.
11. unique
12. plane
13. plane

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Exercises continued

14. line, line
15. plane, angle
16. If C is between A and B and AC = CB then any point D between A and B is such that AD ≠ DB.
17. If two angles are right angles then they have equal measure.
18. If two supplementary angles have equal measure then they are right angles.
19. If two angles have equal measure then their complements have equal measure.
20. If two lines are perpendicular then they form right angles.
21. If two intersecting lines form right angles then they are perpendicular.
22. If two angles have the same complement then they have equal measure.
23. If two lines are perpendicular then they form four right angles.
24. If two rays are noncommon sides of a linear pair then they are opposite rays.
25. If plane m contains two intersecting lines then plane n does not contain them.

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Class Exercises

1. 2. distributive property
   3. distributive property
   4. multiplication commutative
   5. chain rule

2. symmetric and transitive properties.

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Exercises

1. If two addends are odd then the sum is even.
2. If two factors are odd then the product is odd.
3. If the difference of two numbers is multiplied by the sum of the square of the first number, their product, and the square of the second number, then the product is the difference of their cubes.
4. If the sum of two numbers is multiplied by the sum of the square of the first number, the additive inverse of their product, and the square of the second number, then the product is the sum of their cubes.

5. If n is an integer then $n^2 + n$ is an even integer.
6. If an odd integer greater than one is squared, then the square is one more than a multiple of eight.
7. If $5x-13 = x-1$, then $4x = 12$ (adding $-x + 13$)
   If $4x = 12$, then $x = 3$ (multiply by $\frac{1}{4}$)
   If $5x-13 = x-1$, then $x = 3$ (chain rule)

8. $2m + 1$, $2n + 1$ are two odd integers (given)
   $(2m + 1) + (2n + 1) = 2m + 2n + 2$ (associative and commutative properties)
   $2m + 2n + 2 = 2(m + n + 1)$ (distributive property)
   $2(m + n + 1) = 2k$ (integers are closed to addition).
   $2k$ is an even integer (definition of even integer)
   $(2m + 1) + (2n + 1) = 2k$ (transitive property)

9. $2m + 1$, $2n + 1$ are two odd integers (given)
   $(2m + 1)(2n + 1) = (2m + 1)2n + (2m + 1)1$ (distributive property)
   $(2m + 1)2n + (2m + 1)1 = 2n(2m + 1) + (2m + 1)$ (multiplication is commutative and multiplicative identity)
   $2n(2m + 1) + (2m + 1) = 2n \cdot 2m + 2n + 2m + 1$ (distributive property and associative property for addition)

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9. *continued*

   $2n \cdot 2m + 2n + 2m + 1 = 2(n \cdot 2m + n + m) + 1$ (distributive property)
   $2(n \cdot 2m + n + m) + 1$ is an odd integer (definition of odd integer)

10. $(a - b)(a^2 + ab + b^2) = (a - b)a^2 + (a - b)ab + (a - b)b^2$ (distributive property)
    $(a - b)a^2 + (a - b)ab + (a - b)b^2 = a^2(a - b) + ab(a - b) + b^2(a - b)$
    $a^2(a - b) + ab(a - b) + b^2(a - b) = a^3 - a^2b + a^2b - ab^2 + ab^2 - b^3$ (distributive property)
    $a^3 - a^2b + a^2b - ab^2 + ab^2 - b^3 = a^3 - b^3$ (additive inverse and identity properties)
    $(a - b)(a^2 + ab + b^2) = a^3 - b^3$ (transitive property)
    $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ (symmetric property)

11. $(a + b)(a^2 - ab + b^2) = (a + b)a^2 - (a + b)ab + (a + b)b^2$ (distributive property)
    $(a + b)a^2 - (a + b)ab + (a + b)b^2 = a^2(a + b) - ab(a + b) + b^2(a + b)$ (multiplication is commutative)
    $a^2(a + b) - ab(a + b) + b^2(a + b) = a^3 + a^2b - a^2b - ab^2 + ab^2 + b^3$ (distributive property)
    $a^3 + a^2b - a^2b - ab^2 + ab^2 + b^3 = a^3 + b^3$ (additive identity and inverse)
    $(a + b)(a^2 - ab + b^2) = a^3 + b^3$ (transitive property)
    $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ (symmetric property)

12. $n^2 + n = n(n + 1)$ (distributive property)
    If n is even then $n(n + 1)$ is even (if there is an even factor the product is even)
    If n is odd then $(n + 1)$ is even and $n(n + 1)$ is even (same as step above)
    If n is even or odd (any n) $n^2 + n$ is even (two steps above)

13. $2n + 1$ is an odd integer greater than one (definition of odd integer)
    $(2n + 1)^2 = (2n + 1)2n + (2n + 1)1$ (distributive property)
    $(2n + 1)2n + (2n + 1)1 = 2n(2n + 1) + (2n + 1)$ (distributive property and multiplicative identity)
    $2n(2n + 1) + 2n + 1 = 2n \cdot 2n + 2n + 2n + 1$ (distributive property)
    $2n \cdot 2n + 2n + 2n + 1 = 4n^2 + 4n + 1$ (multiplication commutative and distributive property)
    $4n^2 + 4n + 1 = 4n(n + 1) + 1$ (distributive property)
    $n(n + 1)$ is even (exercise 12)
    $4n(n + 1)$ is a multiple of 8 (step above)
    $4n(n + 1) + 1$ is one more than a multiple of 8 (step above)
    $(2n + 1)^2$ is one more than a multiple of 8 (substitution)

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Class Exercises

1. PQ
2. PM = MQ
3. 2

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4. $\frac{1}{2}$ of PQ
5. True

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Exercises

1. 1. If l is a line then it contains two points, R and S.
   2. If P is not in l then R, S, P determine a plane.

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Exercises continued

1. *continued*

    3. If R, S lie in the plane then l lies in the plane.
    4. If l lies in the plane and P lies in the plane then there is exactly one plane containing the point and the line.

2. 1. Two points determine a line.
    2. Three noncollinear points determine a plane.
    3. If two points of a line lie in a plane all points lie in the plane.
    4. Steps 2 and 3.

3. 1. If l and k intersect in point P then they intersect in point R, distinct from P.
    2. If l and k intersect in P and R then two points P, R determine two distinct lines, which is false.
    Thus, l and k intersecting in P and R is false. They intersect in exactly one point P.

4. 1. assumption.
    2. Two points determine exactly one line.

5. 1. If l and k intersect in point R then l contains a point S not on k.
    2. If S is a point no on k then S and k determine exactly one plane.
    3. If S and k determine a plane then the plane contains l.
    4. If the plane contains l and k then l and k determine exactly one plane.

6. 1. Two points of a line determine it.
    2. Theorem 2-5.1.
    3. If two points of a line are in a plane then that line is in the plane.
    4. Definition of "determines"

7. 1. If $\overline{PQ}$ is a segment then it has a measure PQ.
    2. If PQ is the measure of a segment then $\frac{1}{2}$PQ is the measure of a segment.
    3. If $\frac{1}{2}$PQ is the measure of a segment there is a unique point M between P and Q such that PM = $\frac{1}{2}$PQ.
    4. If PM = $\frac{1}{2}$PQ then M is the midpoint of $\overline{PQ}$.

8. 1. Distance Postulate.
    2. Distance Postulate.
    3. Point Uniqueness Postulate.
    4. Definition of midpoint.

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Class Exercises

1. ∠ DBC 2. ∠ FBA 3. ∠ EBA
4. ∠ DBC and ∠ CBE 5. 85
6. They have equal measure.
7. m∠ EBC = m∠ DBC

8. 360 9. 90 10. perpendicular

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Exercises

1. Theorem 2-6.2 supplements of angles of equal measure, or of the same angle, have the same measure.

    Given: m∠ A = m∠ B, ∠ C supplement of ∠ A, ∠ D supplement of ∠ B, ∠ E supplement of ∠ A

    Prove: m∠ C = m∠ D = m∠ E

    Proof:

STATEMENTS	REASONS
1. m∠ A + m∠ C = 180 m∠ B + m∠ D = 180 m∠ A + m∠ E = 180	1. Definition of supplementary angles.
2. m∠ A + m∠ C = m∠ A + + m∠ E	2. Symmetric and Transitive properties.
3. m∠ C = m∠ E	3. Addition property of equality.
4. m∠ A = m∠ B	4. Given
5. m∠ A + m∠ C = m∠ B+ + m∠ D	5. Symmetric and Transitive properties.
6. m∠ C = m∠ D	6. Equals subtracted from equals.
7. m∠ C = m∠ D = m∠ E	7. Steps 3 and 6.

2. m∠ 1 + m∠ 1 = 180 substituting m∠ 1 for m∠ 2 in step 5.

3. The "if" and "then" have been interchanged. That is, they are converses.

4. False

5. True

6. True

7. True

8. False

9. relationships

10. Statement, Given, Prove

11. four

12. perpendicular

13. ∠ 4 14. ∠ 6 15. ∠ 2

16. ∠ NOS 17. ∠ TON 18. ∠ TOS

19. 70 20. 80 21. 30

22. 80 23. 150 24. 100

25. ∠ EBA and ∠ CBD

26. ∠ FBC and ∠ ABG

27. ∠ CBE, ∠ ABE, ∠ ABD

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28. 180 - x 29. x 30. 180 -x

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Exercises continued

31. 60
32. ∠ FBC, ∠ FBA, ∠ CBG, ∠ GBA
33. ∠ FBE, ∠ FBD, ∠ GBE, ∠ GBD
34. 60
35. 140
36. equal measure
37. equal measure

Review Exercises

1. True

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2. True 3. True 4. False 5. True 6. False
7. Point Betweenness Postulate
8. The Plane Intersection Postulate
9. The Angle Sum Postulate
10. terms
11. restrictions
12. statements
13. undefined
14. 4 noncoplanar
15. postulates
16. multiplication property of equality
17. transitive property of equality
18. closure for real numbers
19. Postulate 1-6, Definition 1-28
20. Postulate 2-7
21. Postulate 2-2
22. Postulate 2-1
23. If an integer has one's digit five then it has five for a factor.
24. If a plane contains two intersecting lines then no other plane contains them both.
25. If two lines form equal adjacent angles then they are perpendicular.
26. If $3(x - 5) = x + 7$ then $3x - 15 = x + 7$ (distributive property)
    If $3x - 15 = x + 7$ then $2x = 22$ (addition property of equality)
    If $2x = 22$ then $x = 11$ (multiplication property of equality)
27. If $5(x + 1) = 3x - 11$ then $5x + 5 = 3x - 11$ (distributive property)
    If $5x + 5 = 3x - 11$ then $2x = 16$ (addition property of equality)
    If $2x = -16$ then $x = -8$ (multiplication property of equality)
28. point
29. two
30. segment
31. of equal measure
32. right
33. Hypothesis - 2 lines are perpendicular
    Conclusion - lines form right angles
34. Hypothesis - 2 angles form a linear pair
    Conclusion - the sum of their measures is 180.
35. Hypothesis - opp. angles are formed by intersecting lines
    Conclusion - the angles have equal measure
36. Theorem, given, prove

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Review Exercises continued

37. ∠ FBC and ∠ ABG
38. ∠ EBC
39. perpendicular
40. 30
41. 110
42. 110
43. 150
44. 150
45. 70

Chapter Test

1. 34
2. states more than necessary
3. Definitions are derived from undefined terms. Theorems are proved from unproved statements.
4. One factor may be distributed over the terms of another factor. That is, the first factor can be multiplied by each term of the other factor rather than the factor itself.
5. 170 6. False 7. True 8. True
9. If you come over, then you have to work.
10. ∠ FOC, ∠ BOG, ∠ COG, ∠ FOB
11. $\overleftrightarrow{FG}$ and $\overleftrightarrow{BC}$
12. ∠ AOB, ∠ COD
13. ∠ DOB
14. The Plane Postulate
15. Theorem: The product of an odd integer and an even integer is an even integer.

    Given: Odd integer 2m + 1, even integer $2n$

    Prove: $2n\ (2m + 1)$ is even

    Proof:

STATEMENTS	REASONS
1. $2m + 1$ is odd; $2n$ is even	1. Given
2. $2n(2m + 1) = 2n \cdot 2m + 2n$	2. Distributive Property
3. $2n \cdot 2m + 2n = 2 \cdot (n \cdot 2m + n)$	3. Distributive Property
4. $2(n \cdot 2m + n)$ is even	4. Definition of even number
5. $2n(2m + 1)$ is even	5. Substitution

16. Theorem: If a point does not lie in a given line, then there is exactly one plane containing both the point and line.

    Given: P not on l

    Prove: There is one and only one plane M containing P and l.

    Proof:

STATEMENTS	REASONS
1. line l contains points R, S	1. Two points determine a line.
2. P is not in l	2. Given
3. R, S, P are three noncollinear points	3. Steps 1 and 2
4. There is one and only one plane M containing R, S, P	4. Three noncollinear points determine a plane.
5. Any plane containing l will contain R, S	5. A line lies in a plane when all its points lie in the plane.
6. Any plane containing l and P must contain R,S, P	6. Step 5
7. Any plane containing l and P is the plane M.	7. The noncollinear points determine only one plane.

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Answers for Exercises 1-6 may vary

1. ABC ↔ FDE
2. DEP ↔ SRP
3. APN ↔ RPS
4. ABCDE ↔ RKLMN
5. QPSA ↔ LISA
6. NLS ↔ PAI, or NLIR ↔ PASR

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Class Exercises

1. $\overline{AB}$; symmetry
2. segment congruence
3. $\overline{XY}$; definition of segment congruence
4. symmetry
5. symmetry
6. XY; XY; segment congruence
7. transitive
8. segment congruence
9. transitive
10. transitive
11. reflexive
12. segments; equivalence

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Class Exercises

1. ABC ↔ FED
2. triangles not congruent
3. LION ↔ ASRP
4. ABD ↔ CDB
5. ALR ↔ DEF
6. HOT ↔ HNU, or OSU ↔ NST
7. triangles not congruent
8. DEFGH ↔ MNQKL
9. $\frac{R \leftrightarrow W,\ S \leftrightarrow X}{T \leftrightarrow Y,\ U \leftrightarrow Z}$
10. R ↔ W; S ↔ X; T ↔ Y; U ↔ Z.
11. Q ↔ C; X ↔ B; R ↔ S.
12. T ↔ A; F ↔ E; U ↔ N
13. A ↔ V; B ↔ U; C ↔ T; D ↔ S; E ↔ R.
14. True 15. False 16. False 17. True
18. True 19. False 20. True 21. True

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22. ABC ↔ FED; BCA ↔ DEF; and others
23. MNR ↔ WTS; MRN ↔ WST; and others
24. PQRS ↔ HFMN; RQPS ↔ MFHN; and others
25. ABP ↔ ABQ; APB ↔ AQB; and others
26. $\triangle$ABN ↔ $\triangle$ACM; $\triangle$MPB ↔ $\triangle$NPC; $\triangle$MBC ↔ $\triangle$NCB
27. $\triangle$PSR ↔ $\triangle$QRS; $\triangle$PTS ↔ $\triangle$QTR; $\triangle$PQS ↔ $\triangle$QPR
28. *Given* $\angle$ AOB
    Prove $\angle AOB \cong \angle AOB$
    $m\angle$ AOB is a number (Postulate 1-5)
    $m\angle AOB = m\angle AOB$ (Reflexive property of equality)
    $\angle AOB \cong \angle AOB$ (Definition 3-2).

29. $\angle$ C 30. $\angle$ CMA 31. $\angle$ CAM 32. $m\angle$ B 33. $m\angle$ CAM

34. $\overline{AM}$ 35. $\overline{AC}$ 36. $\overline{CM}$ 37. MB 38. AB

39. Individual student drawings.

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Exercises

1. GF = VU, GH = VW, HF = WU
   $\angle F \cong \angle U$, $\angle G \cong \angle V$, $\angle H \cong \angle W$.
2. EF = XY, FG = YZ, GE = ZX
   $\angle E \cong \angle X$, $\angle F \cong \angle Y$, $\angle G \cong \angle Z$.
3. RS = MN, ST = NO, TR = OM
   $\angle R \cong \angle M$, $\angle S \cong \angle N$, $\angle T \cong \angle O$.
4. AB = CB, BX = BY, XA = YC
   $\angle 1 \cong \angle 2$, $\angle A \cong \angle C$, $\angle X \cong \angle Y$.
5. PO = QO, OS = OR, SP = RQ
   $\angle P \cong \angle Q$, $\angle S \cong \angle R$, $\angle POS \cong \angle QOR$.
6. AE = CE, ED = EF, DA = FC
   $\angle A \cong \angle C$, $\angle D \cong \angle F$, $\angle AED \cong \angle CEF$.
7. PR = QS, RX = SY, XP = YQ
   $\angle P \cong \angle Q$, $\angle R \cong \angle S$, $\angle X \cong \angle Y$.
8. AM = BN, MP = NO, PA = OB
   $\angle A \cong \angle B$, $\angle M \cong \angle N$, $\angle P \cong \angle Q$.
9. MQ = NP, QP = PQ, PM = QN
   $\angle M \cong \angle N$, $\angle MQP \cong \angle NPQ$, $\angle MPQ \cong \angle NQP$.

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10. $\angle$ S
11. $\overline{NP}$
12. *Given* $\triangle$ABC
    $\overline{AB} \cong \overline{AB}$, $\overline{BC} \cong \overline{BC}$, $\overline{CA} \cong \overline{CA}$ (Theorem 3-1.8)
    $\angle A \cong \angle A$, $\angle B \cong \angle B$, $\angle C \cong \angle C$ (Theorem 3-1.8)
    $\triangle ABC \cong \triangle ABC$ (Definition 3-3)
13. If $\triangle ABC \cong \triangle KLM$, $\overline{AB} \cong \overline{KL}$, $\overline{BC} \cong \overline{LM}$, $\overline{CA} \cong \overline{MK}$, $\angle A \cong \angle K$, $\angle B \cong \angle L$, $\angle C \cong \angle M$ (Definition 3-3)
    $\overline{KL} \cong \overline{AB}$, $\overline{LM} \cong \overline{BC}$, $\overline{MK} \cong \overline{CA}$, $\angle K \cong \angle A$, $\angle L \cong \angle B$, $\angle M \cong \angle C$ (Theorem 3-1.8)
    $\triangle KLM \cong \triangle ABC$ (Definition 3-3).
14. If $\triangle ABC \cong \triangle KLM$ and $\triangle KLM \cong \triangle PQR$, $\overline{AB} \cong \overline{KL} \cong \overline{PQ}$, $\overline{BC} \cong \overline{LM} \cong \overline{QR}$, $\overline{CA} \cong \overline{MK} \cong \overline{RP}$, $\angle A \cong \angle K \cong \angle P$, $\angle B \cong \angle L \cong \angle Q$, $\angle C \cong \angle M \cong \angle R$ (Definition 3-3) $\overline{AB} \cong \overline{PQ}$ $\overline{BC} \cong \overline{QR}$, $\overline{CA} \cong \overline{RP}$, $\angle A \cong \angle P$, $\angle B \cong \angle Q$, $\angle C \cong \angle R$ (Theorem 3-1.8) $\triangle ABC \cong \triangle PQR$ (Definition 3-3).
15. $\triangle RST \cong \triangle NML$
16. $\triangle AKF \cong \triangle GHJ$
17. $\angle G \cong \angle M$, $\angle H \cong \angle N$, $\angle I \cong \angle L$, $\overline{GH} \cong \overline{MN}$, $\overline{HI} \cong \overline{NL}$, $\overline{GI} \cong \overline{ML}$
18. $\angle R \cong \angle U$, $\angle S \cong \angle V$, $\angle T \cong \angle W$, $\overline{RS} \cong \overline{UV}$, $\overline{ST} \cong \overline{VW}$, $\overline{RT} \cong \overline{UW}$.
19. $\angle R \cong \angle K$, $\angle C \cong \angle L$, $\angle A \cong \angle H$, $\overline{RC} \cong \overline{KL}$, $\overline{CA} \cong \overline{LH}$ $\overline{RA} \cong \overline{KH}$
20. $\triangle DFE \cong \triangle CAB$
21. $\triangle DFE \cong \triangle ABC$
22. Since $\overline{AB} \perp \overline{CD}$, $m\angle ABC = m\angle ABD$ (Theorem 2-6.5)
    Since $\triangle AMB \cong \triangle ANB$, $m\angle ABM = m\angle ABN$ (Definition 3-3)
    Therefore, $m\angle MBC = m\angle NBD$ (Theorem 2-6.1)
23. Since $\triangle ABD \cong \triangle ACD$, $\angle ABD \cong \angle ACD$ (Definition 3-3)
    Therefore, $\angle ABE \cong \angle ACF$ (Theorem 3-1.4)
24. Since $\triangle KLP \cong \triangle MLP$, $\angle KPL \cong \angle MPL$ (Definition 3-3)
    $\angle KPL \cong \angle SPT$ and $\angle MPL \cong \angle RPT$ (Theorem 3-1.5)
    Therefore $\angle RPT \cong \angle SPT$ (Substitution)
25. $\triangle$ABC is isosceles, since it has two congruent sides.

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Class Exercises

1. SAS
2. SSS
3. SAS
4. insufficient information
5. ASA
6. ASA
7. not congruent
8. ASA
9. insufficient information

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Exercises

1. SSS
2. ASA
3. not congruent
4. SAS
5. SSS
6. SAS
7. ASA
8. SSS
9. SAS
10. SAS
11. SAS
12. SSS
13. not congruent
14. not congruent
15. ASA
16. not congruent

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17. $\angle A \cong \angle D$, and $\overline{AE} \cong \overline{DE}$ (given)
$\angle AEB \cong \angle DEC$ (Theorem 3-1.5)
$\triangle AEB \cong \triangle DEC$ (ASA)

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18. $\overline{AD} \cong \overline{BC}$, and $\overline{AB} \cong \overline{DC}$ (given)
$\overline{AC} \cong \overline{AC}$ (Theorem 3-1.6)
$\triangle CDA \cong \triangle ABC$ (SSS)

19. $\angle Q \cong \angle S$ (Theorem 2-6.5 and Theorem 3-1.1)
QR = SR (Definition 1-15)
$\angle PRQ \cong \angle TRS$ (Theorem 3-1.5)
$\triangle PQR \cong \triangle TSR$ (ASA)

20. $\angle PMQ \cong \angle PMR$ (Theorem 2-6.5 and Theorem 3-1.1)
$\overline{QM} \cong \overline{RM}$ (given)
$\overline{PM} \cong \overline{PM}$ (Theorem 3-1.6)
$\triangle PQM \cong \triangle PRM$ (SAS)

21. $\angle 1 \cong \angle 2$, and $\angle 3 \cong \angle 4$ (given)
$\overline{EG} \cong \overline{EG}$ (Theorem 3-1.6)
$\triangle EFG \cong \triangle GHE$ (ASA)
$\angle H \cong \angle F$ (Definition 3-3)

22. $\angle AMP \cong \angle BMC$ (Theorem 3-1.1)
$\triangle APM \cong \triangle CBM$ (ASA)
$\overline{AP} \cong \overline{CB}$ (Definition 3-3)

23. $\overline{OA} \cong \overline{OB}$, $\overline{OB} \cong \overline{OC}$ (Definition 1-16)
$\triangle OAB \cong \triangle OBC$ (SAS)
$\overline{AB} \cong \overline{BC}$ (Definition 3-3)
AB = BC (Definition 1-16)

24. $\overline{KL} \cong \overline{KN}$, $\overline{LM} \cong \overline{NM}$ (Definition 1-16)
$\overline{KM} \cong \overline{KM}$ (Theorem 3-1.8)
$\triangle KLM \cong \triangle KNM$ (SSS)
$\angle 1 \cong \angle 2$ (Definition 3-3)

25. $\angle 3 \cong \angle 4$, $\overline{PR} \cong \overline{QR}$ (Definition 1-16)
$\overline{RS} \cong \overline{RS}$ (Theorem 3-1.8)
$\triangle PRS \cong \triangle QRS$ (SAS)
$\angle 1 \cong \angle 2$ (Definition 3-3)
$\angle PST \cong \angle QST$ (Theorem 3-1.4)

26. $\angle ABP \cong \angle DCP$ (Theorem 3-1.4)
$\triangle APB \cong \triangle DPC$ (ASA)
$\angle APB \cong \angle DPC$ (Definition 3-3)

27. $\angle DEC \cong \angle BEC$ (Theorem 2-6.5, Theorem 2-5.5)
$\overline{DE} \cong \overline{BE}$ (Definition 1-15, Definition 1-16)
$\overline{EC} \cong \overline{EC}$ (Theorem 3-1.8)
$\triangle DCE \cong \triangle BCD$ (SAS)
DC = BC (Definition 3-3, Definition 1-16).

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28. $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$ (given)
$\angle MST \cong \angle NST$ (Theorem 3-1.4)
$\overline{ST} \cong \overline{ST}$ (Theorem 3-1.6)
$\triangle MST \cong \triangle NST$ (ASA)
MS = NS (Definition 3-3)

29. DF = AF and EF = BF (Definition 1-15)
$\angle DFE \cong \angle AFB$ (Theorem 3-1.5)
$\triangle DFE \cong \triangle AFB$ (SAS)
$\overline{DE} \cong \overline{AB}$ (Definition 3-3)
$\overline{DC} \cong \overline{DE}$ (Transitive property)

30. $\overline{AB} \cong \overline{AC}$, $\overline{AE} \cong \overline{AD}$ (Definition 1-16)
$\angle BAE \cong \angle CAD$ (Theorem 3-1.5)
$\triangle ABE \cong \triangle ACD$ (SAS)
BE = CD (Definition 3-3, Definition 1-16).

31. $\angle EAB \cong \angle DCB$ (Theorem 2-6.5, Theorem 3-1.1)
$\overline{AB} \cong \overline{CB}$ (Definition 1-15, Definition 1-16)
$\triangle EAB \cong \triangle DCB$ (ASA)
AE = CD (Definition 3-3, Definition 1-16)

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32. $\overline{XY} \cong \overline{XZ}$ (Postulate 2-4, Definition 1-16)
$\overline{XR} \cong \overline{XR}$ (Theorem 3-1.8)
$\overline{YR} \cong \overline{ZR}$ (Definition 1-15)
$\triangle XYR \cong \triangle XZR$ (SSS)
$\angle Y \cong \angle Z$ (Definition 3-3).

33. AC - BC = DB - BC (Addition property of equality)
$\overline{AB} \cong \overline{DC}$ (Definition 1-16)
$\overline{AP} \cong \overline{DQ}$, $\overline{BP} \cong \overline{CQ}$ (Definition 1-16)
$\triangle APB \cong \triangle DQC$ (SSS)
$\angle P \cong \angle Q$ (Definition 3-3)

34. $\overline{AF} \cong \overline{CF}$ (Definition 1-15, Definition 1-16)
$\overline{AD} \cong \overline{CE}$ (Definition 1-15, Addition property of equality)
$\triangle ADF \cong \triangle CEF$ (SAS)
DF = EF (Definition 3-3, Definition 1-16).

35. From the given information, $\triangle APC \cong \triangle DPB$ (ASA)
Therefore $\overline{PB} \cong \overline{PC}$ (Definition 3-3)

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36. From the given information, $\triangle APC \cong \triangle DPB$ (ASA),
therefore $\angle APC \cong m\angle DPB$ (Definition 3-3)
$\angle BPC \cong \angle BPC$ (Theorem 3-1.7).
Thus, $m\angle APB = m\angle DPC$ (Subtraction property).

37. $\angle F \cong \angle ABC$ (Theorem 2-6.5, and Theorem 3-1.1)
$\angle 1 \cong \angle 2$, and FB = EC (given)
BE = BE; therefore FE = BC (Addition property)
$\triangle DFE \cong \triangle ABC$ (ASA)
$\angle D \cong \angle A$ (Definition 3-3)

38. ZR = ZS, and ZX = ZY (given)
$\angle Z \cong \angle Z$ (Theorem 3-1.7)
$\triangle RZY \cong \triangle SZX$ (SAS)
$\angle ZRY \cong \angle ZSX$ (Definition 3-3)
$\angle XRY \cong \angle YSX$ (Theorem 3-1.4)

39. From the given information and FE = FE, DF = EB (Subtraction property)
$\triangle ADF \cong \triangle CBE$ (SAS). Therefore $\angle AFD \cong \angle CEB$ (Definition 3-3).
Since $\angle AFB \cong \angle AFD$ (given), $\overline{AF} \perp \overline{DB}$ (Definition 1-25) and $\angle AFD$ is a right angle (Theorem 2-6.5).
Thus $\angle CEB$ is a right angle (Substitution postulate, 2-1)

40. Since $\angle ABF \cong \angle DCG$ (given), $\angle ABC \cong \angle ACB$ (Theorem 3-1.4).
Also $\angle EBC \cong \angle ECB$ (given), and $\overline{BC} \cong \overline{BC}$ (Theorem 3-1.6).
Therefore $\triangle ABC \cong \triangle DCB$ (ASA), and $\angle A \cong \angle D$ (Definition 3-3).

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Class Exercises

1. vertex; opposite
2. perpendicular
3. median
4. obtuse

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Exercises

1. exterior
2. median
3. $\overline{KR}$
4. altitude
5. obtuse
6. isosceles
7. $\angle MKL$, $\angle KML$
8. one, one
9. True
10. False

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11. True
12. False (segment)
13. True
14. False (all triangles)
15. True
16. False

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17. *Given* △ABC is equiangular
Since ∠ B ≅ ∠ C, $\overline{AB} \cong \overline{AC}$ (Theorem 3-4.3)
Similarly, since ∠ A ≅ ∠ C, $\overline{AB} \cong \overline{BC}$ (Theorem 3-4.3)
Therefore △ABC is equilateral (Transitive property)

18. ∠ CBA ≅ ∠ BCA (Theorem 3-4.2)
∠ 1 ≅ ∠ 2 (Theorem 3-1.4)

19. △PQM ≅ △PRM (SSS)
∠ QPM ≅ ∠ PPM (Definition 3-3)
$\overline{PM}$ is an angle bisector (Definition 1-29, Definition 3-8)

20. ∠ PMQ ≅ ∠ PMR (Theorem 2-6.5 and Theorem 3-1.1)
∠ QPM ≅ ∠ RPM (Definition 1-29)
$\overline{PM} \cong \overline{PM}$ (Theorem 3-1.6)
△QPM ≅ △RPM (ASA)
$\overline{PQ} \cong \overline{PR}$ (Definition 3-3)
△ABC is isosceles (Definition 3-12)

21. ∠ A ≅ ∠ B (Definition 3-12, Theorem 3-4.2)
△AKL ≅ △BLM (SAS)
LM = KL (Definition 3-3, Definition 1-16)

22. From the given information, △KAL ≅ △LBM (SSS).
Therefore ∠ A ≅ ∠ B (Definition 3-3), $\overline{AC} \cong \overline{BC}$ (Theorem 3-4.3) and
△ABC is isosceles (Definition 3-12)

23. By subtraction, AD = AE. Thus, m∠ ADE = m∠ AED (Theorem 3-4.2)

24. Since AB = AD (given), ∠ ABD ≅ ∠ ADB (Theorem 3-4.2)
By subtraction, m∠ CBD = m∠ CDB, and BC = DC (Theorem 3-4.3)
Thus △BCD is isosceles (Definition 3-12).

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25. Since AB = AC, ∠ B ≅ ∠ C (Theorem 3-4.2)
∠ PRB ≅ ∠ PSC (Theorem 2-6.5 and Theorem 3-1.1)
△BRP ≅ △CSP (ASA); Therefore $\overline{PR} \cong \overline{PS}$ (Definition 3-3).

26. From the given information, ∠ BRP ≅ ∠ CSP (Theorem 2-6.5 and Theorem 3-1.1)
Therefore △PRB ≅ △PSC (SAS) and ∠ B ≅ ∠ C (Definition 3-3)
Thus $\overline{AB} \cong \overline{AC}$ (Theorem 3-4.3)and △ABC is isosceles.

27. PB = PC (Definition 1-15)
PR = PS (given)
m∠ BPR = m∠ CPS (given)
△PRB ≅ △PSC (SAS) and ∠ B ≅ ∠ C (Definition 3-3)
Thus $\overline{AB} \cong \overline{AC}$ (Theorem 3-4.3) and △ABC is isosceles.

28. Since $\overline{DE} \cong \overline{FE}$ (given), ∠ D ≅ ∠ F (Theorem 3-4.2)
△DEG ≅ △FEH (ASA), and ∠ DGE ≅ ∠ FHE (Definition 3-3).
Therefore ∠ HGE ≅ ∠ GHE (Theorem 3-1.4)
Thus GE = HE (Theorem 3-4.3) and △DEF is isosceles.

29. Since GE = HE, ∠ HGE ≅ ∠ GHE (Theorem 3-4.2).
Therefore ∠ DGE ≅ ∠ FHE (Theorem 3-1.4)
△DGE ≅ △FHE (ASA).
DE = FE (Definition 3-3) and △DEF is isosceles.

30. $\overline{AK} \cong \overline{KC}$, $\overline{AM} \cong \overline{CL}$ (Definition 1-15, Addition property of equality)
∠ A ≅ ∠ C (Corollary 3-4.2a)
△AKM ≅ △CKL (SAS)
$\overline{KM} \cong \overline{KL}$ (Definition 3-3)
Similarly, $\overline{KL} \cong \overline{LM}$
△MKL is equilateral (Theorem 3-1.8, Definition 3-12).

31. $\overline{MQ} \cong \overline{NR}$ (Definition 1-15)
$\overline{QA} \cong \overline{RB}$ (Given, Definition 1-16)
∠ Q ≅ ∠ R (Theorem 3-4.2)
△MQA ≅ △NRB (SAS)
MA = NB (Definition 3-3, Definition 1-16).

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32. ∠ PBC ≅ ∠ PCB (Theorem 3-4.2)
∠ PBA ≅ ∠ PCD (Theorem 3-1.4)
△PBA ≅ △PCD (SAS)
AP = DP (Definition 3-3, Definition 1-16)

33. Since AP = DP (given), ∠ A ≅ ∠ D (Theorem 3-4.2).
Therefore △APB ≅ △DPC (SAS) and PB = PC (Definition 3-3).

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34. ∠ DPG ≅ ∠ FPG (Definition 3-8)
$\overline{PG} \cong \overline{PG}$ (Theorem 3-1.6)
△DPG ≅ △FPG (SAS)
∠ PGD ≅ ∠ PGF (Definition 3-3)
$\overline{PG} \perp \overline{DF}$ (Theorem 3-1.2, Theorem 2-6.6)
$\overline{PG}$ is an altitude of △DPF (Definition 3-10)
$\overline{DG} \cong \overline{FG}$ (Definition 3-3)
$\overline{PG}$ is a median of △DPF (Definition 3-9)

35. ∠ BAD ≅ ∠ CAD (Definition 1-29)
$\overline{AD} \cong \overline{AD}$ (Theorem 3-1.6)
△ABD ≅ △ACD (SAS), and BD = DC (Definition 3-3)
Therefore ∠ DBC ≅ ∠ DCB (Theorem 3-4.3)

36. Since BD = DC (given), m∠ DBC = m∠ DCB (Theorem 3-4.2)
By the addition property m∠ ABC = m∠ ACB.
Therefore, AB = AC (Theorem 3-4.3)
△ABD ≅ △ACD (SAS), and ∠ BAD ≅ ∠ CAD (Definition 3-3)
Thus $\overline{AD}$ bisects ∠ BAC.

37. ∠ Q ≅ ∠ R (Definition 3-3).

38. $\overline{XY} \cong \overline{XZ}$ (Postulate 2-4, Definition 1-16)
$\overline{XR} \cong \overline{XR}$ (Theorem 3-1.8)
$\overline{YR} \cong \overline{ZR}$ (Definition 1-15)
△XYR ≅ △XZR (SSS)
∠ Y ≅ ∠ Z (Definition 3-3)

39. From the given information, △ABC ≅ △AED (SAS)
Therefore, m∠ BCA = ∠ EDA.
Since AC = AD, m∠ ACD = m∠ ADC (Theorem 3-4.2).
By the addition property, m∠ BCD = m∠ EDC.

40. Since △ABC is equilateral, m∠ A = m∠ B = m∠ C.
Since AB = BC = AC, and BF = DC = AE (given),
AB - BF = BC - DC = AC - AE, or AF = CE = BD
(Subtraction property)
Therefore △AFE ≅ △BDF ≅ △CED (SAS), and
FE = DF = ED (Definition 3-3). Thus △DEF is equilateral.

41. ∠ ACD ≅ ∠ ADC (Theorem 3-1.3)
AC = AD (Theorem 3-3.4)
△ABC ≅ △AED (SAS)
∠ BAC ≅ ∠ EAD (Definition 3-3).

42. Since m∠ BAD = m∠ EAC (given), and ∠ CAD ≅ ∠ CAD,
m∠ BAC = m∠ EAD
By substitution m∠ 1 = m∠ 2
Also ∠ ACD ≅ ∠ ADC (Theorem 3-1.3)
Therefore AC = AD (Theorem 3-4.3)
Thus △ABC ≅ △AED (ASA), and ∠ B ≅ ∠ E.

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43. From the given information and $\overline{TR} \cong \overline{TR}$,
△RQT ≅ △RST (SSS), and ∠ QRT ≅ ∠ SRT (Definition 3-3)
Thus △QRP ≅ △SRP (SAS), and ∠ QPR ≅ ∠ SPR (Definition 3-3).

44. △DBC ≅ △ABC (SAS)
AC = DC (Definition 3-3)
∠ ADC ≅ ∠ DAC (Theorem 3-4.2)

45. PQ = PR (Addition property)
∠ PQR ≅ ∠ PRQ (Theorem 3-4.2)
△MQR ≅ △NRQ (SAS)
∠ MRQ ≅ ∠ NQR (Definition 3-3)
KQ = KR (Theorem 3-4.2)

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46. ∠ D ≅ ∠ AFD (Theorem 3-4.2)
∠ C ≅ ∠ BFC (Theorem 3-4.2)
Since ∠ D ≅ ∠ C (given), ∠ AFD ≅ ∠ BFC (Transitive property)
AF = FB (Transitive property)
△AFE ≅ △BFE (SSS)
∠ AFE ≅ ∠ BFE (Definition 3-3)
∠ DFE ≅ ∠ CFE (Addition property)
$\overline{EF} \perp \overline{DC}$ (Definition 1-25)

47. AB = CB (Addition property)
∠ A ≅ ∠ C (Theorem 3-4.2)
△AMN ≅ △CMP (SAS)
∠ ANM ≅ ∠ CPM (Definition 3-3)
∠ BNP ≅ ∠ BPN (Theorem 3-4.2)
∠ 1 ≅ ∠ 2 (Subtraction property)

48. AC = CB = BA (Corollary 3-4.3a)
CD = CE, BE = BF (Theorem 3-4.3)
CA - CD = CB - CE (Subtraction property)
AD = BE (Closure property)
AD = BF (Transitive property).

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Class Exercises

1. △SPR, △RQS
2. △ACD, △DBA; or △ACB, △DBC
3. △SRP, △NMP; or △TRP, △WMP; or QRP, △PNQ; and other pairs.
4. $\overline{IJ}$ and ∠ ILJ
5. ∠ HLJ

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6. Since ∠ ILJ ≅ ∠ ILJ, and ∠ HLI ≅ ∠ KLJ, ∠ HLJ ≅ ∠ KLI (Addition property).
7. Theorem 3-4.2
8. LKI (ASA)

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Exercises

1. △RSQ
2. △PSR
3. ∠ P
4. $\overline{RQ}$
5. median
6. In addition to the given information ∠ A ≅ ∠ A (Theorem 3-1.6)
Then, △ABN ≅ △ACM (ASA)
Therefore $\overline{BN} \cong \overline{CN}$

7. ∠ B ≅ ∠ DEF (Theorem 2-6.5, and Theorem 3-1.1)
Since BE = CF (given), BC = EF (Addition property)
Therefore △ABC ≅ △DEF (ASA), and AB = DE (Definition 3-3)

8. ∠ A ≅ ∠ B (Theorem 2-6.5, and Theorem 3-1.1)
AF = BE (Addition property)
△DAF ≅ △CBE (SAS), and ∠ AFD ≅ ∠ BEC (Definition 3-3)
Therefore in △EGF, EG = FG (Theorem 3-4.3).

9. △AED ≅ △ BAE (SSS), and ∠ AED ≅ ∠ BAE (Definition 3-3)

10. △BDC ≅ △CEB (SSS), and ∠ ECB ≅ ∠ DBC (Definition 3-3)
Therefore, AB = AC (Theorem 3-4.3, Definition 1-16).

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11. BE = CE (Theorem 3-4.2)
∠ BEA ≅ ∠ CED (Theorem 3-1.5)
△BEA ≅ △CED (ASA)
$\overline{AE} \cong \overline{DE}$ (Definition 3-3)
$\overline{DB} \cong \overline{AC}$ (Definition 1-16, and Addition property).

12. ∠ SPR ≅ ∠ PSQ (Addition property)
In △POS, PO = SO (Theorem 3-4.3)
By the addition property, SR = PQ.
Also $\overline{SP} \cong \overline{SP}$.
Therefore △SPR ≅ △PSQ (SAS), and $\overline{RP} \cong \overline{SQ}$ (Definition 3-3)

13. Since ∠ 1 ≅ ∠ 2, PB = PC (Theorem 3-4.3)
Therefore from this and the given information △APC ≅ △DPB (ASA), and AP = DP (Definition 3-3)

14. In △PQR, at the beginning of these exercises, $\overline{SR}$ and $\overline{TQ}$ are the medians to the congruent sides.
△SQR ≅ △TRQ (SAS)
$\overline{SR} \cong \overline{TQ}$ (Definition 3-3)

15. From the given information BC = DC (Subtraction property)
△ACD ≅ △ECB (SAS) and ∠ CBE ≅ ∠ CDA (Definition 3-3)
Therefore ∠ ABE ≅ ∠ EDA (Theorem 3-1.4).

16. ∠ SRQ ≅ ∠ TRP (Postulate 2-10)
△SRQ ≅ △TRP (SAS)
$\overline{PT} \cong \overline{QS}$ (Definition 3-3)

17. Let $\overline{DB}$ intersect $\overline{CA}$ at E.
∠ CAB ≅ ∠ DBA (Theorem 3-1.4)
$\overline{AE} \cong \overline{BE}$ (Theorem 3-4.3)
∠ DEA ≅ ∠ CEB (Theorem 2-6.3)
∠ DEA ≅ ∠ CEB (ASA)
∠ BCA ≅ ∠ ADB (Definition 3-3)

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18. Since m∠ ABP = m∠ CBD (given), and ∠ PBD ≅ ∠ PBD,
m∠ ABD = m∠ PBC (Addition property)
∠ A ≅ ∠ BPC, and AB = PB (Given)
△BAD ≅ △BPC (ASA), and ∠ ADB ≅ ∠ BCP (Definition 3-3)

19. Since ∠ A ≅ ∠ BPA (given), AB = PB (Theorem 3-4.3).
Also since, m∠ ABP = m∠ CBD (given), and∠PBD ≅ ∠ PBD,
m∠ ABD = m∠ PBC (Addition property).
△BAD ≅ △BPC (SAS), and ∠ A ≅ ∠ BPC (Definition 3-3).

20. Since ∠ A ≅ ∠ BPC (Exercise 20), and ∠ A ≅ ∠ BPA (given),
∠ BPC ≅ ∠ BPA (Transitive property).

21. Since ∠ QSR ≅ ∠ RTQ (given), ∠ PSR ≅ ∠ PTQ (Theorem 3-1.4).
Since ∠ PST ≅ ∠ PTS (given), PS = PT (Theorem 3-4.3).
△PTQ ≅ △PSR (ASA), and ∠ Q ≅ ∠ R (Definition 3-3).

22. Since ∠ PST ≅ ∠ PTS (given), PS = PT (Theorem 3-4.3)
Since $\overline{SQ} \cong \overline{TR}$ (given), PQ = PR (Addition property)
△PTQ ≅ △PSR (SAS), and ∠ PSR ≅ ∠ PTQ (Definition 3-3)
By subtraction, m∠ TSW = m∠ STW, and SW = TW (Theorem 3-4.3)

23. Since ∠ PST ≅ ∠ PTS (given), ∠ QST ≅ ∠ RTS (Theorem 3-1.4), also PS = PT (Theorem 3-4.3).
Since ∠ QSR ≅ ∠ RTQ (given), ∠ TSR ≅ ∠ STQ (Subtraction property)
△QST ≅ △RTS (ASA), and $\overline{SQ} \cong \overline{TR}$ (Definition 3-3).

24. Since SW = TW (given), ∠ TSW ≅ ∠ STW (Theorem 3-4.2).
Since PS = PT (given), ∠ PST ≅ ∠ PTS (Theorem 3-4.2)
By addition, ∠ PSW ≅ ∠ PTW.
Therefore, ∠ QSR ≅ ∠ RTQ (Theorem 3-1.4).

25. Since m∠ ABP = m∠ CBD (Given), and ∠ PBD = ∠ PBD (Theorem 3-1.6), m∠ ABD = m∠ PBC (Addition Property)
Since ∠ BCD ≅ ∠ BDC (Given), BD = BC (Theorem 3-4.3)
△ABD ≅ △PBC (ASA), and ∠ A ≅ ∠ BPA (Definition 3-3)

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Review Exercises

1. T	2. APEB ↔ DPFC	3. Identity.
4. ∠ S	5. $\overline{DE}$	6. $\overline{BC}$
7. $\overline{AB}$ and $\overline{BC}$ (or $\overline{BD}$)	8. $\overline{AC}$	9. △RST ≅ △NDF
10. ASA, SAS, or SSS.		

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11. Since BC = BC and AB = DC (given), AC = DB (Addition property)
△PAC ≅ △QDB (SAS).

12. ∠ ABF ≅ ∠ DCG (Theorem 2-6.5 and Theorem 3-1.1)
△ABF ≅ △DCG (ASA).

13. equiangular (or isosceles)
14. interior
15. corollary

16. $\overline{BM}$ = $\overline{CM}$ (Definition 1-15)
$\overline{AM}$ ≅ $\overline{AM}$ (Theorem 3-1.6)
△ABM ≅ △ACM (SSS), and ∠ AMB ≅ ∠ AMC (Definition 3-3).
$\overline{AM}$ ⊥ $\overline{BC}$ (Definition 1-25).

17. Since AB = AD (given), m∠ ABD = m∠ ADB (Theorem 3-4.2)
By subtraction, m∠ CBD = m∠ CDB.
Therefore BC = DC (Theorem 3-4.3)

18. Since ∠ AEP ≅ ∠ AFP (given), ∠ BEP ≅ ∠ CFP (Theorem 3-1.4).
△BEP ≅ △CFP (SAS) and ∠ B ≅ ∠ C (Definition 3-3).
Therefore AB = AC (Theorem 3-4.3)
By subtraction, AE = AF.

19. Since AB = AC (given), ∠ ABC ≅ ∠ ACB (Theorem 3-4.2).
Since AD = AE (given), BD = CE (Subtraction property).
△BDC ≅ △CEB (SAS), and ∠ EBC ≅ ∠ DCB (Definition 3-3).
BF = FC (Theorem 3-4.3), and △BFC is isosceles.

20. Since PI = PS (given), ∠ PIS ≅ ∠ PSI (Theorem 3-4.2).
Also by addition, LS = IA.
△SPL ≅ △IPA (SAS), and ∠ SPL ≅ ∠ IPA (Definition 3-3).

21. Since DE = EC (given), ∠ EDC ≅ ∠ ECD (Theorem 3-4.2)
Since BE = AE (given), AC = BD (Addition property)
$\overline{DC}$ ≅ $\overline{DC}$ (Theorem 3-1.6)
△ADC ≅ △BCD (SAS), and ∠ ADC ≅ ∠ BCD (Definition 3-3).

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Chapter Test

1. Opposite sides are congruent
2. $\overline{AR}$ ≅ $\overline{BS}$, $\overline{RX}$ ≅ $\overline{SZ}$, $\overline{AX}$ ≅ $\overline{BZ}$
∠ R ≅ ∠ S
∠ A ≅ ∠ B
∠ X ≅ ∠ Z

3. SAS

4. Draw isosceles △ABC with angle bisects $\overline{AE}$ and $\overline{CF}$.
Label the intersection of $\overline{AE}$ and $\overline{CF}$ point H.
$\overline{AH}$ ≅ $\overline{CH}$ (Theorem 3-4.3)
∠ FHA ≅ ∠ EHC (Theorem 3-1.5)
△AHF ≅ △CHE (ASA)
$\overline{FH}$ ≅ $\overline{EH}$ (Definition 3-3)
$\overline{FC}$ ≅ $\overline{EA}$ (Addition property).

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5. Draw isosceles right △PQR with right ∠ P.
Let $\overline{PT}$ be the required median.
$\overline{RT}$ ≅ $\overline{QT}$ (Definition 1-15)
∠ R ≅ ∠ Q (Theorem 3-4.2)
△RPT ≅ △QPT (SAS)
∠ RTP ≅ ∠ QTP (Definition 3-3)
m∠ RTP = m∠ QTP = 90 (Theorem 2-5.6)
△RTP and △QTP are right triangles (Definition 1-32)

6. ∠ ACB ≅ ∠ DBC (Theorem 3-1.3)
△ACB ≅ △DBC (ASA), and $\overline{AC}$ ≅ $\overline{DB}$ (Definition 3-3)

7. Since PO = PA (given), ∠ POA ≅ ∠ PAO (Theorem 3-4.2)
Therefore ∠ POJ ≅ ∠ PAN (Theorem 3-1.4).
△JOP ≅ △NAP (ASA), and ∠ J ≅ ∠ N.

8. By subtraction, BF = CF.
Therefore ∠ FBC ≅ ∠ FCB (Theorem 3-4.2)
$\overline{BC}$ ≅ $\overline{BC}$ (Theorem 3-1.6)
△BEC ≅ △CDB (SAS), and ∠ DBC ≅ ∠ ECB (Definition 3-3).
Therefore AB = AC (Theorem 3-4.3) and △ABC is isosceles.

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Mathematical Excursion

1. Yes 2. No, not reflexive 3. No, not reflexive

4. Yes 5. No $1 \cdot 0 \neq 1$

6. 17 - 38 = (- 3) 7

7. 23 - 37 = (- 2) 7

8. 17 + 23 = (5 x 7) + 5
38 + 37 = (10 x 7) + 5
both have remainder 5

9. 17 x 37 ∈[6] and 38 x 23 ∈[6]

10. Prove that the sum of two elements from two classes is an element of the sum of the classes.
Let $a \in [p]$ and $b \in [r]$.
Then $a = 7m + p$, $b = 7n + r$, where m, p, n, r are integers and $0 \leq p < 7$, $0 \leq r < 7$.
$a + b = 7m + p + 7n + r = 7(m + n) + (p + r)$, so $a + b \in [p + r]$.

11. Prove that the product of two elements from two classes is an element of the product of the classes.
Let $m \in [d]$, $n \in [f]$.
Then, $m = 7x + d$, $n = 7y + f$, where x, y, d, and f are integers and $0 \leq d < 7$, $0 \leq f < 7$.
$m \cdot n = (7x + d)(7y + f) = 49xy + 7xf + 7dy + df$.
By the Distributive property, $m \cdot n = 7(7xy + xf + dy) + df$
Thus, $m \cdot n \in [df]$.

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Exercises

1. True	2. True	3. True
4. False	5. False	6. true
7. False	8. True	9. $t \rightarrow r$
10. $x \rightarrow y$	11. contrapositive	12. sometimes
13. p	14. $\sim p \rightarrow \sim q$	

15. If two angles are not vertical angles, then the angles are not congruent.

16. If two lines are not perpendicular, then the lines do not meet to form right angles.

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17. If two distinct lines do not intersect, then the lines do not intersect in exactly one point.

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Exercises continued

18. If two angles are not congruent and supplementary, then the two angles are not right angles.
19. If three points are collinear, then they do not determine exactly one plane.
20. If two angles bisectors of a triangle are not congruent then the triangle is not isosceles.
21. True 22. True 23. True 24. False
25. If two angles are congruent, then the two angles are vertical angles. False.
26. If two lines meet to form right angles, then the two lines are perpendicular. True.
27. If two lines intersect in exactly one point, then the two distinct lines intersect. True.
28. If two angles are right angles then the two angles are congruent and supplementary.
29. If three points determine exactly one plane, then the three points are noncollinear. True.
30. If a triangle is isosceles, then two angle bisectors of the triangle are congruent. True.
31. If the bisectors of the base angles of a triangle are not congruent, then the triangle is not isosceles.
32. If the base angles of a triangle are not congruent, then the triangle is not isosceles.
33. If two angles are not right angles, then the two angles are not congruent and supplementary.
34. If there is not exactly one bisector then there is not exactly one angle.

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35. If triangles are congruent, then corresponding sides are congruent. $p \rightarrow q$
36. If sides are congruent, then triangles are congruent. $q \rightarrow p$
37. Sides are congruent if and only if triangles are congruent. $p \rightarrow q$

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Class Exercises

2. $\cong$; assumption
3. DC; Definition 3-3
4. midpoint; Definition 1-15
5. $\ncong \triangle ACD$; contradiction of hypothesis

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Exercises

1. $\sim p$ 2. p is false 3. q is true

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4. contrapositive 5. true 6. contrapositive
7. the conclusion 8. a known fact 9. negation of the assumption

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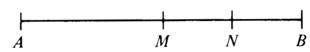
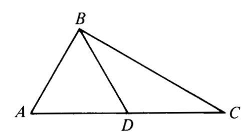
Exercises continued

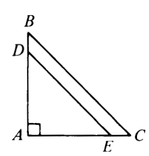
10. Suppose an angle has more than one bisector.
11. Suppose $\angle A \ncong \angle B$.
12. Suppose a segment has two midpoints.
13. Suppose there are at least two altitudes from a vertex to the opposite side.
14. Given $\overline{AB}$ with midpoints M and N (M and N lie between A and B). AM = BM, AN = BN (Definition 1-15)
    AM + MB = AB = AN + NB (Postulate 2-4)
    2AM = 2AN (Transitive property of equality, Postulate 2-1)
    AM = AN (Multiplication property of equality),
    M = N (Postulate 2-2), contradicting our assumption that M $\neq$ N. There is 1 midpoint.



15. The first step is the conjecture as stated.
16. The first step is the conjecture as stated.
17. Three congruent angles implies the triangle is equilateral (Corollary 3-4.3a). This contradicts Definition 3-12). Therefore, it is false that all 3 angles are congruent.
18. Existence. $\overline{BD}$ is a median of $\triangle ABC$.



19. Use the contrapositive of Corollary 3-4.2a with Definition 3-11.
20. Counterexample. $\triangle ABC$ and $\triangle ADE$ are right triangles but are not congruent.



21. Assume $\angle P \cong \angle Q$ is true, then the sides opposite are congruent (Theorem 3-4.3)
    But a scalene triangle has no congruent sides (Definition 3-12)
    Therefore $\angle P \ncong \angle Q$.
22. Assume $\triangle RST$ is isosceles. Then its base angles are congruent (Theorem 3-4.2). This contradicts our hypothesis. Therefore $\triangle RST$ is not isosceles.
23. If $\triangle RST$ is isosceles, then some angles of $\triangle RST$ are congruent. Use Theorem 3-4.2.
24. Assume that a point is on the line perpendicular to the given segment at its midpoint. Then see the proof of Example 4 in section 4-3.
25. If right angles are not formed by two lines, then the lines are not perpendicular. (Contrapositive of Theorem 2-6.5).
26. Assume that two triangles are congruent. By Definition 3-3 the corresponding sides have equal measure. This is the contrapositive of the given statement.

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Exercises continued

27. Assume there is more than one plane containing both the point and the line. Since a line is determined by two points, there must be at least two points, A and B, in the given line. These two points and the given external point determine a unique plane. This contradicts the assumption. Therefore there is *not* more than one plane containing both the point and the line.

28. Assume there is more than one plane containing the two intersecting lines. Since three points determine a unique plane, a point on each line (not at the intersection) and the point of intersection determines a unique plane. This contradicts the assumption.

29. Assume a line segment, $\overline{AB}$, has more than one midpoint, say two midpoints, C and D. By Definition 1-15 AC = CB and AD = DB, this implies that AC = AD or that C and D are the same point. This contradicts the assumption.

30. See the second part of the proof of Theorem 3-4.1. You may wish to use the method of contradiction as in Exercise 29.

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Exercises

1. Draw $\overline{AC}$. $\overline{AC}$ ≅ $\overline{AC}$ (Theorem 3-1.6)
△ADC ≅ △CBA (SSS)
Therefore ∠ D ≅ ∠ B (Definition 3-3)

2. Draw $\overline{SP}$. $\overline{SP}$ ≅ $\overline{SP}$ (Theorem 3-1.6)
△PSQ ≅ △PSR (SSS)
Therefore ∠ Q ≅ ∠ R (Definition 3-3)

3. Draw $\overline{BC}$. $\overline{BC}$ ≅ $\overline{BC}$ (Theorem 3-1.6)
△ABC ≅ △DCB (SSS)
Therefore ∠ A ≅ ∠ D (Definition 3-3)

4. DF = BE (Addition property)
△ADB ≅ △CBD (SSS)
∠ ADB ≅ ∠ CBD (Definition 3-3)
△ADE ≅ △CBF (SAS)
AE = CF (Definition 3-3)

5. DF = BE (Addition property)
△AEB ≅ △CFD (SSS)
∠ ABE ≅ ∠ CDF (Definition 3-3)
△ABD ≅ △CDB (SAS)
AD = BC (Definition 3-3)

6. DE = BF (Subtraction property)
△ADE ≅ △CBF (SSS)
∠ ADE ≅ ∠ CBF (Definition 3-3)
△ADB ≅ △CBD (SAS)
AB = DC (Definition 3-3)

7. ∠ AGB ≅ ∠ DGC (Theorem 3-1.5)
△AGB ≅ △CGD (SAS)
∠ B ≅ ∠ D (Definition 3-3)
∠ DGF ≅ ∠ BGE (Theorem 3-1.5)
△DGE ≅ △BGE (ASA)
EG = FG (Definition 3-3)

8. AG = GC (Definition 1-15)
∠ AGE ≅ ∠ CGF (Theorem 3-1.5)
△AEG ≅ △CFG (ASA)
EG = FG (Definition 3-3)
∠ AEG ≅ ∠ CFG (Definition 3-3)
∠ BEG ≅ ∠ DFG (Theorem 3-1.4)
∠ BGE ≅ ∠ DGF (Theorem 3-1.5)
△BEG ≅ △DFG (ASA)
∠ B ≅ ∠ D (Definition 3-3)

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9. △AEG ≅ △CFG (SSS)
∠ A ≅ ∠ C (Definition 3-3)
∠ AGB ≅ ∠ CGD (Theorem 3-1.5)
△AGB ≅ △CGD (ASA)
AB = DC (Definition 3-3)

10. ∠ AGE ≅ ∠ CGF (Theorem 3-1.5)
△AGE ≅ △CGF (SAS)
∠ AGB ≅ ∠ CGD (Theorem 3-1.5)
△AGB ≅ △CGD (ASA)
BG = DG (Definition 3-3)

11. △ABC ≅ △CDA (SSS)
∠ BAC ≅ ∠ ACD (Definition 3-3)
∠ AMF ≅ ∠ CME (Theorem 3-1.5)
△AMF ≅ △CME (ASA)
FM = EM (Definition 3-3)

12. ∠ AMF ≅ ∠ CME (Theorem 3-1.5)
△AMF ≅ △CME (SAS)
∠ BAC ≅ ∠ ACD (Definition 3-3)
△BAC ≅ △DCA (SAS)
∠ D ≅ ∠ B (Definition 3-3)

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13. continue from Exercise 12:
AF = CE (Definition 3-3)
DE = BF (Subtraction property)

14. △PSR ≅ △PTR (SAS)
∠ PSR ≅ ∠ PTR (Definition 3-3)
△QSR ≅ △QTR (SAS)
∠ QSR ≅ ∠ QTR (Definition 3-3)
m∠ PSQ = m∠ PTQ (Subtraction property)

15. △QSR ≅ △QTR (ASA)
SR = TR (Definition 3-3)
$\overline{PR}$ ≅ $\overline{PR}$ (Theorem 3-1.6)
△PSR ≅ △PTR (SAS)
PS = PT (Definition 3-3)

16. △PSR ≅ △PTR (SSS)
∠ SPR ≅ ∠ TPR (Definition 3-3)
△PSQ ≅ △PTQ (SAS)
∠ PQS ≅ ∠ PQT (Definition 3-3)

17. △ABX ≅ △CBX (SSS)
∠ ABX ≅ ∠ CBX (Definition 3-3)
∠ A ≅ ∠ C (Theorem 3-4.2)
△ABY ≅ △CBY (ASA)
$\overline{AY}$ ≅ $\overline{CY}$ (Definition 3-3)

18. △SQP ≅ △QSR (SSS)
∠ RSQ ≅ ∠ PQS (Definition 3-3)
△SMT ≅ △QNW (ASA)
$\overline{ST}$ ≅ $\overline{QW}$ (Definition 3-3)
$\overline{SW}$ ≅ $\overline{QT}$ (Addition property).

19. Draw $\overline{BC}$.
∠ ABC ≅ ∠ ACB, ∠ EBC ≅ ∠ ECB (Theorem 3-4.2)
∠ DCE ≅ ∠ FBE (Postulate 2-11)
∠ FEB ≅ ∠ DEC (Theorem 2-6.3)
△BEF ≅ △CED (ASA)
$\overline{DE}$ ≅ $\overline{FE}$ (Definition 3-3)

20. △QPW ≅ △RPW (SAS)
∠ PQT ≅ ∠ PRS (Definition 3-3)
QW = RW (Definition 3-3)
∠ SWQ ≅ ∠ TWR (Theorem 3-1.5)
△SWQ ≅ △TWR (ASA)
SW = TW (Definition 3-3)

21. WQ = WR (Theorem 3-4.3)
∠ SWQ ≅ ∠ TWR (Theorem 3-1.5)
△SWQ ≅ △TWR (SAS)
∠ SQW ≅ ∠ TRW (Definition 3-3)
m∠ PQR ≅ m∠ PRQ (Addition property)
PQ = PR (Theorem 3-4.3)
△PQW ≅ △PRW (SAS)
∠ QPW ≅ ∠ RPW

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Exercises continued

22. △ADC ≅ △BCD (SAS)
AC = BD (Definition 3-3)
∠ BDC ≅ ∠ ACD (Definition 3-3)
DE = CE (Theorem 3-4.3)
AE = BE (Subtraction property)
∠ CAB ≅ ∠ DBA (Theorem 3-4.2)

23. △AXY ≅ △BZY (SSS)
∠ XYA ≅ ∠ZYB (Definition 3-3)
∠ XYB ≅ ∠ ZYA (Postulate 2-10)
△XBY ≅ △ZAY (SAS)
XB = ZA (Definition 3-3, Definition 1-16).

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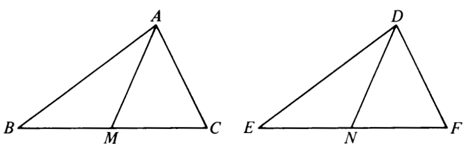
24. Draw $\overline{BD}$.
In △BDC, ∠ CBD ≅ ∠ CDB (Theorem 3-4.2).
By subtraction, ∠ ABD ≅ ∠ ADB and AB = AD
(Theorem 3-4.3).

25.

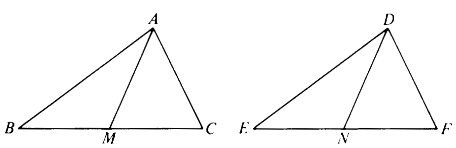


Given: △ABC ≅ △DEF
$\overline{AM}$ is a median of △ABC
$\overline{DM}$ is a median of △DEF

Prove: $\overline{AM} \cong \overline{DN}$

Proof: BC = EF (Definition 3-3)
BM = EN (Division property)
∠ B ≅ ∠ E (Definition 3-3)
AB = DE (Definition 3-3)
△ABM ≅ △DEN (SAS)
AM = DN (Definition 3-3)

26.



Given: △ABC ≅ △DEF
$\overline{AM}$ is an angle bisector of △ABC
DM is an angle bisector of △DEF

Prove: AM = DN
Proof: ∠ BAC ≅ ∠ EDF (Definition 3-3)
∠ BAM ≅ ∠ EDN (Division property)
AB = DE (Definition 3-3)
∠ B ≅ ∠ E (Definition 3-3)
△ABM ≅ △DEN (ASA)
AM = DN (Definition 3-3)

27. Draw $\overline{QS}$.
△PQS ≅ △RSQ (SAS)
∠ RQS ≅ ∠ PSQ (Definition 3-3)
QT = ST (Theorem 3-4.3)

28. △PQT ≅ △PRS (SAS)
SR = TQ (Definition 3-3)
Draw $\overline{RQ}$
SQ = TR (Subtraction property)
△QSR ≅ △RTQ (SSS)
∠ SRQ ≅ ∠ TQR (Definition 3-3)
In △QMR, QM = MR (Theorem 3-4.3)

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29. Draw $\overline{PJ}$.
△PJL ≅ △PJA (SSS)
∠ L ≅ ∠ A (Definition 3-3)
△PIL ≅ △PSA (ASA)
PI = PS (Definition 3-3)
∠ PIS ≅ ∠ PSI (Theorem 3-4.2)

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Exercises

1. right 2. three 3. perpendicular bisector
4. external 5. two or more 6. distance
7. See the figure for Exercises 9-11

Given: AB = AD and BC = DC.
$\overline{AC} \cap \overline{BD}$ at M.

Prove: $\overline{AC}$ is the perpendicular bisector of $\overline{BD}$.
Proof: △ABC ≅ △ADC (SSS)
∠ BAC ≅ ∠ DAC (Definition 3-3)
△BAM ≅ △DAM (SAS)
∠ AMB ≅ ∠ AMD and BM = DM (Definition 3-3)
$\overline{AC} \perp \overline{BD}$ (Definition 1-25)
Therefore $\overline{AC}$ is the perpendicular bisector of $\overline{BD}$.

8. Use the figure for Theorem 4-4.5.
Assume $\overleftrightarrow{PX} \perp m$ (where X is distinct from M).
This contradicts Theorem 4-4.6. Therefore this cannot be, that is there does not exist a triangle such as △PMX.

9. AB = AD and BC = CD (Theorem 4-4.3)
△ABC ≅ △ADC (SSS)
∠ ABC ≅ ∠ ADC (Definition 3-3)

10. See the solution for exercise 7.

11. In △ABD, AB = AD (Theorem 3-4.3)
In △BCD, BC = DC (Theorem 3-4.3)
$\overline{AC}$ is the perpendicular bisector of $\overline{BD}$ (Corollary 4-4.3a)

12. Since △ABC is isosceles AC = BC.
C is on $\overrightarrow{MD}$ (Theorem 4-4.3)

13. $\overline{AM} \cong \overline{BM}$, ∠ AMC ≅ ∠ BMC (Definition 3-3)
△ACM ≅ △BCM (SAS)
$\overline{AC} \cong \overline{BC}$ (Definition 3-3)
△ABC is isosceles (Definition 3-12)

14. TQ = TR (Theorem 3-4.3)
$\overleftrightarrow{TS}$ is the perpendicular bisector of QR (Corollary 4-4.3a)
Since PQ = PR (Theorem 3-4.3), P is on $\overleftrightarrow{TS}$
(Theorem 4-4.3)

15. ∠ PQR ≅ ∠ PRQ (Theorem 3-4.2)
∠ TQR ≅ ∠ TRQ (Multiplication property)
TQ = TR (Theorem 3-4.3)
$\overline{PTS} \perp \overline{QSR}$ (Corollary 4-4.3a)
△PQT ≅ △PRT (SAS)
∠ QPT ≅ ∠ RPT (Definition 3-3)
△QPS ≅ △RPS (SAS)
QS = SR (Definition 3-3)

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16. BM = CN (Multiplication property)
∠ ABC ≅ ∠ ACB (Theorem 3-4.2)
△BMC ≅ △CNB (SAS)
∠ NBC ≅ ∠ MCB (Definition 3-3)
BG = CG (Theorem 3-4.3)
$\overline{AGH}$ is the perpendicular bisector of $\overline{BC}$. (Corollary 4-4.3a)

17. SR = SQ and SQ = SP (Theorem 4-4.3)
Therefore SR = SP (Transitive property)

18. ∠ RSQ ≅ ∠ RQS (Theorem 3-4.2)
∠ NSQ ≅ ∠ MQS (Multiplication property)
TS = TQ (Theorem 3-4.3)
$\overline{RTW}$ is the perpendicular bisector of SWQ (Theorem 4-4.3)

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Exercises continued

19. P is on $\overleftrightarrow{RS}$ (Theorem 4-4.3)
P is on $\overleftrightarrow{QT}$ (Theorem 4-4.3)
Therefore $\overleftrightarrow{RS} \cap \overleftrightarrow{QT}$ at P.

20. Since $\overline{RP}$ is the perpendicular bisector of $\overline{AB}$, AP = BP.
Since $\overline{SP}$ is the perpendicular bisector of $\overline{AC}$, CP = AP.
Therefore BP = CP (Transitive property)
P is on $\overleftrightarrow{QT}$ (Theroem 4-4.3)

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Exercises

1. perpendicular   2. perpendicular segment.

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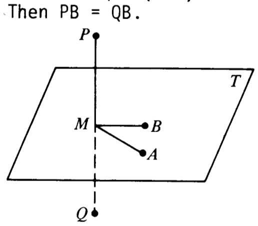
3. perpendicular bisector plane
4. a point   5. coplanar

6. Use Theorem 4-5.5.
7. Use Theorem 4-5.5 and prove by contradiction.
8. Use Theorem 4-5.6
9. Use Theorem 4-5.6 and prove by contradiction.
10. Use Definition 4-10 and Theorem 4-5.9

11. $\overline{PQ} \perp$ plane T and PM = QM (Given)
$\overline{PQ} \perp \overline{MB}$ (Definition 4-7)
$\angle$ PMB $\cong \angle$ QMB
$\triangle$MPB $\cong \triangle$MQB (SAS)
Then PB = QB.



12. Use Definition 4-10 and contradiction.

13. Let the projection of R be R'
Then $\triangle$RSR' $\cong \triangle$RTR' (SAS) and $\overline{SR} \cong \overline{TR}$ (Definition 3-3).

14. Let the midpoints of $\overline{RQ}$ and $\overline{RP}$ be T and S, respectively.
PX = RX and RX = QX (Theorem 4-4.3)
$\triangle$PXA $\cong \triangle$RXA $\cong \triangle$QXA (SAS)
$\overline{AP} \cong \overline{AQ} \cong \overline{AR}$ (Definition 3-3)

15. $\triangle$ARS $\cong \triangle$BRS (ASA)
$\overline{RA} \cong \overline{RB}$ (Definition 3-3)

16. Simply apply Theorem 4-5.1

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Review Exercises

1. *Converse:* If two angles of a triangle are congruent, then the sides opposite these angles are congruent.

   Inverse: If two sides of a triangle are not congruent, then the angles opposite these sides are not congruent.

   Contrapositive: If two angles of a triangle are not congruent, then the sides opposite these angles are not congruent.

2. *Converse:* If two triangles are congruent, then the corresponding sides of the two triangles are congruent.

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2. *continued*

   Inverse: If the corresponding sides of two triangles are not congruent, then the triangles are not congruent.

   Contrapositive: If two triangles are not congruent, then the corresponding sides of the two triangles are not congruent.

3. *Converse:* If $\angle$ B $\cong \angle$ Y, then $\triangle$ABC $\cong \triangle$XYZ.

   Inverse: If $\triangle$ABC $\not\cong \triangle$XYZ, then $\angle$ B $\not\cong \angle$ Y.

   Contrapositive: If $\angle$ B $\cong \angle$ Y, then $\triangle$ABC $\cong \triangle$XYZ.

4. *Converse:* If $\overline{AB}$ has exactly one midpoint, then $\overline{AB}$ is a segment.

   Inverse: If $\overline{AB}$ is not a segment, then $\overline{AB}$ does not have exactly one midpoint.

   Contrapositive: If $\overline{AB}$ does not have exactly one midpoint, then it is not a segment.

5. original implication
6. negation of the hypothesis
7. *q*
8. For a proof by contradiction assume two triangles are not congruent.
For a proof by contrapositive use the hypothesis: "the corresponding angles are not congruent" and conclude "the two triangles are not congruent."

9. For a proof by contradiction assume two angles are not vertical angles.
For a proof by contrapositive use the hypothesis: "the two angles are not congruent" and conclude "the two angles are not vertical angles."

10. Draw $\overline{AC}$. $\overline{AC} \cong \overline{AC}$ (Theorem 3-1.6)
$\triangle$ABC $\cong \triangle$ADC (SSS)
$\angle$ B $\cong \angle$ D (Definition 3-3)

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11. $\triangle$PSQ $\cong \triangle$RSQ (SAS)
PQ = RQ (Definition 3-3)
$\angle$ PQS $\cong \angle$ RQS (Definition 3-3)
$\triangle$PQT $\cong \triangle$RQT (SAS)
$\angle$ QPT $\cong \angle$ QRT (Definition 3-3)

12. Because there would then be *two* perpendiculars to a given line from a given external point. This violates Theorem 4-4.6.

13. AB = AC (Theorem 3-4.3)
m$\angle$ DBE = m$\angle$ DCB (Subtraction property)
BD = CD (Theorem 3-4.3)
$\overline{ADE}$ is the perpendicular bisector of $\overline{BEC}$ (Corollary 4-4.3a)

14. perpendicular
15. exactly one
16. perpendicular

Chapter Test

1. If BC $\neq$ AC, then $\angle$ A $\cong \angle$ B.
2. Suppose $\triangle$ABC is scalene.
AB $\neq$ BC $\neq$ CA (Definition 3-12)
$\angle$ A $\not\cong \angle$ B $\not\cong \angle$ C (contrapositive of Corollary 3-4.3a)
Contradiction.

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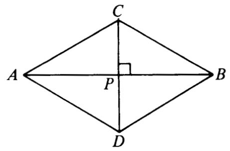
Chapter Test continued

3.



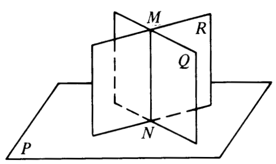
Given $\overline{AB}$ and $\overline{CD}$ are perpendicular and bisect each other.

Prove Quadrilateral ADBC is equilateral

Proof AC = CB (Theorem 4-4.3)
BD = CB (Theorem 4-4.3)
AD = BD (Theorem 4-4.3)

Therefore quadrilateral ADBC is equilateral (Transitive property)

4.



Given $\overline{MN} \perp$ Plane P
Planes R and Q intersect at $\overline{MN}$

Prove Plane R $\perp$ Plane P
Plane Q $\perp$ Plane P

Proof If $\overline{MN} \perp$ plane P, then plane R $\perp$ plane P and plane Q $\perp$ plane P (Theorem 4-5.9)

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5. AM = DM and BM = CM (Definition 1-15)
$\angle$ AMB $\cong$ $\angle$ DMC (Theorem 3-1.5)
$\triangle$AMB $\cong$ $\triangle$DMC (SAS)
$\angle$ B $\cong$ $\angle$ C (Definition 3-3)
$\triangle$EBM $\cong$ $\triangle$FCM (SAS)
EM = FM (Definition 3-3)

6. $\angle$ ABC $\cong$ $\angle$ ACB (Theorem 3-4.2)
MB = NC (Multiplication property)
$\triangle$MBC $\cong$ $\triangle$NCB (SAS)
$\angle$ GBC $\cong$ $\angle$ GCB (Definition 3-3)
In $\triangle$GBC, BG = CG (Theorem 3-4.3)
$\overline{AGE}$ is the perpendicular bisector of $\overline{BEC}$.
(Corollary 4-4.3a)

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1. Multiplication property.
2. Subtraction property.
3. Transitive property.
4. Trichotomy property.
5. Transitive property.
6. Addition property.
7. Trichotomy property.
8. Division property.
9. Subtraction property.
10. Multiplication property

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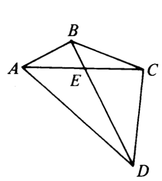
1. Corollary 5-1.1a
2. Multiplication property.
3. Trichotomy property.
4. Division property.
5. Transitive property.
6. Addition property.
7. Subtraction property.
8. Transitive property.
9. Less than; Transitive property.
10. Greater than; Addition property.
11. Less than; Trichotomy property.
12. Less than; Subtraction property.
13. Less than; Theorem 3-4.3
    Postulate 2-1
14. Greater than; Transitive property.
15. Greater than; Addition property.
16. Less than; Division property.
17. Complement ∠ A < complement ∠ B.
18. Supplement ∠ A > supplement ∠ B.
19. Let a be any negative number, and b be any positive number.
    Therefore $a < 0$. However $0 < b$
    Thus, $a < b$ by the Transitive Property of Inequality.

20. Let m be the measure of the acute angle.
    Let s be the measure of its supplement.
    $s + m = 180$, and $m < 90$
    $s > 90$ (Subtraction property).

21. AB = AP + BP (Point Betweenness Postulate, 2-4)
    AB - AP = BP (Subtraction property)
    Since BP > 0,
    AB - AP > 0 (Substitution Postulate, 2-1)
    AB - AP + AP > AP (Addition property of inequalities)
    AB > AP (Additive inverse and identity properties)
    To prove AB > BP interchange AP and BP in the above proof.

22. m∠ ABC = m∠ APB + m∠ CBP (Point Betweenness Postulate, 2-4)
    m∠ ABC - m∠ APB = m∠ CBP (Subtraction property)
    Since m∠ CBP > 0,
    m∠ ABC - m∠ APC > 0 (Substitution Postulate, 2-1)
    m∠ ABC - m∠ APC + m∠ APC > m∠ APC (Addition property of inequalities)
    m∠ ABC > m∠ APC (Additive inverse and identity properties)
    To prove m∠ ABC > m∠ CBP interchange m∠ APC with m∠ CBP in the above proof.

23. BE > EC (Given)
    DE > EC (Given)
    EC = AE (Given)
    DE > AE (Postulate 2-1)
    BE + DE > EC + AE (Addition property)
    BD > AC (Postulate 2-4).



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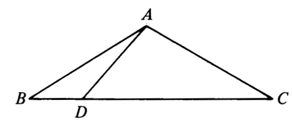
24. m∠ C = m∠ B (Given, Theorem 3-4.2)
    m∠ BAC > m∠ DAC (Corollary 5-1.1b)
    m∠ BAC > m∠ B (Transitive property).



25. △AED ≅ △AFD (ASA)
    AE = AF (Definition 3-3)
    EB < FC (Subtraction property).

26. △ABH ≅ △ADH (SAS)
    m∠ B = m∠ ADB (Definition 3-3)
    m∠ C < m∠ B (Postulate 2-1).

27. △ADB ≅ △CDP (SAS)
    m∠ A = m∠ DCP
    m∠ ACR > m∠ DCP (Corollary 5-1.1b)
    m∠ ACR > m∠ A (Postulate 2-1).

28. ED = DC (Theorem 3-4.3)
    m∠ AED = m∠ BCD (Addition property)
    EG = HC (Subtraction property)
    m∠ DGH = m∠ DHG (Theorem 3-4.2)
    m∠ AGE = m∠ BHC (Transitive property)
    △AEG ≅ △BCH (ASA)
    AE = BC (Definition 3-3)
    △AED ≅ △BCD (SAS)
    AD = BD (Definition 3-3)
    m∠ 6 = m∠ ABD (Theorem 3-4.2)
    m∠ 5 < m∠ ABD (Corollary 5-1.1b)
    m∠ 5 < m∠ 6 (Postulate 2-1).

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1. ∠A, ∠B
2. Theorem 5-2.1
3. They are less than m∠ ACD.
4. Definition 1-17
5. Acute, by Exercises 3 and 4.

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1. ∠ABK, ∠CBH, ∠ACF, ∠BCG.
2. ∠DAB, ∠EAC, ∠BCG, ∠ACF.
3. ∠ABC, ∠BAC.
4. Greater than.
5. Equal.
6. Supplementary.
7. By Theorem 5-2.1, m∠ GCB > m∠ CAB and m∠ GCB > m∠ ABC.
8. By Theorem 5-2.1, m∠ EAC > m∠ ABC amd m∠ EAC > m∠ ACB.
9. ∠s 2, 4, 14, 12.
10. ∠s 1, 2, 15, 16.
11. ∠6, ∠7, ∠ AEB, ∠ ABJ, ∠ DGB. ∠ AHC, ∠4, ∠3.
12. ∠6, ∠5, ∠ AHC, ∠ AFD, ∠ ABJ.
13. ∠ 15 or ∠ 9.
14. ∠ 11

15. *(I)* For △EGB: m∠ ABJ > m∠ 11 (Theorem 5-2.1)
    For △AGE: m∠ 11 > m∠ 8 (Theorem 5-2.1)
    Therefore: m∠ ABJ > m∠ 8 (Transitive property)

    (II) For △AEB: m∠ ABJ > m∠ AEB (Theorem 5-2.1)
    However m∠ AEB > m∠ 8 (Corollary 5-1.1b)
    Therefore: m∠ ABJ > m∠ 8 (Transitive property)

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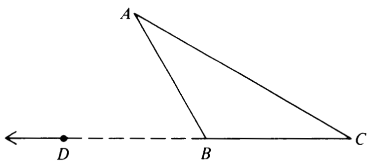
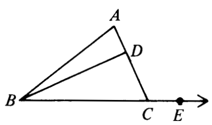
16. *Given:* Obtuse $\triangle ABC$ with obtuse $\angle ABC$
Prove: $\angle A$ and $\angle C$ are acute.



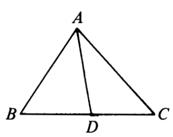
$\angle ABD$ and $\angle ABC$ are supplementary (Supplementary Angles Postulate, 1-6)
$\angle ABD$ is acute (Definition 1-28)
$m\angle A < m\angle ABD$ (Theorem 5-2.1)
$m\angle C < m\angle ABD$ (Theorem 5-2.1)
Therefore $\angle A$ and $\angle C$ are acute (Definition 1-24)

17.



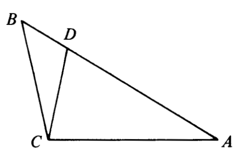
$m\angle ACE > m\angle ABC$ (Theorem 5-2.1)
$m\angle ABC > m\angle ABD$ (Corollary 5-1.1b)
Therefore $m\angle ACE > m\angle ABD$ (Transitive property)

18.



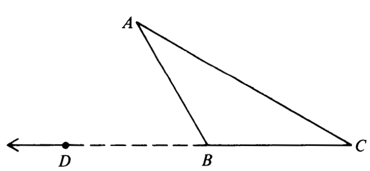
$m\angle ABD > m\angle ACB$ (Theorem 5-2.1)
Therefore $m\angle ABD \neq m\angle ACB$ (Trichotomy property)

19.



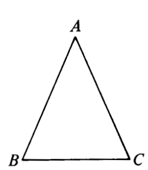
$m\angle ACD > m\angle ADC$ (Given)
$m\angle ADC > m\angle B$ (Theorem 5-2.1)
Therefore $m\angle ACD > m\angle B$ (Transitive property)

20. *Given:* $\triangle ABC$
Prove: $m\angle A + m\angle ABC < 180$



$m\angle ABD + m\angle ABC = 180$ (Supplementary Angles Postulate, 1-6)
$m\angle A < m\angle ABD$ (Theorem 5-2.1)
By adding the above two relationships:
$m\angle ABD + m\angle ABC + m\angle A < m\angle ABD + 180$.
(Addition property of inequalities)
By subtracting $m\angle ABD$ from both sides of the inequality:
$m\angle ABC + m\angle A < 180$.

21. *Given:* Isosceles $\triangle ABC$, with $AB = AC$
Prove: $\angle B$ and $\angle C$ are acute angles



From Exercise 20 we know that $m\angle B + m\angle C < 180$.
Since $m\angle B = m\angle C$ (Theorem 3-4.2), $2\, m\angle B < 180$, or

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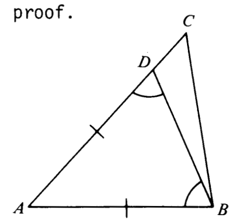
21. *(continued)*

$m\angle B < 90$. In a similar way we show that $\angle C$ is also acute.

22. Let $\angle A$, $\angle B$, and $\angle C$ be the 3 angles of a triangle.
$m\angle A + m\angle B <$ the measure of the exterior angle at $C < 180$.
$m\angle A + m\angle C <$ the measure of the exterior angle at $B < 180$.
$m\angle B + m\angle C <$ the measure of the exterior angle at $A < 180$.
Therefore $2m\angle A + 2m\angle B + 2m\angle C <$ the sum of the measures of the exterior angles at A, B, and $C < 3\cdot(180)$ (Addition property).
Thus $m\angle A + m\angle B + m\angle C < \frac{1}{2}(540)$ or 270.

23. Students should discover that an exterior angle of a quadrilateral may be acute while a remote interior angle is obtuse. Better students may wish to present a formal proof.

24.

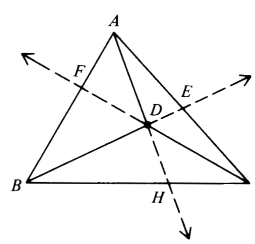


$\overline{AD} \cong \overline{AB}$ (Given)
$m\angle ADB = m\angle ABD$ (Theorem 3-4.2)
$m\angle ADB > m\angle C$ (Theorem 5-2.1)
$m\angle ABD > m\angle C$ (Substitution postulate)
$m\angle ABC > m\angle ABD$ (Corollary 5-1.1b)
$m\angle ABC > m\angle C$ (Transitive Property of Inequalities)

25. $m\angle 1 = m\angle 2$ (Definition 3-8)
$m\angle 3 > m\angle 1$ (Theorem 5-2.1)
$m\angle 3 > m\angle 2$ (Substitution postulate)

26. $m\angle CED > m\angle A$ (Theorem 5-2.1)
$m\angle BDC > m\angle CED$ (Theorem 5-2.1)
$m\angle BDC > m\angle A$ (Transitive property)

27.



$m\angle BHD > m\angle ACB$ (Theorem 5-2.1)
$m\angle BDA > m\angle BHD$ (Theorem 5-2.1)
*$m\angle BDA > m\angle ACB$ (Transitive property)
$m\angle AFD > m\angle ABC$ (Theorem 5-2.1)
$m\angle ADC > m\angle AFD$ (Theorem 5-2.1)
*$m\angle ADC > m\angle ABC$ (Transitive property)
$m\angle CED > m\angle BAC$ (Theorem 5-2.1)
$m\angle BDC > m\angle CED$ (Theorem 5-2.1)
*$m\angle BDC > m\angle BAC$ (Transitive property)

Now add the (*) inequalities:
$m\angle BDA + m\angle ADC + m\angle BDC > m\angle ACB + m\angle ABC + m\angle BAC$
(Addition Property of Inequalities)

28. $\angle A$ is the same as $\angle BAD$, $\angle C$ is the same as $\angle BCD$
$m\angle EBC > m\angle BAC$ (Theorem 5-2.1)
$m\angle EBC > m\angle ACB$ (Theorem 5-2.1)
$m\angle FDC > m\angle DAC$ (Theorem 5-2.1)
$m\angle FDC > m\angle DCA$ (Theorem 5-2.1)

Now add the above inequalities:
$2\,(m\angle EBC) + 2\,(m\angle FDC) > m\angle BAC + m\angle ACB + m\angle DAC + m\angle DCA$ (Addition Property of Inequalities)
$2\,(m\angle EBC) + 2(m\angle FDC) > m\angle BAD + m\angle BCD$
(Substitution postulate)
$m\angle EBC + m\angle FDC > \frac{1}{2}[m\angle BAD + m\angle BCD]$ (Division by 2 or multiply by $\frac{1}{2}$).

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29. *Given:* $\triangle ABC$ with obtuse $\angle ABC$
$\overline{AD} \perp \overleftrightarrow{BC}$.
Prove: D is in the exterior of $\triangle ABC$.

Indirect Proof: Suppose D is not in the exterior of $\triangle ABC$.
Then D is either in $\overline{BC}$ or coincident with B.
If D is in $\overline{BC}$, then $m\angle ADC > m\angle ABC$.
But a right angle is not greater than an obtuse angle.
Therefore, D is not in $\overline{BC}$.
If D is coincident with B, then $\overline{AB} \perp \overleftrightarrow{BC}$.
This is also not true because $\angle ABC$ is obtuse (not a right angle).
Therefore D is not in $\overleftrightarrow{BC}$ or coincident with B.
Instead it is in the exterior of $\triangle ABC$.

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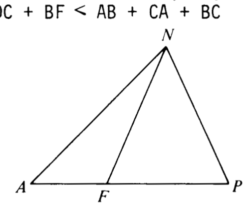
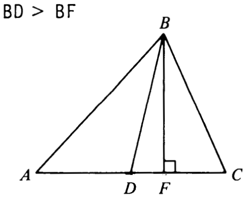
1. $\angle B$; (Theorem 5-3.1)
2. $\angle C$; (Theorem 5-3.1)
3. $\overline{RS}$; (Theorem 5-3.2)
4. $\overline{PR}$; (Theorem 5-3.2)
5. $\angle F$; (Theorem 5-3.1)
6. $\angle E$; (Theorem 5-3.1)
7. $\overline{MN}$; (Theorem 5-3.2)
8. $\overline{MK}$; (Theorem 5-3.2)
9. $\overline{AP}$; (Theorem 5-3.2)
10. *Given:* $\triangle ABC$ with altitudes $\overline{DC}$, $\overline{AE}$, and $\overline{FB}$.
Prove: $AE + DC + BF < AB + CA + BC$



From the result of Exercise 9:
$AE < AB$
$DC < CA$
$BF < BC$
By adding these inequalities we get:
$AE + DC + BF < AB + CA + BC$.

11. *Given:* $\triangle ABC$ with median $\overline{BD}$ and altitude $\overline{BF}$.
Prove: $BD > BF$



From Exercise 9 applied to $\triangle BDF$: $BD > BF$.
By this point students should realize that the hypotenuse is the longest side of a right triangle.

12. In $\triangle PBA$, $\overline{PB}$ is the longest side (Theorem 5-3.2).
Since $\triangle CPB$ is isosceles, $PB = PC$ (Theorem 3-4.2).
However $CE > PC$ (Theorem 5-3.2)
Therefore $CE > PB$ (Transitive property)
Lastly, $CD > CE$ (Theorem 5-3.2) which lets us conclude that $\overline{CD}$ is the longest segment in the figure.

13. $m\angle ACD > m\angle A$ (Theorem 5-2.1)
$m\angle ACB > m\angle A$ (Transitive property)
$BC < AB$ (Theorem 5-3.2).

14. In $\triangle AMB$, $AM < BM$ (Theorem 5-3.2)
In $\triangle CMD$, $CM < DM$ (Theorem 5-3.2)
$AC < BD$ (Addition property).

15. $m\angle ADB < m\angle DAC$ (Theorem 5-2.1)
$m\angle DAC = m\angle BAD$
$m\angle ADB > m\angle BAD$ (Postulate 2-1)
$AB > BD$ (Theorem 5-3.2).

16. $m\angle ACB < m\angle ABC$ (Theorem 5-3.1)
$m\angle PCB < m\angle PBC$ (Division property)
$PC > PB$ (Theorem 5-3.2).

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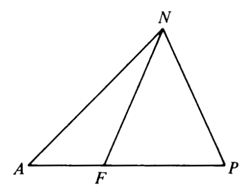
17. $m\angle ABD > m\angle ADB$ (Theorem 5-3.1)
$m\angle CBD < m\angle CDB$ (Subtraction property)
In $\triangle BDC$, $DC < BC$ (Theorem 5-3.2).

18. Since $\overline{AD}$ is the shortest side, $AD < AB$
In $\triangle ADB$, $m\angle ABD < m\angle ADB$.
Similarly, since $\overline{BC}$ is the longest side, $DC < BC$.
In $\triangle DBC$, $m\angle CBD < m\angle CDB$.
Therefore, $m\angle ABC < m\angle ADC$ (Addition property).

19. $AC = AB$ (Given)
$CD < BE$ (Given)
$AC + CD < AB + BE$ (Addition Property of Inequalities)
$AD < AE$ (Substitution postulate)
$m\angle E < m\angle D$ (Theorem 5-3.1)
or, $m\angle D > m\angle E$.

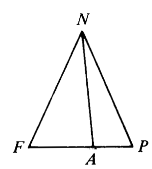
20. $AD < DC$ (Postulate 2-1)
$m\angle ACD < m\angle CAD$ (Theorem 5-3.1)
$\triangle DAC \cong \triangle BCA$ (SSS)
$m\angle ACD = m\angle BAC$ (Definition 3-3)
$m\angle BAC < m\angle CAD$ (Postulate 2-1)
$\overline{AC}$ does not bisect $\angle BAD$ (Definition 1-29).

21.



$m\angle NFP > m\angle A$ (Theorem 5-2.1)
$m\angle NFP = m\angle NPF$ (Theorem 3-4.2)
$m\angle NPF > m\angle A$ (Postulate 2-1)
In $\triangle ANP$, $AN > NP$ (Theorem 5-3.2).

22.



$m\angle NAP > m\angle NFA$ (Theorem 5-2.1)
$m\angle NFA = m\angle P$ (Theorem 3-4.2)
$m\angle NAP > m\angle P$ (Postulate 2-1)
$AN < NP$ (Theorem 5-3.2).

23. In right $\triangle ABD$, $AB > BD$ (Exercise 9).
In right $\triangle ACD$, $AC > DC$ (Postulate 2-1 and Postulate 2-4).
$AB + AC > BD + DC$ (Addition property)
$AB = AC$ (Given)
$2(AB) > BC$.

24. $m\angle ADC > m\angle B$ (Theorem 5-2.1)
$m\angle ADC = m\angle ACD$ (Theorem 3-4.2)
$m\angle ACD > m\angle B$ (Postulate 2-1)
However, $m\angle ACB > m\angle ACD$ (Postulate 2-11)
$m\angle ACB > m\angle B$ (Transitive property).

25. $\triangle EAD \cong \triangle CAD$ (SAS)
$m\angle C = m\angle AED$ (Definition 3-3)
$m\angle AED > m\angle B$ (Theorem 5-2.1)
$m\angle C > m\angle B$ (Postulate 2-1).

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Class Exercises

1. Yes	2. No
3. No	4. Yes
5. AD	6. HE
7. BD	8. CE
9. AE	10. HD

Exercises

1. BC	2. GD
3. GC	4. FC

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Exercises continued

5. AG
6. GF
7. FB
8. DG
9. BD
10. DG
11. Yes
12. No
13. No
14. No

15. Let the sides of the triangle be:
$5, 7, x$, such that $x > 0$.
$5 + 7 > x$, $12 > x$ (Theorem 5-4.1)
$5 + x > 7$, $x > 2$ (Theorem 5-4.1)
$7 + x > 5$ (Theorem 5-4.1) already true.
Therefore $2 < x < 12$, or
$\{x: 2 < x < 12\}$

Exercises 16-18 are done in a similar manner.

16. $\{y: 8 < y < 14\}$
17. $\{z: 1 < z < 3\}$
18. $\{w: 11 < w < 25\}$
19. $AE < AB$ and $AE < AC$.
20. $CD < AC$ and $CD < BC$.
21. $BF < AB$ and $BF < BC$.

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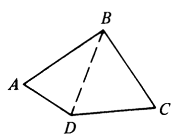
22.



In quadrilateral ABCD, draw $\overline{BD}$.
$BC + CD > BD$ (Theorem 5-4.1)
$AB + BC + CD > BD + AB$ (Addition property)
$BD + AB > AD$ (Theorem 5-4.1)
$AB + BC + CD > AD$ (Transitive property).

23. $AB + AC > BC$ (Theorem 5-4.1)
$AB = AC$ (Given)
$2AB > BC$ (Postulate 2-1), or
$AB > \frac{1}{2}BC$ (Division property).

24. In $\triangle APB$, $PB < AB + AP$ (Theorem 5-4.1)
In $\triangle BPC$, $PB < BC + PC$ (Theorem 5-4.1)
$2PB < AB + AP + BC + PC$ (Addition property)
$PB < \frac{1}{2}(AB + BC + CA)$ (Division property).

25. In $\triangle ABD$, $BD < AD + AB$ (Theorem 5-4.1)
In $\triangle CBD$, $BD < CD + BC$ (Theorem 5-4.1)
$2BD < AB + BC + CD + DA$ (Addition property)
$BD < \frac{1}{2}(AB + BC + CD + DA)$ (Division property).

26. $BP < AP + AB$ (Theorem 5-4.1)
$BP < BC + DC + DP$ (see Exercise 22).
$2BP < AB + BC + CD + DA$ (Addition property)
$BP < \frac{1}{2}(AB + BC + CD + DA)$ (Division property)

27. $AB + DB > AD$ (Theorem 5-4.1)
$BC + DB > AD$ (Postulate 2-1), or
$DC > AD$ (Postulate 2-4).

28. $AP + AR > PR$ (Theorem 5-4.1)
$AP + AR + PB + RC > PR + PB + RC$ (Addition property)
$AB + AC > PR + PB + RC$ (Postulate 2-4).

29. $\triangle ABE \cong \triangle ADE$ (SAS)
$BE = ED$ (Definition 3-3)
$DC < ED + EC$ (Theorem 5-4.1)
$DC < BE + EC$ (Postulate 2-1)
$DC < BC$ (Postulate 2-4).

30. *Given:* $\triangle ABC$
$AB + AC > BC$ (Theorem 5-4.1)
$AB > BC - AC$ (Subtraction property).

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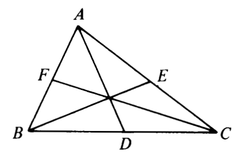
31. $AB + BM > AM$ (Theorem 5-4.1)
$AC + MC > AM$ (Theorem 5-4.1)
$AB + AC + 2BM > 2AM$ (Addition property, Postulate 2-1)
$AB + AC > 2BM$ (Theorem 5-4.1)
$2(AB + AC) > 2AM$
$AB + AC > AM$ (Division property).

32.



Given: $\triangle ABC$ with midpoints F, D, and E respectively.
$AB + AE > BE$
$BC + EC > BE$ (Theorem 5-4.1)
$AB + AC + BC > 2BE$ (Addition property)
$AB + AC + BC > 2AD$, $AB + AC + BC > 2CF$
(Theorem 5-4.1, Addition property)
$3(AB + AC + BC) > 2(BE + AD + CF)$
(Addition property)
$BE + AD + CF < 1\frac{1}{2}(AB + AC + BC)$
(Division property).

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33. $PN + NC > PC$ (Theorem 5-4.1)
$BP + PN + NC > PC + BP$ (Addition property)
$BN + NC > PC + BP$ (Postulate 2-4)
$AB + AN > BN$ (Theorem 5-4.1)
$AB + AN + NC > BN + NC$ (Addition property), or
$AB + AC > PC + BP$ (Transitive property).

34. $AP + PB > AB$ (Theorem 5-4.1)
$AP + PC > AC$ (Theorem 5-4.1)
$PB + PC > BC$ (Theorem 5-4.1)
$2(AP + PB + PC) > AB + AC + BC$ (Addition property)
$AP + PB + PC > \frac{1}{2}(AB + AC + BC)$ (Division property).

35. $BP + PC < AB + AC$ (Exercise 33)
$AP + BP < AC + BC$ (Exercise 33)
$AP + PC < AB + BC$ (Exercise 33)
$2(AP + BP + PC) < 2(AB + AC + BC)$ (Addition property), or
$AP + BP + PC < AB + AC + BC$ (Division property)

36. Choose any point in the plane and use the proof of Theorem 5-4.2 to show that the length of the perpendicular segment from the external point to the plane is less than the distance from the external point to the chosen point in the plane.

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Class Exercises

1. Greater than; (Theorem 5-5.1)
2. Greater than; (Theorem 5-5.1)
3. Greater than; (Theorem 5-5.1)
4. Less than; (Theorem 5-5.1)
5. Greater than; (Theorem 5-5.1)
6. Less than; (Theorem 5-5.1)
7. Greater than; (Theorem 5-5.1)
8. Greater than; (Theorem 5-5.1)

Exercises

For exercises 1-9 use Theorem 5-5.1

1. Greater than
2. Less than
3. Greater than
4. Greater than

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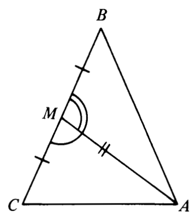
Exercises continued

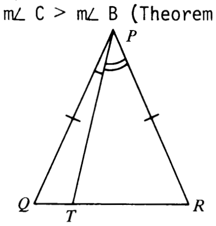
5. Greater than
6. Greater than
7. Less than
8. Greater than
9. Less than

10. AB > AC (Theorem 5-3.2)
    BM = MC (Definition 3-9)
    m∠ AMB > m∠ AMC (Theorem 5-5.2)
    ∠ AMB is obtuse (Definition 1-28, Postulate 2-11).

11.


m∠ AMC < m∠ AMB (Given)
CM = BM (Definition of Median, 3-9)
AM = AM (Reflexive property)
AB > AC (Theorem 5-5.1)

Now consider △ABC:
m∠ C > m∠ B (Theorem 5-3.1)

12.


Since PT = PT (Reflexive property), we apply Theorem 5-5.1 to △QPT and △RPT to conclude QT < RT.

13. m∠ BAD > m∠ CAD (Corollary 5-1.1b).
    In △BAD and △CAD, AB = AC and AD = AD.
    BD > CD (Theorem 5-5.1).

14. △AED ≅ △CEB (SAS)
    AD = BC (Definition 3-3)
    m∠ ADC < m∠ BCD (Definition 1-24, Definition 1-28)
    AD < BD (Theorem 5-5.1)

15. In △ABC, m∠ ABC > m∠ ACB (Theorem 5-3.1)
    Consider △PBC and △QCB, PC > QB (Theorem 5-5.1)

16. Consider △PBC and △QCB, m∠ ABC > m∠ ACB (Theorem 5-5.2)
    In △ABC, AC > AB (Theorem 5-3.2)

17. PR > RS (Corollary 5-1.1a)
    PR > QP (Postulate 2-1)
    m∠ PQR > m∠ PRQ (Theorem 5-3.1)

18. See diagram in text. Assume m∠ B = m∠ E. △ABC ≅ △DEF (SAS).
    This is impossible since AC > DF and not AC = DF.
    Now assume that m∠ B < m∠ E. AC < DF (Theorem 5-5.1).
    This, too, is impossible since AC > DF.
    m∠ B > m∠ E (Trichotomy property).

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Review Exercises

1. Greater than; (Corollary 5-1.1a)
2. Greater than; (Addition property)
3. Greater than; (Division property)
4. Less than; (Trichotomy property)
5. Greater than (Transitive property)
6. Less than (Corollary 5-1.1b)
7. m∠ M < m∠ N (Subtraction property)
8. m∠ ACD = m∠ ADC (Theorem 3-4.2)
   m∠ CAD = m∠ CAD (Reflexive property)
   m∠ BAC + m∠ CAD = m∠ EAD + m∠ CAD (Addition property)
   m∠ BAD = m∠ EAC (Substitution Postulate)
   △BAD ≅ △EAC (ASA)
   BD = CE (Definition of congruent triangles, 3-3)
   CF > CE (Corollary 5-1.1a); CF > BD (Postulate 2-1).

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9. ∠ DBE and ∠ DEB
10. ∠ DBC and ∠ DCB
11. ∠ GFC and ∠ GCF
12. ∠ GDB and ∠ DBG
13. Yes, because of Theorem 5-2.1.
14. No, because of Theorem 5-2.1.
15. Apply Theorem 5-2.1 to △ BDG:
    m∠ BGC > m∠ DBG *(1)*
    m∠ BGC > m∠ BDG *(2)*
    However, m∠ BDG > m∠ GDH (Corollary 5-1.1b)
    and m∠ BDG > m∠ HDB (Corollary 5-1.1b).
    Therefore, m∠ BGC > m∠ GDH (Transitive property) *(3)*
    and m∠ BGC > m∠ HDB (Transitive property) *(4)*
    Apply Theorem 5-2.1 to △GFC:
    m∠ BGC > m∠ GCF *(5)*
    m∠ BGC > m∠ GFC *(6)*
    However, m∠ GFC > m∠ GFJ (Corollary 5-1.1b)
    and m∠ GFC > m∠ JFC (Corollary 5-1.1b)
    Therefore, m∠ BGC > m∠ GFJ (Transitive property) *(7)*
    m∠ BGC > m∠ JFC (Transitive property) *(8)*
    Consider Theorem 5-2.1 relative to △GJF:
    m∠ BGC > m∠ GJF *(9)*
    Consider Theorem 5-2.1 relative to △GHD:
    m∠ BGC > m∠ DHG *(10)*
    Consider Theorem 5-2.1 relative to △ABF:
    m∠ GFC > m∠ A
    However, from *(6)* above, m∠ BGC > m∠ GFC
    Therefore, m∠ BGC > m∠ A (Transitive property) *(11)*
    Since m∠ DHG = m∠ BHE (Theorem 2-6.3)
    From *(10)*: m∠ BGC > m∠ BHE (Substitution Postulate, 2-1) *(12)*
    Since m∠ GJF = m∠ EJC (Theorem 2-6.3)
    From *(9)*: m∠ BGC > m∠ EJC (Substitution Postulate, 2-1) *(13)*
    Consider Theorem 5-2.1 relative to △DJE:
    m∠ EJC > m∠ DEJ
    However, from *(13)*: m∠ BGC > m∠ DEJ (Substitution postulate, 2-1) *(14)*
    Here are *14* angles of measure less than that of ∠ BGC.

16. From Corollary 5-1.1b the following angles have measure less than that of ∠ ADE:
    ∠ ADF, ∠ FDE, ∠ FDC, ∠ EDC, ∠ ADC.
    Apply Theorem 5-2.1 to △DBE:
    m∠ ADE > m∠ ABC
    m∠ ADE > m∠ DEB
    Similarly for △DFB:
    m∠ ADF > m∠ ABF (Theorem 5-2.1) *(a)*
    m∠ ADF > m∠ DFB (Theorem 5-2.1) *(b)*
    For △BDC:
    m∠ ADC > m∠ ABC (Theorem 5-2.1) *(c)*
    m∠ ADC > m∠ BCD (Theorem 5-2.1) *(d)*
    For △DGB:
    m∠ ADG > m∠ DGB (Theorem 5-2.1) *(e)*
    Since m∠ ADE is greater than that of each of the *left* members of inequalities *a, b, c, d* and *e*, m∠ ADE is also greater than each of the *right* members.

    Consider DHB:
    m∠ ADE > m∠ DHB (Theorem 5-2.1)
    Therefore since m∠ FHE = m∠ DHB (Theorem 2-6.3),
    m∠ ADE > m∠ FHE.

17. In △ACD, m∠ ACB > m∠ ADC (Theorem 5-2.1)
    In △DAE, m∠ ADC > m∠ DAE (Theorem 5-2.1)
    Therefore, m∠ ACB > m∠ DAE (Transitive property)

18. m∠ AKB > m∠ ANK (Theorem 5-2.1)
    m∠ ANK > m∠ NCB (Theorem 5-2.1)
    m∠ NCB > m∠ Q (Theorem 5-2.1)
    m∠ Q < m∠ AKB (Transitive property).

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19. ∠R, ∠P, ∠Q.
20. $\overline{NK}$, $\overline{KM}$, $\overline{MN}$
21. m∠ DBC > m∠ ABC (Corollary 5-1.1b)
    m∠ DCB < m∠ ACB (Corollary 5-1.1b)
    m∠ ABC = m∠ ACB (Theorem 3-4.2)
    m∠ DBC > m∠ DCB (Transitive property)
    DC > DB (Theorem 5-3.2).

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Review Exercises continued

22. m∠ B > m∠ C (Theorem 5-3.1)
m∠ AMC > m∠ B (Theorem 5-2.1)
m∠ AMC > m∠ C (Transitive property)
AC > AM (Theorem 5-3.2).

23. m∠ B > m∠ BAM (Theorem 5-3.1)
AM > CM (Transitive property)
m∠ C > m∠ CAM (Theorem 5-3.1)
m∠ B + m∠ C > m∠ BAM + m∠ CAM, or
m∠ B + m∠ C > m∠ BAC (Addition property, Postulate 2-10)

24. m∠ PCB > m∠ PBC (Theorem 5-3.1)
m∠ ACB > m∠ ABC (Addition property)
AB > AC (Theorem 5-3.2).

25. FC
26. BC
27. AB
28. CF
29. Yes.
30. No.
31. Yes.
32. Use Theorem 5-4.1:
$5 + 6 > x \rightarrow 11 > x$
$5 + x > 6 \rightarrow x > 1$
$6 + x > 5$ already true.
Therefore $1 < x < 11$.

33. Use Theorem 5-4.1:
$28 + 28 > x \rightarrow 56 > x$
$28 + x > 28 \rightarrow x > 0$
Therefore $0 < x < 56$.

34. The shortest of the segments is $\overline{PC}$ (Theorem 5-4.2).

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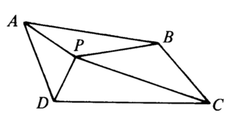
35. *Given:* △ABC with AB = AC
AB + AC > BC (Theorem 5-4.1)
2AB > BC (Postulate 2-1)
AB > ½BC (Division property).

36.



Given quadrilateral ABCD with interior point P.
AP + BP > AB (Theorem 5-4.1)
AP + DP > AD (Theorem 5-4.1)
DP + PC > DC (Theorem 5-4.1)
BP + PC > BC (Theorem 5-4.1)
2(AP + BP + DP + PC) > AB + AD + DC + BC (Addition property)
AP + BP + CP + DP > ½(AB + AD + DC + BC) (Division property).

37. AD + DB > AB (Theorem 5-4.1)
AD + DC > AC (Theorem 5-4.1)
2 AD + BC > AB + AC (Addition property).

38. Less than
39. Less than
40. Less than
41. Greater than
42. $\overline{AD} \cong \overline{DC}$ and BD > AC (Given)
$\overline{BC} \cong \overline{BC}$ (Theorem 3-1.6)
m∠ DCB > m∠ ABC (Theorem 5-5.2)

Chapter Test

1. Since the exterior angle must have greater measure than either remote interior angle, the exterior angle at R must have at least measure 64.

2. Use Theorem 5-3.1 to get:
(1) ∠ KMN has the greatest measure, and
(2) ∠ KNM has the smallest measure.

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Chapter Test continued

3. In △AED, $\overline{AE}$ is longest side (Theorem 5-5.2)
In △ADB, $\overline{AD}$ is longest side (Theorem 5-5.2)
In △BCD, $\overline{BD}$ is longest side (Theorem 5-5.2)
AE > AD (Theorem 5-3.2)
AD > BD (Theorem 5-3.2)
AE > AD > $\overline{BD}$ (Transitive property)
Therefore $\overline{AE}$ is longest segment in the diagram.

4. In △AED, $\overline{AD}$ is shortest side (Theorem 5-3.2)
In △ADB, $\overline{BD}$ is shortest side (Theorem 5-3.2)
In △BCD, $\overline{DC}$ is shortest side (Theorem 5-3.2)
AD > BD (Theorem 5-3.2)
BD > DC (Theorem 5-3.2)
AD > BD > $\overline{DC}$ (Transitive property)
Therefore $\overline{DC}$ is shortest segment in the diagram.

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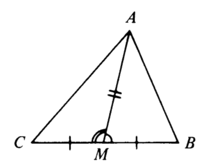
5. No; Theorem 5-4.1
6. Yes; Theorem 5-4.1
7. Yes; Theorem 5-4.1
8. No; Theorem 5-4.1
9. Let x be the third side of the triangle;
17, 17, x are the third sides of the triangle
where $x > 0$
$17 + 17 > x$, $34 > x$ (Theorem 5-4.1)
$17 + x > 17$, $x > 0$
Therefore $0 < x < 34$

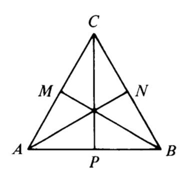
10.



CM = MB (Definition of Median, 3-9)
AM = AM
For △AMB and △AMC:
AB > AC (Theorem 5-5.1)

11. Show that two of the three possibilities (of <, >, or =) are not true.

12.

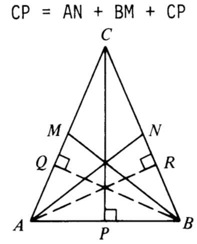


Case I.

If AB = BC = AC,

Altitude		Median	
$\overline{AN}$	=	$\overline{AN}$	$\overline{AN}$ is an altitude and a median
$\overline{BM}$	=	$\overline{BM}$	$\overline{BM}$ is an altitude and a median
$\overline{CP}$	=	$\overline{CP}$	$\overline{CP}$ is an altitude and a median

BN = BM = CP
AN + BM + CP = AN + BM + CP (Addition property)



Case II.

If AC = BC

CP = CP
AN > AR (Theorem 5-4.2)
BM > BQ (Theorem 5-4.2)

Altitude	Median
$\overline{CP}$	$\overline{CP}$
$\overline{AR}$	$\overline{AN}$
$\overline{BQ}$	$\overline{BM}$

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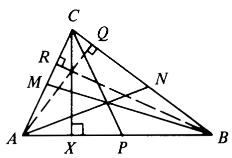
12. *continued*

CP + AN + BM > CP + AR + BQ (Addition Property of Inequality).



Case III.

If AC ≠ BC ≠ AB

	Altitude	Median
CP > CX (Theorem 5-4.2)	$\overline{CX}$	$\overline{CP}$
AN > AQ (Theorem 5-4.2)	$\overline{AQ}$	$\overline{AN}$
BM > BR (Theorem 5-4.2)	$\overline{BR}$	$\overline{BM}$

CP + AN + BM > CX + AQ + BR (Addition Property of Inequality).

13. $m\angle$ BAD = $m\angle$ BCD (see figure)(Theorem 5-5.1)
$m\angle$ ACD > $m\angle$ CAD (Theorem 5-5.1)
$m\angle$ BAC > $m\angle$ BCA (Subtraction property)
BC > AB (Theorem 5-3.2).

14. $m\angle$ APQ > $m\angle$ B (Theorem 5-2.1)
$m\angle$ B = $m\angle$ C (Theorem 3-4.2)
$m\angle$ C > $m\angle$ CQD (Theorem 5-2.1)
$m\angle$ APQ > $m\angle$ CQD (Transitive property)
$m\angle$ APQ > $m\angle$ AQP (Postulate 2-1)
AP < AQ (Theorem 5-3.2).

15. $\overline{AP} \cong \overline{AP}$ (Theorem 3-1.6)
In △APB and △APD:
$\overline{BP} < \overline{PD}$ (Theorem 5-5.1)
$\overline{CP} \cong \overline{CP}$ (Theorem 3-1.6)

In △BCP and △PCD:
$\angle$ BCP < $\angle$ PCD (Theorem 5-5.2)

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Mathematical Excursion

1. The sum is 2.
2. $|1 - n/(n+1)| = |(n + 1 - n)/(n+1)| = |1/(n+1)| < M$ for $n < (1/M - 1)$.
3. No. We cannot make any part of the sequence approach a predetermined number.
4. Divide the circle into isosceles triangles whose sides are radii of the circle. The sum of the bases of these triangles approaches the circumference as the length of each base decreases. Use the areas of these triangles to approach the area of the circle.
5. Divide the rectangle by a commensurable unit. The base has b units and the altitude a units. Consequently, α and β above are both zero. The area is $a \cdot b$.

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Class Exercises

1. No, skew lines fit this.
2. More restrictive than Definition 6-1.
3. No, might be collinear.
4. Adequate definition.
5. All right, if we agree on what is meant by "share."

Exercises

1. $\overline{SR}$, $\overline{TW}$, $\overline{VU}$
2. $\overline{QV}$, $\overline{PU}$, $\overline{ST}$
3. $\overline{TU}$, $\overline{SP}$, $\overline{RQ}$
4. $\overline{QV}$, $\overline{PU}$, $\overline{RW}$
5. $\overline{TU}$, $\overline{SP}$, $\overline{RQ}$
6. $\overline{TW}$, $\overline{PQ}$, $\overline{SR}$
7. No, they intersect at B.

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Exercises continued

8. Because they always intersect.
9. Corresponding
10. Alternate interior.
11. Alternate interior.
12. Interior angles on the same side of the transversal.
13. $\angle$ 7 and $\angle$ 4; $\angle$ 8 and $\angle$ 3.
14. $\angle$ 15 and $\angle$ 20; $\angle$ 13 and $\angle$ 19.
15. $\angle$ 27 and $\angle$ 30; $\angle$ 28 and $\angle$ 29.
16. $\angle$ 1 and $\angle$ 3; $\angle$ 7 and $\angle$ 6; $\angle$ 2 and $\angle$ 4; $\angle$ 8 and $\angle$ 5.
17. $\angle$ 17 and $\angle$ 13; $\angle$ 20 and $\angle$ 16; $\angle$ 18 and $\angle$ 15; $\angle$ 14 and $\angle$ 19
18. $\angle$ 25 and $\angle$ 29; $\angle$ 27 and $\angle$ 32; $\angle$ 26 and $\angle$ 30; $\angle$ 28 and $\angle$ 31
19. $\angle$ 8 and $\angle$ 4; $\angle$ 7 and $\angle$ 3.
20. $\angle$ 20 and $\angle$ 13; $\angle$ 19 and $\angle$ 15.
21. $\angle$ 27 and $\angle$ 29; $\angle$ 28 and $\angle$ 30.
22. $\angle$ 2 and $\angle$ 27; $\angle$ 8 and $\angle$ 25.
23. $\angle$ 4 and $\angle$ 32; $\angle$ 5 and $\angle$ 29.
24. $\angle$ 21 and $\angle$ 11; $\angle$ 10 and $\angle$ 24.
25. $\angle$ 2 and $\angle$ 26; $\angle$ 8 and $\angle$ 28; $\angle$ 7 and $\angle$ 27; $\angle$ 1 and $\angle$ 25.
26. $\angle$ 3 and $\angle$ 29; $\angle$ 4 and $\angle$ 30; $\angle$ 5 and $\angle$ 31; $\angle$ 6 and $\angle$ 32.
27. $\angle$ 9 and $\angle$ 21; $\angle$ 10 and $\angle$ 23; $\angle$ 12 and $\angle$ 24; $\angle$ 11 and $\angle$ 22.
28. $\angle$ 2 and $\angle$ 25; $\angle$ 8 and $\angle$ 27.
29. $\angle$ 4 and $\angle$ 29; $\angle$ 5 and $\angle$ 32.
30. $\angle$ 10 and $\angle$ 21; $\angle$ 11 and $\angle$ 24.
31. $\overleftrightarrow{EF}$ 32. $\overleftrightarrow{HG}$ 33. $\overleftrightarrow{HG}$ 34. $\overleftrightarrow{CD}$ 35. $\overleftrightarrow{EF}$
36. $\overleftrightarrow{HG}$ 37. $\overleftrightarrow{MN}$ 38. $\overleftrightarrow{HG}$

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39. $\angle$ 4; $\angle$ 11
40. $\angle$ 28; $\angle$ 22
41. $\angle$ 14; $\angle$ 19; $\angle$ 24
42. $\angle$ 32

43. *Given:* $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$; $\overleftrightarrow{AB} \perp \overleftrightarrow{EF}$.
Prove: $\overleftrightarrow{CD} \perp \overleftrightarrow{EF}$

Assume $\overleftrightarrow{CD}$ not perpendicular to $\overleftrightarrow{EF}$.
P is in $\overleftrightarrow{CD}$ so that $\overleftrightarrow{PH} \perp \overleftrightarrow{EF}$ (Theorem 4-4.2)
$\overleftrightarrow{PH} \parallel \overleftrightarrow{AB}$ (Theorem 6-1.1)
$\overleftrightarrow{PH}$ and $\overleftrightarrow{CPD}$ cannot both be parallel to $\overleftrightarrow{AB}$ (Postulate 6-1)
Our assumption is false and $\overleftrightarrow{CD} \perp \overleftrightarrow{EF}$.

44. *Given:* $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$; $\overleftrightarrow{EF} \parallel \overleftrightarrow{CD}$.
Prove: $\overleftrightarrow{AB} \parallel \overleftrightarrow{EF}$.

Assume $\overleftrightarrow{AB} \nparallel \overleftrightarrow{EF}$.
$\overleftrightarrow{AB}$ meets $\overleftrightarrow{EF}$ at P
$\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$ contain P and are not parallel to $\overleftrightarrow{CD}$
Impossible (Postulate 6-1)
Thus, $\overleftrightarrow{AB} \parallel \overleftrightarrow{EF}$.

45. *Given:* $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$; $\overleftrightarrow{EF} \perp \overleftrightarrow{AB}$; $\overleftrightarrow{GH} \perp \overleftrightarrow{CD}$
Prove: $\overleftrightarrow{EF} \parallel \overleftrightarrow{GH}$
$\overleftrightarrow{GH} \perp \overleftrightarrow{AB}$ (Corollary 6-1.1b)
$\overleftrightarrow{EF} \parallel \overleftrightarrow{GH}$ (Theorem 6-1.1).

46. *Given:* $\overleftrightarrow{AB}$ is not parallel to $\overleftrightarrow{CD}$, intersecting at H
$\overleftrightarrow{EG} \perp \overleftrightarrow{CD}$
$\overleftrightarrow{JF} \perp \overleftrightarrow{AB}$.
Prove: $\overleftrightarrow{GE}$ is not parallel to $\overleftrightarrow{JF}$.

Assume $\overleftrightarrow{GE} \parallel \overleftrightarrow{JF}$.
$\overleftrightarrow{CD} \parallel \overleftrightarrow{AB}$ (Corollary 6-1.1d)
This contradicts the Given
Thus, $\overleftrightarrow{GE}$ is not parallel to $\overleftrightarrow{JF}$.

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Class Exercises

1. Yes; Corollary 6-2.1a.
2. Yes; Corollary 6-2.1b.
3. Yes; Theorem 6-2.1.
4. $\overline{AB} \parallel \overline{CD}$; Theorem 6-2.1.
5. $\overline{AD} \parallel \overline{BCE}$; Theorem 6-2.1.
6. $\overline{AB} \parallel \overline{FE}$ and $\overline{AC} \parallel \overline{FD}$; Theorem 6-2.1.

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Exercises

1. $\overleftrightarrow{BC} \parallel \overleftrightarrow{FE}$; Theorem 6-2.1
2. $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$; Corollary 6-2.1a.
3. $\overleftrightarrow{AC} \parallel \overleftrightarrow{BE}$; Corollary 6-2.1a.
4. $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$; Theorem 6-2.1
5. $\overleftrightarrow{BC} \parallel \overleftrightarrow{FD}$; Theorem 6-2.1
6. $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$; Corollary 6-2.1b

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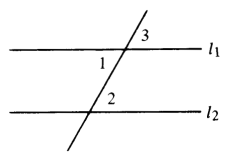
Exercises continued

7. $\angle BEC \cong \angle BCE$ (Theorem 3-4.2)
   $\angle ABC \cong \angle DCF$ (Transitive property)
   $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$ (Corollary 6-2.1a).

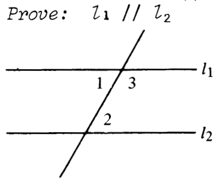
8. $\overleftrightarrow{CB} \parallel \overleftrightarrow{DE}$ (Theorem 6-2.1)
   $\angle ABD \cong \angle FDB$ (Postulate 2-10)
   $\overleftrightarrow{AB} \parallel \overleftrightarrow{DE}$ (Theorem 6-2.1).

9. *Given:* $\angle 2 \cong \angle 3$
   Prove: $l_1 \parallel l_2$



   $\angle 1 \cong \angle 3$ (Theorem 3-1.5)
   $\angle 1 \cong \angle 2$ (Transitive property)
   $l_1 \parallel l_2$ (Theorem 6-2.1)

10. *Given:* $\angle 2$ is supplementary to $\angle 3$.
    Prove: $l_1 \parallel l_2$



    $\angle 1$ is supplementary to $\angle 3$ (Postulate 1-6)
    $\angle 1 \cong \angle 2$ (Theorem 3-1.4)
    $l_1 \parallel l_2$ (Theorem 6-2.1)

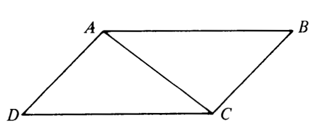
11. $m\angle ACE = 126$ (Definition 1-29).
    $m\angle CAD = 54$ (Definition 1-29).
    $\overleftrightarrow{AD} \parallel \overleftrightarrow{CE}$ (Corollary 6-2.1b).

12. BC = QP (Addition property)
    $\triangle ABC \cong \triangle RQP$ (SAS)
    $\angle ACB \cong \angle RPQ$ (Definition 3-3)
    $\overline{AC} \parallel \overline{PR}$ (Theorem 6-2.1).

13. $\angle EDF \cong \angle BCA$ (Theorem 2-6.2)
    $\overline{AC} \cong \overline{FD}$ (Addition property)
    $\triangle ABC \cong \triangle FED$ (SAS)
    $\angle A \cong \angle F$ (Definition 3-3)
    $\overline{AB} \parallel \overline{EF}$ (Theorem 6-2.1)

14. $\overline{AM} \cong \overline{DM}$ (Definition 1-15)
    $\overline{BM} \cong \overline{EM}$ (Definition 1-15)
    $\angle AMB \cong \angle DME$ (Theorem 2-6.3)
    $\triangle AMB \cong \triangle DME$ (SAS)
    $\angle A \cong \angle MDE$ (Definition 3-3)
    $\overleftrightarrow{ABC} \parallel \overleftrightarrow{DE}$ (Theorem 6-2.1).

15. *Given:* In quadrilateral ABCD, $\overline{AB} \cong \overline{CD}$; $\overline{AD} \cong \overline{CB}$.
    Prove: $\overline{AB} \parallel \overline{CD}$
    $\overline{AD} \parallel \overline{CB}$.
    Draw $\overline{AC}$.



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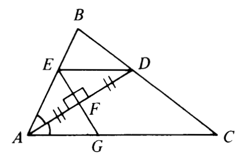
Exercises continued

15. *continued*

    $\triangle ABC \cong \triangle CDA$ (SSS)
    $\angle BAC \cong \angle DCA$ (Definition 3-3)
    $\overline{AB} \parallel \overline{CD}$ (Theorem 6-2.1)
    Similarly, $\overline{AD} \parallel \overline{CB}$.

16. $\triangle PHA \cong \triangle PHB$ (SAS)
    $\angle PBA \cong \angle PAB$ (Definition 3-3)
    $\angle PAB \cong \angle BAC$ (Definition 1-29)
    $\angle PBA \cong \angle BAC$ (Transitive property)
    $\overline{PB} \parallel \overline{AC}$ (Theorem 6-2.1)

17. The perpendicular bisector of $\overline{AD}$ meets $\overline{AD}$ at F.
    $\triangle AFE \cong \triangle DFE$ (SAS)
    $\angle ADE \cong \angle DAE$ (Definition 3-3)
    $\angle ADE \cong \angle CAD$ (Transitive property)
    $\overline{AC} \parallel \overline{DE}$ (Theorem 6-2.1)



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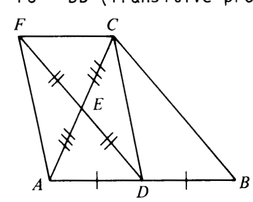
Exercises continued

18. $\overline{CE} \cong \overline{AE}$ (Definition 1-15)
    $\angle FEA \cong \angle DEC$ (Theorem 2-6.3)
    $\triangle AEF \cong \triangle CED$ (SAS)
    $\angle AFE \cong \angle CDE$ (Definition 3-3)
    $\overline{AF} \parallel \overline{CD}$ (Theorem 6-2.1)
    $\triangle FEC \cong \triangle DEA$ (SAS)
    $\angle CFE \cong \angle ADE$ (Definition 3-3)
    $\overline{CF} \parallel \overline{AB}$ (Theorem 6-2.1)
    $\overline{FC} \cong \overline{DA}$)Definition 3-3)
    $\overline{DA} \cong \overline{DB}$ (Definition 3-9)
    $\overline{FC} = \overline{BD}$ (Transitive property).



19. Consider $\overline{DB}$.
    $\triangle AED \cong \triangle CFB$ (SAS)
    $\overline{ED} \cong \overline{FB}$ (Definition 3-3)
    $\angle AED \cong \angle CFB$ (Definition 3-3)
    $\angle BED \cong \angle DFB$ (Theorem 3-1.4)
    $\triangle BED \cong \triangle DFB$ (SAS)
    $\angle EDB \cong \angle FBD$ (Definition 3-3)
    $\overline{DE} \parallel \overline{BF}$ (Theorem 6-2.1)

Class Exercises

1. $m\angle$ 65.
2. $m\angle$ 65.

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Class Exercises continued

3. Congruent.
4. Corresponding angles.
5. $m\angle$ 115.
6. Supplementary.
7. Interior angles on same side of transversal.
8. The conclusions are stated in Corollary 6-3.1a and Corollary 6-3.1b.

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Exercises

1. 40; Corollary 6-3.1a.

2. $67\frac{1}{2}$; Corolllary 6-3.1b.

3. Since $\overline{HB}$ // $\overline{DG}$, m∠ DCA = m∠ ABC = 45 (Theorem 6-2.1)
   Since $\overline{AF}$ // $\overline{EC}$, m∠ FAC = m∠ ECA = 45 + 25 = 70.
   By subtraction m∠ FAB = 25.

4. $3x - 2 + 7x + 2 = 180.$
   $x = 18.$

5. $13x + 2 = 15x - 6$
   $x = 4.$

6. $15x - 7 = 10x + 3$
   $x = 2.$

7. ∠ D is supplementary to ∠ A (Corollary 6-3.1b)
   ∠ B is supplementary to ∠ A (Corollary 6-3.1b)
   ∠ D ≅ ∠ B (Theorem 3-1.4).

8. ∠ A ≅ ∠ C (Theorem 6-3.1)
   ∠ C ≅ ∠ D (Theorem 6-3.1)
   ∠ A ≅ ∠ D (Transitive property)

9. ∠ A ≅ ∠ C (Theorem 6-3.1)
   ∠ B ≅ ∠ D (Theorem 6-3.1)
   △AEB ≅ △CED (ASA)
   $\overline{AE}$ ≅ $\overline{CE}$ (Definition 3-3)
   $\overline{BE}$ ≅ $\overline{DE}$ (Definition 3-3)

10. ∠B ≅ ∠ C (Theorem 6-3.1)
    ∠ AFB ≅ ∠ DEC (Theorem 6-3.1)
    $\overline{BF}$ ≅ $\overline{CE}$ (Closure property)
    △AFB ≅ △DEC (ASA)
    $\overline{AF}$ ≅ $\overline{DE}$ (Definition 3-3)

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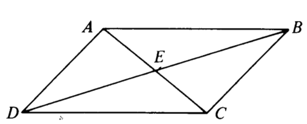
Exercises continued

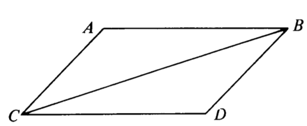
11. ∠ APE ≅ ∠ CEP (Theorem 6-3.1)
    ∠ CEP ≅ ∠ FEP (Definition 1-29)
    ∠ APE ≅ ∠ FEP (Transitive property)
    △PFE is isosceles (Theorem 3-4.3, Definition 3-12).

12. ∠ FAD ≅ ∠ FBC (Corollary 6-3.1a)
    ∠ DAC ≅ ∠ BCA (Theorem 6-3.1)
    ∠ B ≅ ∠ C (Theorem 3-4.2)
    ∠ FAD ≅ ∠ CAD (Transitive property)
    $\overrightarrow{AD}$ bisects ∠ FAC (Definition 1-29)

13. $\overline{AC}$ and $\overline{BD}$ intersect at E.
    ∠ BAC ≅ ∠ DCA (Theorem 6-3.1)
    ∠ ABD ≅ ∠ CDB (Theorem 6-3.1)
    ∠ BCA ≅ ∠ DAC (Theorem 6-3.1)
    △ABC ≅ △CDA (ASA)
    $\overline{AB}$ ≅ $\overline{CD}$ (Definition 3-3)
    △AEB ≅ △CED (ASA)
    $\overline{AE}$ ≅ $\overline{CE}$ (Definition 3-3)
    $\overline{DE}$ ≅ $\overline{BE}$ (Definition 3-3)
    $\overline{AC}$ and $\overline{BD}$ bisect each other (Definition 1-15).



14.



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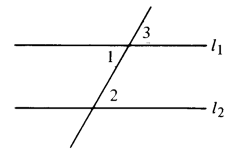
Exercises continued

14. *continued*

    ∠ ABD ≅ ∠ CDB (Theorem 6-3.1)
    ∠ ADB ≅ ∠ CBD (Theorem 6-3.1)
    △ABD ≅ △CDB (ASA).

15. *Given:* l_1 // l_2

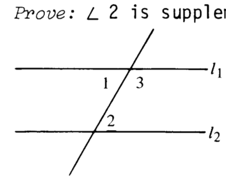
    Prove: ∠ 2 ≅ ∠ 3



    l_1 // l_2 (Given)
    ∠ 1 ≅ ∠ 2 (Theorem 6-3.1)
    ∠ 1 ≅ ∠ 3 (Theorem 3-1.5)
    ∠ 2 ≅ ∠ 3 (Transitive property)

16. *Given:* l_1 // l_2

    Prove: ∠ 2 is supplementary to ∠ 3.



    l_1 // l_2 (Given)
    ∠ 1 ≅ ∠ 2 (Theorem 6-3.1)
    ∠ 1 is supplementary to ∠ 3 (Postulate 1-6)
    ∠ 2 is supplementary to ∠ 3 (Substitution Postulate 2-1)

17. x = 35 (Corollary 6-3.1a)
    ∠ FEG = 90 (Given)
    ∠ FEB + ∠ BEG = 90
    x + ∠ BEG = 90
    35 + ∠ BEG = 90
    ∠ BEG = 55
    ∠ HEG + ∠ BEG = 180 (Postulate 1-6)
    y + 55 = 180
    y = 125

18. m∠ AED = m∠ FCE = 55 (Corollary 6-3.1a)
    x = m∠ FCE = 55 (Theorem 6-3.1)
    ∠ D is supplementary to ∠ BCD (Corollary 6-3.1b)
    ∠ B is supplementary to ∠ BCD (Corollary 6-3.1b)
    ∠ D ≅ ∠ B (Theorem 2-6.2)
    m∠ D = y = m∠ B = 35

19. m∠ BED = x = m∠ BEF + m∠ FED
    ∠ ABE is supplementary to ∠ BEF (Corollary 6-3.1b)
    (90 + 25) + m∠ BEF = 180
    m∠ BEF = 65
    ∠ CDE is supplementary to ∠ FED (Corollary 6-3.1b)
    (90 + 25) + m∠ FED = 180
    m∠ FED = 65
    Therefore, x = 130 = m∠ BED
    y + m∠ BED = 180
    y + 130 = 180
    y = 50

20. m∠ AEC = x = m∠ AEG + m∠ CEG
    m∠ AEG = m∠ BAE = 65 (Theorem 6-3.1)
    m∠ CEG = m∠ DCE = 35 (Theorem 6-3.1)
    Therefore, x = 65 + 35 = 100
    ∠ FCE ≅ ∠ AEC (Theorem 6-3.1)
    y + 35 = 100
    y = 65

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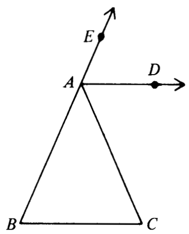
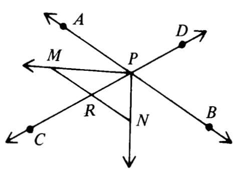
Exercises continued

21. *Given:* $\overleftrightarrow{EAB}$
$\overrightarrow{AD}$ bisects exterior ∠ EAC of △ABC
$\overrightarrow{AD}$ // $\overline{BC}$.

Prove: △ABC is isosceles.
∠ EAD ≅ ∠ EBC (Corollary 6-3.1a)
∠ DAC ≅ ∠ ACB (Theorem 6-3.1)
∠ EAD ≅ ∠ DAC (Definition 1-2)
∠ ACB ≅ ∠ EBC (Transitive property)
△ABC is isosceles (Theorem 3-4.3).

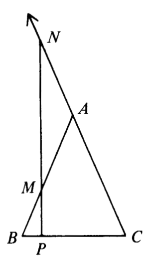


22.



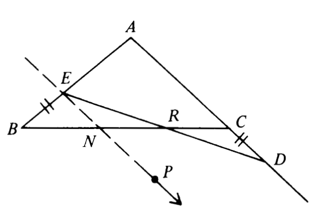
∠ BPN ≅ ∠ PNR (Theorem 6-3.1)
∠ NPC ≅ ∠ PNR (Transitive property)
$\overline{PR}$ = $\overline{NR}$ (Theorem 3-4.3)
Similarly, $\overline{MR}$ ≅ $\overline{PR}$
$\overline{MR}$ ≅ $\overline{NR}$ (Transitive property).

23.



Let $\overline{AH}$ ⊥ $\overline{NP}$.
$\overline{AH}$ // $\overline{BC}$ (Theorem 6-1.1)
∠ NAH ≅ ∠ C (Corollary 6-3.1a)
∠ HAM ≅ ∠ B (Theorem 6-3.1)
∠ C ≅ ∠ B (Theorem 3-4.2)
∠ NAH ≅ ∠ HAM (Transitive property)
△NAH ≅ △MAH (ASA)
$\overline{NA}$ ≅ $\overline{MA}$ (Definition 3-3)
△AMN is isosceles (Definition 3-12).

24.



Draw $\overleftrightarrow{EP}$ // $\overleftrightarrow{AD}$
$\overleftrightarrow{EP}$ intersects $\overline{BC}$ at N.
∠ ADE ≅ ∠ DEN (Theorem 6-3.1)
∠ ACB ≅ ∠ ENB (Corollary 6-3.1a)
∠ ACB ≅ ∠ ABC (Theorem 3-4.2)
∠ ENB ≅ ∠ ABC (Transitive property)
$\overline{EB}$ ≅ $\overline{EN}$ (Theorem 3-4.3)
$\overline{EN}$ ≅ $\overline{DC}$ (Transitive property)

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24. *continued*

∠ ENC ≅ ∠ BCD (Theorem 6-3.1)
△ENR ≅ △DCR (ASA)
$\overline{ER}$ ≅ $\overline{DR}$ (Definition 3-3)

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1. m∠ A = 100; Theorem 6-4.2
2. m∠ A = 110; Theorem 6-4.2
3. m∠ A + m∠ C = 128 (Theorem 6-4.2).
Therefore m∠ A = 64.

4. m∠ B + m∠ A = 132 (Theorem 6-4.2)
2m∠ A + m∠ A = 132 (Substitution postulate, 2-1)
3m∠ A = 132
m∠ A = 44

5. m∠ A = 90 - 20 = 70 (Corollary 6-4.2b)

6. Applying Theorem 6-4.2 to △ADC we get
m∠ ADC = 180 - (29 + 40)
Therefore x = 111.
Applying Theorem 6-4.1 to △ABD we get
m∠ ADC = y + 57
x = y + 57
111 = y + 57
y = 54

7. Since △ABC is isosceles, m∠ ABC = m∠ ACB (Theorem 3-4.2)
Therefore m∠ ABC = x.
By Theorem 6-4.2: $2x + 48 = 180$,
and $x = 66$
$y = x + 48$ (Theorem 6-4.1)
$y = 66 + 48 = 114$.

8. Since △ABC is equilateral:
m∠ A = m∠ B = m∠ C (Corollary 3-4.2a)
$3x = 180$; and
$x = 60$
$x = y = 60$.

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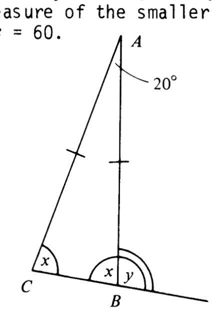
1. m∠ B = m∠ C (Theorem 3-4.2)
m∠ A + m∠ B + m∠ C = 180 (Theorem 6-4.2)
m∠ A + 2m∠ B = 180 (Substitution postulate, 2-1)
m∠ A + 2 (60) = 180
m∠ A = 60

Exercises 2-20 are done the same way as Exercise 1.

2. 61
3. 34
4. 45
5. $12\frac{1}{2}$
6. $53\frac{1}{2}$
7. $90 - x$
8. $\frac{180 - 5x}{2}$
9. $95 - 2x$
10. $\frac{177 - 7x}{2}$
11. 60
12. 130
13. 90
14. 178
15. 1
16. $180 - 2x$
17. $180 - 6x$
18. $4x - 180$
19. $186 - 10x$
20. $540 - 14x$
21. By Corollary 6-4.2b, $x + 2x = 90$, where x represents the measure of the smaller acute angle. Then $x = 30$, and $2x = 60$.

22.



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22. *continued*

$x + x + 20 = 180$ (Theorem 6-4.2)
$x = 80$
$x + y = 180$
$y = 100$

Exercises 23 - 26 are done the same way as Exercise 22.

23. 120
24. 135
25. $147\frac{1}{2}$
26. $180 - 4x$

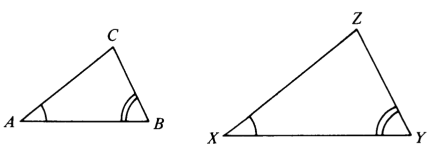
27. $m\angle A = m\angle ABC = x$ (Theorem 3-4.2)
$m\angle A + m\angle ABC = 90$ (Corollary 6-4.2b)
$x + x = 90$
$x = 45$
$y = m\angle A + m\angle C$ (Theorem 6-4.1)
$y = 45 + 90 = 135$

28. $x + 53 + 90 = 180$ (Theorem 6-4.2)
$x = 37$
$x + y = 90$ (Corollary 6-4.2b)
$37 + y = 90$
$y = 53$

29. $m\angle DAC = m\angle DCA = x$ (Theorem 3-4.2)
$m\angle BDA = m\angle DAC + m\angle DCA = 2x$ (Theorem 6-4.1)
$m\angle B = m\angle BDA = 2x$ (Theorem 3-4.2)
In $\triangle ABD$, $4x + y = 180$ (Theorem 6-4.2)
Then $y = 180 - 4x$ *(I)*
$\overleftrightarrow{EAC}$, $m\angle EAB + m\angle BAC = 180$
$87 + m\angle BAD + m\angle DAC = 180$
$87 + y + x = 180$
$x + y = 93$
Then $y = 93 - x$ *(II)*
Equating equations *(I)* and *(II)* gives us
$180 - 4x = y = 93 - x$
$3x = 87$
$x = 29$
$y = 64$

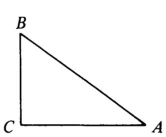
30. *Given:* $\triangle ABC$ and $\triangle XYZ$
$\angle A \cong \angle X$
$\angle B \cong \angle Y$

Prove: $\angle C \cong \angle Z$



$m\angle A + m\angle B + m\angle C = 180$ (Theorem 6-4.2)
$m\angle X + m\angle Y + m\angle Z = 180$ (Theorem 6-4.2)
$m\angle C - m\angle Z = 0$ (Subtraction property)
$m\angle C = m\angle Z$ (Addition property)

31. *Given:* Right $\triangle ABC$
$m\angle C = 90$
Prove: $\angle A + \angle B = 90$



$m\angle A + m\angle B + m\angle C = 180$ (Theorem 6-4.2)
$m\angle A + m\angle B + 90 = 180$ (Substitution postulate, 2-1)
$m\angle A + m\angle B = 90$
Therefore $\angle A$ is complementary to $\angle B$

32. *Given:* $\triangle ABC$ is isosceles.
Angle bisectors $\overline{CD}$ and $\overline{BE}$ intersect at F.

Prove: $m\angle BFC = m\angle ACG$

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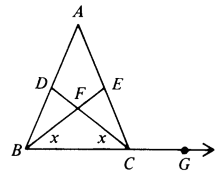
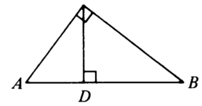
32. *continued*



Let $m\angle FBC = m\angle FCB = x$.
$m\angle BFC = 180 - 2x$ (Theorem 6-4.2)
$m\angle FCE = x$ (Definition 3-8)
$m\angle ACG = 180 - 2x$ (Definition 1-28)
$m\angle BFC = m\angle ACG$ (Transitive property)

33. If x and y are the measures of two angles of the triangle, then the third angle has measure $x + y$.
$x + y + (x + y) = 180$ (Theorem 6-4.2)
$2(x + y) = 180$ (Distributive property)
$x + y = 90$ (Division property)
The triangle is a right triangle (Definition 1-24).

34.



$\angle DCB$ is complementary to $\angle DCA$ (Definition 1-27)
$\angle A$ is complementary to $\angle DCA$ (Corollary 6-4.2b)
$\angle DCB \cong \angle A$ (Theorem 3-1.3).

35. $\triangle DAE \cong \triangle EBC$ (SAS)
$\overline{DE} \cong \overline{EC}$ (Definition 3-3)
$\triangle DEC$ is isosceles (Definition 3-12)
$\angle ADE \cong \angle BEC$ (Definition 3-3)
$m\angle AED + m\angle ADE = 90$ (Corollary 6-4.2b)
$m\angle BEC + m\angle AED = 90$ (Transitive property)
$m\angle BEC + m\angle AED + m\angle DEC = 180$ (Postulate 2-10, Definition 1-28)
$m\angle DEC = 90$ (Subtraction property)
$\triangle DEC$ is an isosceles right triangle (Definition 1-32, Definition 3-12).

36. Consider $\overrightarrow{PG} \parallel \overrightarrow{AE}$.
$\angle ACP \cong \angle CPG$ (Theorem 6-3.1)
$\angle APC \cong \angle ACP$ (Theorem 3-4.2)
$\angle APC \cong \angle CPG$ (Transitive property)
Similarly, $\angle GPD \cong \angle DPB$ (Definition 1-28)
$m\angle APC + m\angle CPG + m\angle DPB = 180$ (Definition 1-28)
$2m\angle CPG + 2m\angle GPD = 180$ (Postulate 2-1)
$m\angle 3 + m\angle 4 = 90$.

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37. $m\angle K + m\angle N + m\angle M = 180$ (Theorem 6-4.2)
$(x + 10) + (2x) + (2x - 30) = 180$ (Substitution Postulate, 2-1)
$x = 40$
$m\angle K = x + 10 = 50$
$m\angle N = 2x = 80$
$m\angle M = 2x - 30 = 50$
$\triangle KNM$ is isosceles.

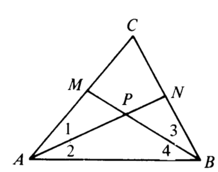
Exercises 38-40 are done in the same way as Exercise 37.

38. $m\angle K = 90$, $m\angle N = 45$, $m\angle M = 45$; isosceles right.

39. $m\angle K = 50$, $m\angle N = 40$, $m\angle M = 90$; right.

40. $m\angle K = 30$, $m\angle N = 90$, $m\angle M = 60$; right.

41.



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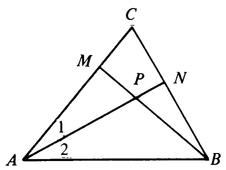
41. *continued*

m∠ CAB = 50, m∠ CBA = 60
m∠ 1 = m∠ 2 = 25 (Definition 3-8)
m∠ 3 = m∠ 4 = 30 (Definition 3-8)
m∠ MPB = m∠ 2 + m∠ 4 = 55 (Theorem 6-4.1)

42.



m∠ CAB = 50, m∠ CBA = 60
m∠ CNA = m∠ CBA + m∠ 2 (Theorem 6-4.1)
90 = 60 + m∠ 2
m∠ 2 = 30
m∠ 1 + m∠ 2 = m∠ CAB
m∠ 1 + 30 = 50
m∠ 1 = 20
m∠ 1 + m∠ MPA = 90 (Corollary 6-4.2b)
20 + m∠ MPA = 90
m∠ MPA = 70 the acute angle.

43. m∠ ABC = m∠ ACB (Theorem 3-4.2)
m∠ ABC + m∠ ACB + m∠ A = 180 (Theorem 6-4.2)
2(m∠ ACB) + 48 = 180
m∠ ACB = 66
y + m∠ ACB = 90 (Corollary 6-4.2b)
y + 66 = 90
y = 24
m∠ ABE + m∠ AEB + m∠ A = 180
m∠ ABE + 90 + 48 = 180
m∠ ABE = 42
x = m∠ ABE + m∠ FDB (Theorem 6-4.1)
x = 42 + 90 = 132

44. Draw $\overline{BC}$, m∠ ACD = x = m∠ ACB + m∠ DCB
m∠ DBC = m∠ DCB (Theorem 3-4.2)
m∠ DBC + m∠ DCB + m∠ D = 180
2(m∠ DCB) + 60 = 180
m∠ DCB = 60 = m∠ DBC
m∠ ABC = m∠ ACB (Theorem 3-4.2)
m∠ ABC + m∠ ACB = 90 (Corollary 6-4.2b)
2(m∠ ABC) = 90
m∠ ABC = 45 = m∠ ACB
m∠ ACD = x = m∠ ACB + m∠ DCB
x = 45 + 60
x = 105
y + m∠ DBC + m∠ ABC = 180
y + 60 + 45 = 180
y = 75

45. 70 + 30 + m∠ ECB = 180
m∠ ECB (or m∠ ACB) = 80
m∠ BAC + m∠ ACB + m∠ ABC = 180 (Theorem 6-4.2)
50 + 80 + m∠ ABC = 180
m∠ ABC = 50
m∠ CBE = m∠ ABE = 25
y = m∠ DEC = m∠ BCE + m∠ CBE (Theorem 6-4.1)
y = m∠ DEC = 80 + 25 = 105

46. m∠ BAC = 123
m∠ BAE = $\frac{1}{2}$(m∠ BAC) = $61\frac{1}{2}$
m∠ BAD = 90 - 32 = 58
m∠ DAE = m∠ BAE = m∠ BAD = $3\frac{1}{2}$.

47. In △BFE
$2x + x = 100$
$x = 33\frac{1}{3}$
m∠ ADE = $33\frac{1}{3}$

48. Use Theorem 6-4.1 and Theorem 3-4.2 in △EDC, △BED, △BEC, △ABE.
m∠ A = m∠ ABC (Theorem 3-4.2)
$3x = 2x + (180 - 6x)$
$3x = 180 - 4x$
$x = 25^5/_7$.

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49. Let m∠ ACE = x
m∠ E = m∠ DCE = x (Theorem 3-4.2)
m∠ ADE = m∠ E + m∠ DCE (Theorem 6-4.1)
m∠ ADE = $2x$ (Substitution Postulate 2-1)
m∠ BAD = m∠ ADE = $2x$ (Theorem 6-3.1)
But m∠ B = m∠ ACB (Theorem 3-4.2)
2m∠ ACB = 180 - $2x$
m∠ ACB = 90 - x
Therefore, m∠ BCE = m∠ ACB + m∠ ACE
m∠ BCE = 90 - x + x = 90.

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50. m∠ EAB = m∠ B (Theorem 6-3.1)
m∠ FAC = m∠ C (Theorem 6-3.1)
m∠ EAB + m∠ FAC + m∠ BAC = 180 (Postulate 2-10, Definition 1-28)
m∠ B + m∠ C + m∠ BAC = 180 (Postulate 2-1).

51. m∠ DBA + m∠ ABC + m∠ ECA + m∠ ACB = 2(90) = 180 (Def.1-27).
m∠ DBA = m∠ BAF (Theorem 6-3.1)
m∠ ECA = m∠ CAF (Theorem 6-3.1)
m∠ BAF + m∠ ABC + m∠ CAF + m∠ ACB = 180 (Postulate 2-1)
m∠ BAF + m∠ CAF = m∠ BAC (Postulate 2-10)
m∠ BAC + m∠ ABC + m∠ ACB = 180 (Postulate 2-1).

52. m∠ FED = m∠ BDE (Theorem 6-3.1)
m∠ BDE = m∠ A (Corollary 6-3.1a)
m∠ FED = m∠ A (Transitive property)
m∠ FEC = m∠ B (Corollary 6-3.1a)
m∠ BED = m∠ C (Corollary 6-3.1a)
m∠ FEC + m∠ FED + m∠ BED = 180 (Postulate 2-10, Definition 1-28)
m∠ B + m∠ A + m∠ C = 180 (Postulate 2-1).

53. m∠ ACF = m∠ A + m∠ ABC (Theorem 6-4.1)
m∠ DCF = $\frac{1}{2}$(m∠ ACF) = $\frac{1}{2}$(m∠ A + m∠ ABC) = $\frac{1}{2}$(m∠ A) + $\frac{1}{2}$(m∠ ABC) (Postulate 2-1, Distributive property)
m∠ DBC = $\frac{1}{2}$(m∠ ABC) (Definition 1-29)
m∠ DCF = $\frac{1}{2}$(m∠ A) + m∠ DBG (Transitive property)
m∠ DCF = m∠ DBC + m∠ D (Theorem 6-4.1)
m∠ DBC + $\frac{1}{2}$(m∠ A) = m∠ DBC + m∠ D (Transitive property)
m∠ D = $\frac{1}{2}$(m∠ A) (Subtraction property).

54. m∠ ACE = m∠ BCF = 60 (Given, Theorem 6-4.2)
m∠ BCE = m∠ FCA (Addition property)
EC = AC (Theorem 3-4.3)
BC = CF (Theorem 3-4.3)
△ECB ≅ △ACF (SAS)
$\overline{BE}$ ≅ $\overline{FA}$ (Definition 3-3)
Similarly, $\overline{FA}$ ≅ $\overline{CD}$.

55. m∠ MQN = 60 = m∠ PQR (Theorem 6-4.2, Definition 3-12)
MQ = NQ (Theorem 6-4.2, Definition 3-12)
PQ = RQ (Theorem 6-4.2, Definition 3-12)
△MQP ≅ △NQR (SAS)
$\overline{PM}$ ≅ $\overline{RN}$ (Definition 3-3)

56. AM = BM = CM.
Let $\overline{FM}$ bisect ∠ AMB and $\overline{EM}$ bisect ∠ AMC.
△AMF ≅ △BMF (SAS)
∠ BMF ≅ ∠ FMA (Definition 3-3)
Similarly, ∠ AME ≅ ∠ EMC.
m∠ BMF + m∠ FMA + m∠ AME + m∠ EMC = 180 (Definition 1-28)
2m∠ FMA + 2m∠ AME = 180 (Postulate 2-1)
m∠ FMA + m∠ AME = 90 (Division property)
m∠ 6 + m∠ 3 = 90 (Theorem 6-4.2)
m∠ 5 + m∠ 1 = 90 (Theorem 6-4.2)
m∠ 6 + m∠ 3 + m∠ 5 + m∠ 1 = 180 (Postulate 2-10)
m∠ 6 + m∠ 5 = 90 (Subtraction property)
m∠ BAC = 90 (Postulate 2-1).

57. Let m∠ MNC = x, m∠ KNB = y, m∠ B = a = m∠ C.
The angles of △KMN each have measure 60 (Postulate 2-1)
m∠ AMN = m∠ C + m∠ MNC (Theorem 6-4.1)
m∠ AMK = $x + a - 60$ (Postulate 2-1)
m∠ A = 180 - $2a$ (Theorem 6-4.2)
m∠ BMK = m∠ A + m∠ AMK (Theorem 6-4.1)
m∠ BKM = 180 - $2a + x + a - 60 = x - a + 120$ (Postulate 2-1)
m∠ BKN = m∠ BKM = m∠ NKM = m∠ BKM - 60 (Postulate 2-11)
m∠ BKN = $x - a + 120 - 60 = x - a + 60$ (Postulate 2-1)
m∠ BKN + m∠ AMK = $x - a + 60 + x + a - 60 = 2x$ (Add. prop.)
m∠ MNC = $\frac{1}{2}$(m∠ BKN + m∠ AMK) (Transitive property).

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1. Yes, it is consistent with Definition 6-3.
2. Yes, it is consistent with Definition 6-3.
3. Yes, it is consistent with Definition 6-3.

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Exercises

1. Yes, concave.
2. No, Definition 6-3, part 1.
3. Yes, convex.
4. Yes, concave.
5. Yes, concave.
6. 5 diagonals; $\overline{AC}$, $\overline{AD}$, $\overline{EB}$, $\overline{EC}$, $\overline{DB}$.

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7. Equilateral.
8. Square.
9. No, could be rhombus.
10. No, could be rectangle.

11. By theorem 6-5.1, when $n = 3$, the sum of the measures of the angle is $(n - 2)\,180 = (3 - 2)\,180 = 180$.

Exercises 12-18 are done in the same way as Exercise 11.

12. 540
13. 720
14. 900
15. 1080
16. 1440
17. 1800
18. 3240

19. *Method I*
By Corollary 6-5.1a, when $n = 4$, each interior angle has measure:
$$\frac{(n - 2)\,180}{n} = \frac{(4 - 2)\,180}{4} = 90.$$
Method II
By Corollary 6-5.2b, when $n = 4$, each interior angle has measure:
$$180 - \frac{360}{n} = 180 - \frac{360}{4} = 90.$$

Exercises 20-24 are done in the same way as Exercise 19.

20. 108
21. 120
22. 135
23. 140
24. 144

25. By Corollary 6-5.2a, when $n = 4$, the measure of an exterior angle is:
$$\frac{360}{n} = \frac{360}{4} = 90.$$

Exercises 26-28 are done in the same way as Exercise 25.

26. 10
27. 5
28. ½

29. Theorem 6-5.2, Definition 6-5.

30. Since there are n congruent exterior angles in a regular polygon of n-sides, each has a measure of $\frac{1}{n}$ of the sum of measures of all the exterior angles (i.e. 360). Thus the measure of each interior angle is $\frac{360}{n}$.

31. Since the measure of each exterior angle of a regular polygon of n sides is $\frac{360}{n}$ (Corollary 6-5.2a) and each exterior angle is supplementary to an interior angle, the measure of an interior angle is $180 - \frac{360}{n}$.

32. Both are right angles since they are congruent and supplementary; therefore the regular polygon is a square.

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33. Let x = the exterior angle.
$\frac{1}{2}x$ = the interior angle
$x + \frac{1}{2}x = 180$ (Postulate 1-6)
$x = 120$
Since $\frac{360}{n} = 120$ (Corollary 6-5.2a), $n = 3$ and the regular polygon is an equilateral triangle.

Exercises 34-37 are done in the same way as Exercise 33.

34. 6
35. 8
36. 5
37. 10

38. Let x = the measure of an exterior angle of the regular polygon.
$x + mx = 180$ (Postulate 1-6)
$x(1 + m = 180$
$x = \frac{180}{1 + m}$
$x = \frac{360}{n}$ (Corollary 6-5.2a)
Therefore $\frac{180}{1 + m} = \frac{360}{n}$
$n = 2\,(1 + m)$

39. $S = (n - 2)\,180$ (Theorem 6-5.1)
$540 = (\,n - 2)\,180$
$3 = n - 2$
$n = 5$ (sides)

Exercises 40-42 are done in the same way as Exercise 39.

40. 16 sides
41. 12 sides
42. 17 sides
43. $S = (n - 2)\,180$ (Theorem 6-5.1)
$S = (7 - 2)\,180$
$S = 900$
The remaining angle has measure $900 - 755 = 145$.

44. $S = (n - 2)\,180$ (Theorem 6-5.1)
$S = (4 - 2)\,180 = 360$
$3x + 4x + 5x + 6x = 360$
$x = 20$
Therefore $3x = 60$, $4x = 80$, $5x = 100$, $6x = 120$.

45. This angle is found in the regular polygon with the fewest number of sides.
Thus when $n = 3$, $\frac{(n - 2)\,180}{n} = 60$.

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46. Increases
47. None
48. 108
49. No, $\frac{360}{50}$ is not an integer.

50. Yes, $\frac{360}{40}$ is an integer.

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Class Exercises

1. $\triangle ABC \cong \triangle EDF$ (HL)
2. $\triangle ADB \cong \triangle ADC$ (HL)
3. $\triangle BAE \cong \triangle CDE$ (AAS)
$\triangle ABC \cong \triangle DCB$ (HL)
4. $\triangle DEC \cong \triangle BFA$ (HL)
$\triangle EDA \cong \triangle FBC$ (ASA)
5. $\triangle ABC \cong \triangle DEC$ (AAS)
6. $\triangle CDF \cong \triangle BEF$ (AAS)

Exercises

1. $\angle BAD \cong \angle BCD$ (Theorem 3-1.4)
$\triangle ABD \cong \triangle CBD$ (AAS).

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Exercises continued

2. $m\angle B = 90 = m\angle C$ (Theorem 2-6.5)
$\angle BEA \cong \angle CED$ (Theorem 3-1.5)
$\overline{AE} \cong \overline{DE}$ (Definition 1-15)
$\triangle ABE \cong \triangle DCE$ (AAS).

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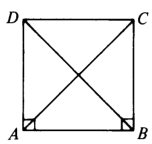
3. $\triangle FEB$ and $\triangle FDB$ are right triangles (Definition 1-32).
$\triangle FEB \cong \triangle FDB$ (HL)
$\angle FBE \cong \angle FBD$ (Definition 3-3).
Thus $\overline{FB}$ bisects $\angle ABC$ (Definition 1-29).

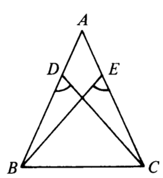
4. $\angle ADB \cong \angle CBD$ (Theorem 6-3.1)
$\overline{DE} \cong \overline{BF}$ (Addition property)
$\triangle ADE \cong \triangle CBF$ (AAS).

5.



$\triangle ABD$ and $\triangle BAC$ are right triangles (Definition 1-32)
$\triangle ABD \cong \triangle BAC$ (HL)
$\overline{AD} \cong \overline{BC}$ (Definition 3-3)

6.



1: $\angle ABC \cong \angle ACB$ (Theorem 3-4.2)
$\triangle BDC \cong \triangle CEB$ (AAS)
$\overline{CD} \cong \overline{BE}$ (Definition 3-3)

2: $\angle ADC \cong \angle AEB$ (Theorem 3-1.4)
$\triangle ADC \cong \triangle AEB$ (AAS)
$\overline{CD} \cong \overline{BE}$ (Definition 3-3).

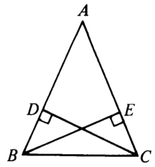
7. $m\angle FEC = 90 = m\angle GDB$ (Theorem 2-6.5)
FC = GB (Addition property)
$\angle ABC \cong \angle ACB$ (Theorem 3-4.1)
$\triangle FEC \cong \triangle GDB$ (AAS)
$\angle F \cong \angle G$ (Definition 3-3)

8. $\triangle FEC$ and $\triangle GDB$ are right triangles (Definition 1-32)
FC = GB (Addition property)
$\triangle FEC \cong \triangle GDB$ (HL)
$\angle F \cong \angle G$ (Definition 3-3).

9. $\triangle FEC \cong \triangle GDB$ (HL)
$\angle ABC \cong \angle ACB$ (Exercise 8)
$\overline{AB} \cong \overline{AC}$ (Theorem 3-4.3)
$\triangle ABC$ is isosceles (Definition 3-12).

10. BF = CG (Subtraction property)
$\triangle DFB \cong \triangle EGC$ (HL)
$\angle B \cong \angle C$ (Definition 3-3)
$\overline{AB} \cong \overline{AC}$ (Theorem 3-4.3)
$\triangle ABC$ is isosceles (Definition 3-12).

11.



$\overline{BE}$ and $\overline{CD}$ are altitudes of $\triangle ABC$, $\overline{BE} \cong \overline{CD}$ (Given)
$\triangle AEB \cong \triangle ADC$ (AAS)
$\overline{AB} \cong \overline{AC}$ (Definition 3-3)
$\triangle ABC$ is isosceles (Definition 3-12)

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Exercises continued

12. Isosceles $\triangle ABC$ (Given)
$\overline{AB} \cong \overline{AC}$ (Given)
Altitudes $\overline{BE}$ and $\overline{CD}$ (Given)
$\triangle AEB \cong \triangle ADC$ (AAS)
$\overline{BE} \cong \overline{CD}$ (Definition 3-3)

13. $\overline{BE} \cong \overline{CE}$ (Definition 1-15)
$\triangle AEB$ and $\triangle DEC$ are right triangles (Definition 1-32)
$\triangle AEB \cong \triangle DEC$ (HL)
$\angle AEB \cong \angle DEC$ (Definition 3-3)
$\angle AEB$ and $\angle AEC$ form a linear pair (Definition 1-26)
$\angle DEC$ and $\angle AEC$ also form a linear pair (Transitive property)
Thus, A, E, and D are collinear (Definition 1-11).

14. Quadrilateral ABCD, $\overline{AB} \cong \overline{CD}$, $\angle B$ and $\angle D$ are right angles (Given)
$\triangle ABC \cong \triangle CDA$ (HL)
$\angle BAC \cong \angle DCA$ (Definition 3-3)
$\angle BCA \cong \angle DAC$ (Definition 3-3)
$\overline{AB} \parallel \overline{CD}$ (Theorem 6-2.1)
$\overline{AD} \parallel \overline{BC}$ (Theorem 6-2.1)

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15. $\overline{AD}$ is an altitude of isosceles $\triangle ABC$, $\overline{AB} \cong \overline{AC}$ (Given)
$\overline{AB} \cong \overline{AC}$ (Definition 1-15)
$\triangle ADB$ and $\triangle ADC$ are right triangles (Definition 1-32)
$\triangle ADB \cong \triangle ADC$ (HL)
$\overline{BD} \cong \overline{CD}$ (Definition 3-3)
$\overline{AD}$ is a median of $\triangle ABC$ (Definition 3-9)
$\overline{AD} \perp \overline{BC}$ (Corollary 4-4.2a)
$\overline{AD}$ is an altitude of $\triangle ABC$ (Definition 3-10).

16. Construct $\angle PAB \cong \angle FDE$
$\angle PBA \cong \angle FED$
$\triangle ABP \cong \triangle DEF$ (ASA)
$\overline{AP} \cong \overline{DF}$ (Definition 3-3)
$\angle F \cong \angle APB$ (Definition 3-3)
$\overline{AC} \cong \overline{AP}$ (Transitive property)
$\angle APB \cong \angle ACB$ (Transitive property)
$m\angle ACP = m\angle APC$ (Theorem 3-4.2)
$m\angle PCB = m\angle CPB$ (Postulate 2-11)
$\overline{CB} \cong \overline{PB}$ (Theorem 3-4.3)
$\triangle ABC \cong \triangle ABP$ (SAS)
$\triangle ABC \cong \triangle DEF$ (Transitive property).

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Review Exercises

1. False; they may be skew lines.
2. False; A, B, C, D could be collinear.

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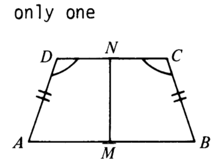
3. True.
4. $\overleftrightarrow{EF}$
5. $\overrightarrow{RS}$
6. only one
7.



$\triangle ADN \cong \triangle BCN$ (SAS)
$\overline{AN} \cong \overline{BN}$ (Definition 3-3)
$\overline{AB} \perp \overline{MN}$ (Corollary 4-4.2a)
$\triangle AMN \cong \triangle BMN$ (SAS or SSS)
$m\angle ANM = m\angle BNM$ (Definition 3-3)
$m\angle MND = m\angle MNC$ (Postulate 2-10, Addition property)

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Review Exercises continued

7. *continued*

m∠ MND = 90 = m∠ MNC (Theorem 3-1.2)
$\overline{MN} \perp \overline{DC}$ (Theorem 2-6.6)
$\overline{AB} \parallel \overline{DC}$ (Theorem 6-1.1)

8. We now have five methods of proving that lines are parallel:
   1. Prove a pair of alternate interior angles congruent.
   2. Prove a pair of corresponding angles congruent.
   3. Prove a pair of interior angles on the same side of the transversal supplementary.
   4. Prove that the lines are parallel to the same line or to parallel lines.
   5. Prove that the lines are coplanar and perpendicular to the same line or to parallel lines.

9. m∠ ABC = 2m∠ EBC (Given)
m∠ BCD = 2m∠ ECB (Given)
m∠ ABC + m∠ BCD = 2(90) = 180 (Addition property)
$\overline{AB} \parallel \overline{CD}$ (Corollary 6-2.1b)

10. BC = FD (Addition property)
△ABC ≅ △EFD (SAS)
∠ ACD ≅ ∠ EDF (Definition 3-3)
$\overline{AC} \parallel \overline{DE}$ (Theorem 6-2.1)

11. BC = CD (Definition 1-15)
EC = CA (Definition 1-15)
△BCA ≅ △DCE (SAS)
∠ B ≅ ∠ D (Definition 3-3)
$\overline{AB} \parallel \overline{DE}$ (Theorem 6-2.1)

12. m∠ ABD = m∠ BDC = m∠ DCE = 25 (Theorem 6-3.1)
m∠ ADB = m∠ ADC - m∠ BDC
m∠ ADB = 116 - 25 = 91
From Theorem 6-4.2, m∠ A = 180 - (m∠ ABD + m∠ ADB)
m∠ A = 180 - (25 + 91) = 64.

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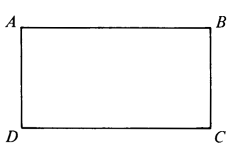
13. $\overline{AC} \cong \overline{FD}$ (Addition property)
∠ A ≅ ∠ F (Theorem 6-3.1)
△ABC ≅ △FED (SAS)
∠ ACB ≅ ∠ FDE (Definition 3-3)
$\overline{DE} \parallel \overline{BC}$ (Theorem 6-2.1)

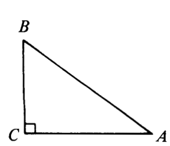
14.



Quadrilateral ABCD (Given)
$\overline{AD} \parallel \overline{BC}$ (Given)
$\overline{AB} \parallel \overline{DC}$ (Given)
Draw $\overline{AC}$
∠ DAC ≅ ∠ BCA (Theorem 6-3.1)
∠ BAC ≅ ∠ DCA (Theorem 6-3.1)
∠ A ≅ ∠ C (Postulate 2-10)
Similarly, ∠ B ≅ ∠ D.

15. 60.
16. Complementary.

17.



If m∠ C = m∠ A + m∠ B
m∠ C + m∠ A + m∠ B = 180 (Theorem 6-4.2)
2(m∠ C) = 180
m∠ C = 90
Therefore, △ABC is right triangle.

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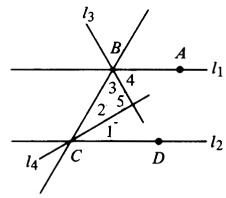
18.



$l_1 \parallel l_2$ (Given)
l_3 bisects ∠ ABC (Given)
l_4 bisects ∠ DCB (Given)
∠ 1 = ∠ 2, ∠ 3 = ∠ 4 (Definition 1-29)
∠ DCB is supplementary to ∠ ABC (Corollary 6-3.1b)
m∠ DCB + m∠ ABC = 180
m∠ 1 + m∠ 2 = m∠ 3 + m∠ 4 = 180
2m∠ 2 + 2m∠ 3 = 180
m∠ 2 + m∠ 3 = 90
m∠ 2 + m∠ 3 + m∠ 5 = 180 (Theorem 6-4.2)
90 = m∠ 5 = 180
m∠ 5 = 90
Therefore $l_3 \perp l_4$

19. m∠ ACE = m∠ BCE = 45 (Definition 1-29)
m∠ CBE + m∠ EBD = 180 (Definition 1-26, 1-28)
m∠ CBE + 140 = 180
m∠ CBE = 40
m∠ AEC = m∠ CBE + m∠ BCE (Theorem 6-4.1)
m∠ AEC = 40 + 45 = 85

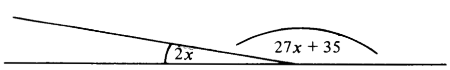
20. $(4x + 9) + (3x + 18) + 10x = 180$ (Theorem 6-4.2)
$17x = 153$
$x = 9$
The sides have lengths:
$4x + 9 = 45$
$3x + 18 = 45$
$10x = 90$
Therefore the triangle is an isosceles right triangle.

21. See Definition 6-4.

22. $180 - \frac{360}{n} = 180 - \frac{360}{26}$ (Corollary 6-5.2b)
$= 180 - 13\frac{11}{13}$
$= 166\frac{2}{13}$

23. $\frac{360}{n} = \frac{360}{720} = \frac{1}{2}$ degree (Corollary 6-5.2a)

24.



$2x + 27 + 35 = 180$ (Postulate 1-6)
$x = 5$ and $2x = 10$
Therefore $10 = \frac{360}{n}$, and

$n = 36$

25. $(n-2)$ triangles formed if all diagonals are drawn from any single vertex
$n = 17$ $(n-2)$ triangles = 15 triangles
$n = 10$ $(n-2)$ triangles = 8 triangles
15 triangles - 8 triangles = 7 triangles, each of whose measure sum = 7 (180) = 1260.

26. Quadrilateral ABCD, ∠ A ≅ ∠ C (Given)
∠ B ≅ ∠ D (Given)
m∠ A + m∠ B + m∠ C + m∠ D = 360 (Theorem 6-5.1)
2m∠ A + 2m∠ D = 360 (Postulate 2-1)
m∠ A + m∠ D = 180 (Division property)
$\overline{AB} \parallel \overline{DC}$ (Corollary 6-2.1b)
Similarly, AD ∥ BC.

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27. Sometimes (only right triangles)

28. Always (AAS)

29. $\triangle ADC \cong \triangle AEB$ (AAS)
$\overline{AC} \cong \overline{AB}$ (Definition 3-3)
$\triangle ABC$ is isosceles (Definition 3-12).

30. $\triangle APQ \cong \triangle BPQ$ (HL)
$\overline{AQ} \cong \overline{BQ}$ (Definition 3-3).

Chapter Test

1. $\frac{1}{2}(180 - 30)$

2. $360 - 360 = 0$.

3. Let z = the measure of non-adjacent exterior angle of a triangle for interior angles of measure x and y.
$2x = z$ and $z = x + y$ (Theorem 6-4.1)
Therefore $2x = x + y$, and $x = y$.
Therefore two sides of the triangle are congruent (Theorem 3-4.3) and the triangle is isosceles.

4. perpendicular

5. Since the larger triangle is equilateral $x = 60$ (Corollary 3-4.2a)
Since the smaller triangle is an isosceles right triangle, $y = 60 - 45 = 15$.

6. Use Theorem 6-3.1 and Corollary 6-3.1b to get $x = 24$ and $y = 85$.

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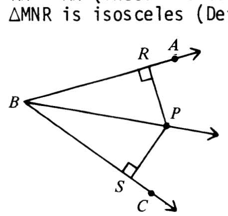
7.- 10. Only 8 (AAS); the others give insufficient information.

11. By Theorem 6-4.2, $m\angle BAC + m\angle B + m\angle C = 180$.
Therefore $(11x - 7) + (3x + 4) + (10x + 15) = 180$
Then $x = 7$, and $m\angle C = 3x + 4 = 25$.
However, $m\angle PAC = 6x - 17 = 25$.
Therefore since $m\angle C = m\angle PAC$,
$\overrightarrow{AP} \parallel \overline{BC}$ (Theorem 6-2.1)

12. $\triangle QPR \cong \triangle SPT$ (HL)
$\angle R \cong \angle T$ (Definition 3-3)
$\angle T \cong \angle TMR$ (Theorem 6-3.1)
$\angle R \cong \angle TMR$ (Transitive property)
$\overline{RN} \cong \overline{MN}$ (Theorem 3-4.3)
$\triangle MNR$ is isosceles (Definition 3-12)

13.



$\angle PRB \cong \angle PSB$ (Theorem 2-6.5, Theorem 3-1.1)
$\triangle PRB \cong \triangle PSB$ (AAS)
$\overline{PR} \cong \overline{PS}$ (Definition 3-3)

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Mathematical Excursion Exercises

1. Construct the altitude from the vertex of the largest angle, forming two right triangles. The sum of the angles of the original triangle is the sum of the angles of the right triangles minus the two right angles at the foot of the altitude.

2. 360

3. The sum of the angles of a triangle is less than 180 implies the sum of the angles of a quadrilateral is

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3. *continued*
less than 360. This makes it impossible for a quadrilateral to have four right angles.

4. It is greater than 360.

5. It is less than 360.

6. Square.

7. Joining the ends of the perpendicular segments, we have four congruent isosceles right triangles. Hence, each base angle is greater than 45 degrees. Now draw the perpendiculars at A, B, C and D. We have four more congruent isosceles triangles with each base angle less than 45 degrees. Thus, the vertex angles are greater than 90 degrees.

8. Reverse the inequalities in Exercise 7.

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Class Exercises

1. parallelogram.
2. BC.
3. AD.
4. BC.
5. They are equal.

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Exercises

1. $\overline{MN} \parallel \overline{LK}$, $\overline{ML} \parallel \overline{NK}$:
$\overline{MP} \cong \overline{KP}$,
$\overline{MN} \cong \overline{LK}$, $\overline{LP} \cong \overline{NP}$
$\overline{ML} \cong \overline{NK}$

2. Eight pairs;
$\angle LMN \cong \angle LKN$, $\angle MLK \cong \angle MNK$
$\angle LMK \cong \angle NKM$, $\angle NMK \cong \angle LKM$
$\angle MNL \cong \angle KLN$, $\angle KNL \cong \angle MLN$
$\angle MPN \cong \angle KPL$, $\angle MPL \cong \angle KPN$.

3. $\angle MLK$, $\angle MNK$

4. $\angle LMN$, $\angle NKL$

5. Use Theorem 7-1.4 to find:
$m\angle X = 133$
$m\angle Y = 47$
$m\angle Z = 133$

6. $m\angle D = 145$
$m\angle A = 35$
$m\angle B = 145$
$m\angle C = 35$

7. Since EH = FG (Theorem 7-1.2)
$3x - 5 = 7x - 17$
$x = 3$
Therefore EH = FG = 4; and GH = 7.

8. Since WP = YP (Theorem 7-1.5)
$3x - 5 = 12x - 41$
$x = 4$
Therefore $WY = 2(WP) = 2(3x - 5) = 14$
and $xz = 2(xP) = 2(3x + 11) = 46$.

9. Since HP = AP (Theorem 7-1.5)
$5x + y = 4y - x$, or $4x = 3y$
Similarly, MP = TP (Theorem 7-1.5)
$3y + 7 = x + 6y$
$x + 3y = 7$
$x = 1$, and $y = 2$
$HA = 2(HP) = 2(5x + y) = 14$
$MT = 2(MP) = 2(3y + 7) = 26$.

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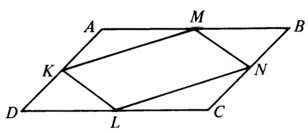
Exercises continued

10. m∠ A + m∠ B = 180 (Theorem 7-1.4)
m∠ C + m∠ B = 180 (Theorem 7-1.4)
∠ A ≅ ∠ C (Theorem 3-1.4)
Similarly, ∠ B ≅ ∠ D.

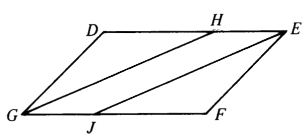
11. ∠ A ≅ ∠ C (Theorem 7-1.3)
∠ B ≅ ∠ D (Theorem 7-1.3)
m∠ A + m∠ B + m∠ C + m∠ D = 360 (Theorem 6-5.1)
2m∠ A + 2m∠ B = 360 (Postulate 2-1)
m∠ A + m∠ B = 180 (Division property)

12.



MB = $(\frac{1}{2})$AB (Definition 1-15)
DL = $(\frac{1}{2})$DC (Definition 1-15)
MB = DL (Transitive property)
Similarly, BN = DK
∠ B ≅ ∠ D (Theorem 7-1.3)
△MBN ≅ △LDK (SAS)
$\overline{MN}$ ≅ $\overline{LK}$ (Definition 3-3)

13.



m∠ DGH = $(\frac{1}{2})$ m∠ DGF (Definition 1-29)
m∠ FEJ = $(\frac{1}{2})$ m∠ FED (Definition 1-29)
m∠ DGF = m∠ FED (Theorem 7-1.3)
m∠ DGH = m∠ FEJ (Transitive property)
DG = FE (Theorem 7-1.2)
∠ D ≅ ∠ F (Theorem 7-1.3)
△DGH ≅ △FEJ (ASA)
DH = FJ (Definition 3-3)
DE = FG (Theorem 7-1.2)
HE = GJ (Subtraction property).

14. $\overline{BC}$ ≅ $\overline{DA}$ (Theorem 7-1.2)
$\overline{BC}$ ∥ $\overline{DA}$ (Definition 7-1)
∠ BCA ≅ ∠ DAC (Theorem 6-3.1)
m∠ BGC = 90 = m∠ DHA (Theorem 3-1.1)
△BGC ≅ △DHA (AAS)
$\overline{HD}$ ≅ $\overline{GB}$ (Definition 3-3)
$\overline{AH}$ ≅ $\overline{CG}$ (Definition 3-3)

15. $\overline{MA}$ ∥ $\overline{HT}$ (Definition 7-1)
∠ AMT ≅ ∠ HTM (Theorem 6-3.1)
∠ MEG ≅ ∠ TEO (Theorem 3-1.5)
△MEG ≅ △TEO (ASA)
$\overline{GE}$ ≅ $\overline{OE}$ (Definition 3-3)
$\overline{MT}$ bisects $\overline{GO}$ at E (Definition 1-15)

16. Parallelogram MATH, $\overline{MGH}$, $\overline{HOT}$, $\overline{MT}$ bisects $\overline{GO}$ at E (Given)
$\overline{MA}$ ∥ $\overline{HT}$ (Definition 7-1)
∠ MGO ≅ ∠ TOG (Theorem 6-3.1)
∠ MEG ≅ ∠ TEO (Theorem 3-1.5)
△MEG ≅ △TEO (ASA)
$\overline{ME}$ ≅ $\overline{TE}$ (Definition 3-3)
$\overline{GO}$ bisects $\overline{MT}$ at E (Definition 1-15)

17. MB = $(\frac{1}{2})$ AB (Definition 1-15)
ND = $(\frac{1}{2})$ DC (Definition 1-15)
AB = DC (Theorem 7-1.2)
MB = ND (Postulate 2-1)
AD ≅ CB (Theorem 7-1.2)
∠ ADN ≅ ∠ CBM (Theorem 7-1.3)
△ADN ≅ △CBM (SAS)
∠ AND ≅ ∠ CMB (Definition 3-3)
$\overline{AB}$ ∥ $\overline{DC}$ (Definition 7-1)
∠ ABD ≅ ∠ CDB (Theorem 6-3.1)
△DFN ≅ △BEM (ASA)
$\overline{DF}$ ≅ $\overline{BE}$ (Definition 3-3)

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18. AB = DC (Theorem 7-1.2)
AP = CQ (Addition property)
$\overline{AB}$ ∥ $\overline{DC}$ (Definition 7-1)
∠ PAC ≅ ∠ QCA (Theorem 6-3.1)
∠ APQ ≅ ∠ CQP (Theorem 6-3.1)
△AMP ≅ △CMQ (ASA)
AM = MC (Definition 3-3)
PM = MQ (Definition 3-3)
$\overline{AC}$ and $\overline{PQ}$ bisect each other (Postulate 1-15).

19. ED = AE = BC (Definition 3-12)
AF = BF = DC (Definition 3-12)
m∠ EDA = 60 = m∠ FBA (Theorem 7-1.3)
m∠ ADC = m∠ ABC (Theorem 7-1.3)
m∠ EDC = m∠ FBC (Addition property)
△EDC ≅ △CBF (SAS)
$\overline{EC}$ ≅ $\overline{CF}$ (Definition 3-3)
m∠ DAB = 180 - m∠ ADC (Corollary 6-3.1b)
m∠ EAF = 360 - (60 + 60 + m∠ DAB)(Postulate 2-1)
m∠ EAF = 240 - (180 - m∠ADC) (Postulate 2-1)
m∠ EAF = 60 + m∠ ADC (Postulate 2-1)
m∠ EAF = m∠ EDC (Postulate 2-1)
△EDC ≅ △EAF (SAS)
$\overline{EC}$ ≅ $\overline{EF}$ (Definition 3-3)
△EFC is equilateral (Definition 3-12)

20. Draw $\overline{PE}$ ⊥ $\overline{DC}$ (Definition 3-10)
$\overline{AB}$ ⊥ $\overline{DC}$ (Definition 3-10)
$\overline{PE}$ ∥ $\overline{AB}$ (Theorem 6-1.1)
∠ EPC ≅ ∠ B (Corollary 6-3.1a)
∠ B ≅ ∠ ACB (Theorem 3-4.2)
∠ EPC ≅ ∠ ACB (Transitive property)
△EPC ≅ △ NCP (AAS)
PN = EC (Definition 3-3)
$\overline{DC}$ ∥ $\overline{MP}$ (Theorem 6-1.1)
Quadrilateral DEPM is a parallelogram (Definition 7-1)
PM = ED (Theorem 7-1.2)
PM + PN = ED + EC = CD (Addition property).

21. Use Definition 7-3 and the proof of Theorem 5-4.2.

22. Converse. The length of the shortest segment which has an endpoint on each of two parallel lines is the distance between the parallel lines.
Proof outline: Use an indirect proof to show that the perpendicular segment is shorter than any other such segment. Apply Definition 7-3.

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Exercises

1. No.
2. Yes, Theorem 7-2.1.
3. Yes, Theorem 7-2.4.
4. Parallelogram ABCD
M, N, P, Q are the midpoints of sides $\overline{AD}$, $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, respectively (Given)
AM = $(\frac{1}{2})$AD (Definition 1-15)
PC = $(\frac{1}{2})$ BC (Definition 1-15)
AD = BC (Theorem 7-1.2)
AM = PC (Postulate 2-1)
Similarly, AN = QC
∠ A ≅ ∠ C (Theorem 7-1.3)
△MAN ≅ △PCQ (SAS)
$\overline{MN}$ ≅ $\overline{PQ}$ (Definition 3-3)
Similarly △MDQ ≅ △PBN (SAS)
$\overline{MQ}$ ≅ $\overline{PN}$ (Definition 3-3)
Quadrilateral MNPQ is a parallelogram (Theorem 7-2.1)

5. AB = CD (Theorem 7-1.2)
FB = DE (Subtraction property)
AF = CE (Subtraction property)
$\overline{AB}$ ∥ $\overline{CD}$ (Definition 7-1)
Quadrilateral AFCE is a parallelogram (Theorem 7-2.2)

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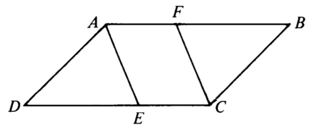
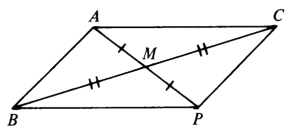
6.



$\angle D \cong \angle B$ (Theorem 7-1.3)
$\overline{AD} \cong \overline{CB}$ (Theorem 7-1.2)
$\triangle ADE \cong \triangle CBF$ (ASA)
$\overline{AE} \cong \overline{CF}$ (Definition 3-3)
$\overline{DE} \cong \overline{BF}$ (efinition 3-3)
$\overline{AF} \cong \overline{CE}$ (Definition 3-3)
$\overline{AB} \cong \overline{CD}$ (Theorem 7-1.2)
Quadrilateral AFCE is a parallelogram (Theorem 7-2.1)

7.



CM = MB (Definition 3-9)
AM = MP (Definition 3-9)
Quadrilateral ABPC is a parallelogram (Theorem 7-2.6)

8. $\overline{PQ} \parallel \overline{SR}$ (Theorem 6-2.1)
$m\angle 1 + m\angle 3 = m\angle 2 + m\angle 4$ (Definition 1-28)
$m\angle RQS = m\angle PSQ$ (Subtraction property)
PS $\parallel$ QR (Theorem 6-2.1)
Quadrilateral PQRS is a parallelogram (Definition 7-1)

9. $\angle FNP \cong \angle RSA$ (Theorem 3-1.4)
$\triangle FNP \cong \triangle RSA$ (AAS)
$\overline{NP} \cong \overline{SA}$ (Definition 3-3)
Quadrilateral ASPN is a parallelogram (Theorem 7-2.5).

10. $\overline{ST} \parallel \overline{QH}$ (Theorem 6-1.1)
$\overline{PS} \cong \overline{RQ}$ (Theorem 7-1.2)
$\overline{PS} \parallel \overline{RQ}$ (Definition 7-1)
$\angle SPR \cong \angle QRP$ (Theorem 6-3.1)
$\triangle STP \cong \triangle QHR$ (AAS)
$\overline{ST} \cong \overline{QH}$ (Definition 3-3)
Quadrilateral TSHQ is a parallelogram (Theorem 7-2.2)
$\overline{QT} \parallel \overline{HS}$ (Definition 7-1)
$\angle TQH \cong \angle TSH$ (Theorem 7-1.3).

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11. $\overline{MN} \cong \overline{RS}$ (Theorem 7-1.2)
$\overline{MN} \parallel \overline{RS}$ (Definition 7-1)
$\angle MNR \cong \angle MRS$ (Theorem 6-3.1)
$\triangle MNP \cong \triangle RSQ$ (ASA)
$\overline{NP} \cong \overline{SQ}$ (Definition 3-3)
$\angle MNP \cong \angle RQS$ (Definition 3-3)
$\angle NPQ \cong \angle SQP$ (Theorem 3-1.4)
$\overline{NP} \parallel \overline{SQ}$ (Theorem 6-2.1)
Quadrilateral PNQS is a parallelogram (Theorem 7-2.2)
$\overline{PS} \parallel \overline{QN}$ (Definition 3-3).

12. $\overline{MN} \cong \overline{RS}$ (Theorem 7-1.2)
MN $\parallel$ RS (Definition 7-1)
$\angle MNR \cong \angle MRS$ (Theorem 6-3.1)
MP = RQ (Subtraction property)
$\triangle MPN \cong \triangle RQS$ (SAS)
(continue as in the solution of Exercise 11).

13. $\overline{PN} \cong \overline{QS}$ (Theorem 7-1.2)
$\overline{PN} \parallel \overline{QS}$ (Definition 7-1)
$\angle NPO \cong \angle SQP$ (Theorem 6-3.1)
$\angle MPN \cong \angle RQS$ (Theorem 3-1.4)
$\overline{MP} \cong \overline{RQ}$ (Subtraction property)
$\triangle MPN \cong \triangle RQS$ (SAS)
$\overline{MN} \cong \overline{RS}$ (Definition 3-3)
$\angle NMR \cong \angle SRM$ (Theorem 6-3.1)
$\overline{MN} \parallel \overline{RS}$ (Theorem 6-2.1)
Quadrilateral PNQS is a parallelogram (Theorem 7-2.2)
$\angle PSQ \cong \angle PNQ$ (Theorem 7-1.3)

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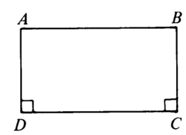
14. Parallelogram ABCD with M and N midpoints of sides $\overline{AB}$ and $\overline{CD}$, respectively (Given)
AN = ($\frac{1}{2}$)AB (Definition 1-15)
DM = ($\frac{1}{2}$)CD (Definition 1-15)
$\overline{AB} \cong \overline{CD}$ (Theorem 7-1.2)

$\overline{AB} \cong \overline{DC}$ (Definition 7-1)
AN = DM (Postulate 2-1)
Quadrilateral ANMD is a parallelogram (Theorem 7-2.2)
$\overline{AD} \cong \overline{NM}$ (Definition 7-1)(Theorem 7-1.2)
AD $\parallel$ NM (Definition 7-1)(Theorem 7-1.2)
Similarly $\overline{BC} \cong \overline{NM}$
$\overline{BC} \parallel \overline{NM}$

15. Quadrilateral ABCD with $\overline{AP}$ and $\overline{DP}$ the bisectors of $\angle DAB$ and $\angle ADC$, respectively
$\overline{AP}$ is not perpendicular to $\overline{DP}$ (Given)
$m\angle 1 + m\angle 2 \neq 90$
$2m\angle 1 + 2m\angle 2 \neq 180$ ($\overline{AP}$ is not perpendicular to $\overline{DP}$)
$\overline{AB}$ is not parallel to $\overline{DC}$ (Corollary 6-3.1b)
ABCD is not a parallelogram (Definition 7-1).

16. AM =($\frac{1}{2}$) SA (Definition 1-15)
AN = ($\frac{1}{2}$) AQ (Definition 1-15)
SA = AQ (Theorem 7-1.5)
AM = AN (Postulate 2-1)
$\angle KPA \cong \angle LRA$ (Theorem 6-3.1)
PA = RA (Theorem 7-1.5)
$\angle PAK \cong \angle RAL$ (Theorem 3-1.5)
$\triangle PAK \cong \triangle RAL$ (ASA)
KA = LA (Definition 3-3)
Quadrilateral MKNL is a parallelogram (Theorem 7-2.6)

17.



$\overline{AD} \parallel \overline{BC}$ (Theorem 6-1.1)
DC is the distance between $\overline{AD}$ and $\overline{BC}$ (Definition 7-3)
AB is also the distance between $\overline{AD}$ and $\overline{BC}$ (Given)
$\overline{AB} \perp \overline{AD}$ (Definition 7-3)
$\overline{AB} \perp \overline{BC}$ (Definition 7-3)
Quadrilateral ABCD is a parallelogram (Theorem 7-2.3).

18. MB = MC (Definition 1-15)
Quadrilateral GCQB is a parallelogram (Theorem 7-2.6)
AN = NC (Definition 1-15)
Quadrilateral GCPA is a parallelogram (Theorem 7-2.6)
$\overline{QC} \parallel \overline{BGN}$ (Definition 7-1)
PC $\parallel$ AGM (Definition 7-1)
Quadrilateral GPCQ is a parallelogram (Definition 7-1)
GM = ($\frac{1}{2}$)GQ (Definition 1-15)
GQ = PC (Theorem 7-1.2)
PC = AG (Theorem 7-1.2)
GM = ($\frac{1}{2}$) AB (Postulate 2-1)
GM = ($\frac{1}{2}$)AM (Division property)
GN = ($\frac{1}{2}$)GP (Definition 1-15)
GP = QC (Theorem 7-1.2)
QC = BG (Theorem 7-1.2)
GN = ($\frac{1}{2}$)BG (Postulate 2-1)
GN = ($\frac{1}{3}$)BN (Division property).

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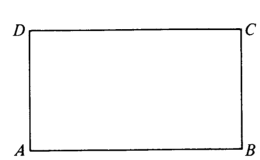
Class Exercises

1.



2. Congruent.
3. Right angles.
4. Congruent.

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Class Exercises continued

5. SAS.
6. Congruent.
7. Congruent.

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1. Yes, Theorem 7-3 3, Theorem 6-5.1.
2. Yes, Theorem 7-3.4.
3. No.

4. Since AC = BD (Theorem 7-3.2)
   $7x + 5 = 3x + 17$
   $x = 3$
   AC = BD = $3x + 17 = 26$

5. PM = $x + 3y = 20$
   SM = $4y - 2x = 20$
   $x = 2$, and $y = 6$

6. MP = PQ (Theorem 7-1.5)
   $15x - 11 = 7x + 21$
   $x = 4$
   MQ = MP + PQ = $22x + 10 = 98$
   RN = $20x + 18 = 98$
   Therefore parallelogram MNQR is a rectangle (Theorem 7-3.4)

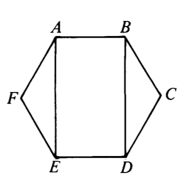
7. $7\frac{1}{2}$; Theorem 7-3.5.
8. 30; Theorem 7-3.5.
9. m∠ A = 90 = m∠ N (Theorem 7-3.1)
   $\overline{AF} \cong \overline{NF}$ (Definition 1-15)
   $\overline{AS} \cong \overline{NP}$ (Theorem 7-1.2)
   △FAS ≅ △FNP (SAS)
   $\overline{FS} \cong \overline{FP}$ (Definition 3-3)
   △SFP is isosceles (Definition 3-12).

10. CN = EM (Addition property)
    m∠ C = 90 = m∠ E (Theorem 7-3.1)
    $\overline{CT} \cong \overline{ER}$ (Theorem 7-1.2)
    △NCT ≅ △MER (SAS)
    TN = RM (Definition 3-3)
    m∠ CTN = m∠ ERM (Theorem 7-3.1)
    m∠CTR = m∠ ERT (Theorem 7-3.1)
    m∠ PTR = m∠ PRT (Subtraction property)
    PT = PR (Theorem 3-4.3)
    △TPR is isosceles (Theorem 3-4.3)
    m∠ CNT = m∠ EMR (Theorem 3-4.3)
    MP = NP (Theorem 3-4.3)
    △MPN is isosceles (Theorem 3-4.3).

11. Draw rectangle ABCD.
    ABCD is a parallelogram with one right angle, ∠ A (Definition 7-5)
    m∠ A + m∠ B = 180 = m∠ A + m∠ D (Theorem 7-1.4)
    m∠ B = m∠ D = 90 (Transitive property)
    ∠ A ≅ ∠ C (Theorem 7-1.3)
    m∠ C = 90 (Transitive property).

12. The quadrilateral is a parallelogram (Theorem 7-2.3).
    It is also a rectangle (Theorem 7-2.3, Definition 7-5).

13. Draw parallelogram ABCD with $\overline{AC} \cong \overline{BD}$.
    $\overline{AD} \cong \overline{BC}$ (Theorem 7-1.2)
    △ADC ≅ △BCD (SSS)
    ∠ ADC ≅ ∠ BCD (Definition 3-3)
    m∠ ADC + m∠ BCD = 180 (Theorem 7-1.4)
    m∠ ADC = m∠ BCD = 90 (Theorem 2-5.6)
    Parallelogram ABCD is a rectangle (Definition 7-5).

14. 

    m∠ F = 120 (Corollary 6-5.1a)
    $\overline{AF} \cong \overline{FE}$ (Definition 6-5)
    m∠ FAE = m∠ AEF = 30 (Theorem 3-4.3)
    m∠ FAB = 120 (Corollary 6-5.1a)

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14. *continued*

    m∠ BAE = 90 (Subtraction property)
    Similarly, m∠ AED = m∠ EDB = m∠ ABD
    Quadrilateral ABDE is a rectangle (Theorem 7-3.3)

15. $\overline{PM}$, $\overline{QM}$, $\overline{RM}$, and $\overline{SM}$ are medians of right triangles, APB, AQB, ARB, ASB;
    Each has length ($\frac{1}{2}$) AB (Theorem 7-3.5)
    PM = QM = RM = SM (Postulate 2-1).

16. AE = BE (Definition 1-15)
    AE = EC (Def nition 1-15)
    BE = EC (Transitive property)
    m∠ A = m∠ ABE (Theorem 3-4.2)
    m∠ C = m∠ EBC (Theorem 3-4.2)
    m∠ A + m∠ ABE + m∠ C + m∠ EBC = 180 (Definition 1-28)
    2 m∠ ABE + 2 m∠ EBC = 180 (Postulate 2-1)
    m∠ ABC = m∠ ABE + m∠ EBC = 90 (Division property).

17. After proving △AMR ≅ △NCP and △MBN ≅ △PDR, we prove that quadrilateral MNPR is a parallelogram (Theorem 7-2.1);
    By proving that quadrilateral ABNR is a parallelogram (Theorem 7-2.2), RN = AB = AD;
    Quadrilateral ADPM is a parallelogram (Theorem 7-2.2)
    MP = AD (Definition 7-1);
    RN = MP (Transitive property).

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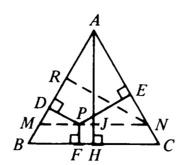
18. AC = BD (Theorem 7-3.2)
    DP = ($\frac{1}{2}$)BD (Definition 1-15)
    DP = ($\frac{1}{2}$)AE (Postulate 2-1)
    m∠ APE = 90 (Exercise 16)
    $\overline{APC} \perp \overline{PE}$ (Theorem 2-6.6)

19. 

    Draw $\overline{MPN} \parallel \overline{BC}$ where $\overline{MPN}$ intersects $\overline{AH}$ at J (Theorem 7-1.6)
    Draw $\overline{NR} \perp \overline{AB}$ with $\overline{ARD}$, $\overline{ENC}$.
    NR = PD + PE (See Exercise 20, Section 7-1)
    We can easily show that AJ = PD + PE
    AH = AJ + JH = PD + PE + PF (Addition property).

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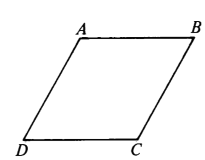
Class Exercises

1. 

2. Congruent.
3. Theorem 7-4.1.
4. No.
5. SSS
6. ∠ CBD.
7. ∠ CDB.
8. Angle bisector.
9. Each diagonal bisects two angles.
10. $\overline{AC} \perp \overline{BD}$ (Corollary 4-4.3a).
11. They are perpendicular.

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Exercises

1. No.
2. Yes, Theorem 7-4.5.
3. Yes, Theorem 7-4.6.

4. m∠ ABD + m∠ BAC = 90 (Corollary 6-4.2b)
   $(3x - 5) + (11x - 3) = 90$
   $x = 7$
   m∠ ABD = $3x - 5 = 16$
   m∠ ABC = m∠ ADC = 32 (Theorem 7-4.2; Theorem 7-1.3)
   m∠ BAC = $11x - 3 = 74$
   m∠ BAD = m∠ BCD = 148 (Theorem 7-4.2; Theorem 7-1.1)

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5. AB = DC and AD = BC (Theorem 7-1.2)
   $17x - 3 = 4x + 23$, and $x = 2$
   AB = 31 = BC
   Therefore parallelogram ABCD is a rhombus (Definition 7-6)

6. Draw rhombus ABCD with diagonal $\overline{BD}$.
   △ABD ≅ △CBD (Theorem 7-1.1)
   ∠ ABD ≅ ∠ CBD (Definition 3-3)
   ∠ ADB ≅ ∠ CDB (Definition 3-3)
   $\overline{BD}$ bisects ∠ B and ∠ D (Definition 1-29)
   Similarly for $\overline{AC}$.

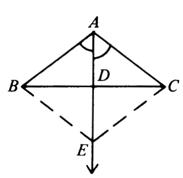
7. Draw rhombus PQRS with diagonals $\overline{PR}$ and $\overline{QS}$ meeting at M.
   PS = PQ (Theorem 7-4.1)
   SR = RQ (Theorem 7-4.1)
   $\overline{PR}$ is the perpendicular bisector of $\overline{QS}$ (Corollary 4-4.2a)
   $\overline{PR}$ ⊥ $\overline{QS}$ (Definition 4-4)

8. Conclusion follows from Theorem 7-1.2 and Theorem 7-4.1.

9. $\overline{PR}$ ⊥ $\overline{QS}$ (Theorem 7-4.3)
   $\overline{PR}$ bisects $\overline{QS}$ (Theorem 7-1.5)
   $\overline{PQ}$ ≅ $\overline{PS}$ (Theorem 4-4.2)
   Quadrilateral PQRS is a rhombus (Definition 7-6).

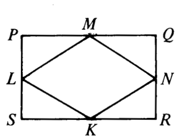
10. Draw parallelogram PQRS where $\overline{PR}$ bisects ∠ SPQ.
    ∠ QPR ≅ ∠ SRP (Theorem 6-3.1)
    ∠ SRP ≅ ∠ SPR (Transitive property)
    $\overline{SP}$ ≅ $\overline{SR}$ (Theorem 3-4.3)
    Parallelogram PQRS is a rhombus (Definition 7-6).

11.



△CAD ≅ △BAD (SAS)
$\overline{CD}$ ≅ $\overline{BD}$ (Definition 3-3)
$\overline{AD}$ ≅ $\overline{ED}$ (Definition 1-15)
Quadrilateral ABEC is a parallelogram (Theorem 7-2.6)
Parallelogram ABEC is a rhombus (Definition 7-6)
$\overline{AB}$ ≅ $\overline{EB}$ (Theorem 7-4.1)

12.



△LPM ≅ △NQM ≅ △NRK ≅ △LSK (SAS)
$\overline{LM}$ ≅ $\overline{NM}$ ≅ $\overline{NK}$ ≅ $\overline{LK}$ (Definition 3-3)
Quadrilateral MNKL is a rhombus (Theorem 7-4.4)

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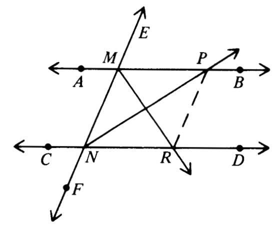
13.



m∠ QMN = (½) m∠ NMP (Definition 1-29)
m∠ QNM = (½) m∠ MNR (Definition 1-29)
m∠ NMP + m∠ MNR = 180 (Corollary 6-3.1b)
(½) m∠ NMP + (½) m∠ MNR = (½) (180) = 90 (Multiplication property)
m∠ QMN + m∠ QNM = 90 (Postulate 2-1)
$\overline{MQR}$ ⊥ $\overline{NQP}$ (Theorem 2-6.6)
△MQP ≅ △ MQN ≅ △RQN(ASA)
$\overline{MQ}$ ≅ $\overline{RQ}$ (Definition 3-3)
$\overline{NQ}$ ≅ $\overline{PQ}$ (Definition 3-3)
Quadrilateral MPRN is a parallelogram (Definition 7-1)
Parallelogram MPRN is a rhombus (Theorem 7-4.3).

14. m∠ Y = 180 - m∠ X.
    Since CY = BY,
    m∠ YCB = m∠ CBY = ½[180 - (180 - m∠ X)] = ½ m∠ X
    (Theorem 3-4.2 and Theorem 6-4.2)
    Similarly, since AX = BX,
    m∠ XAB = m∠ XBA = ½(180 - m∠ X) = 90 - ½ m∠ X.
    m∠ ABC = 180 - [m∠ XAB + m∠ CBY]
    = 180 - [90 - ½ m∠ X + ½ m∠ X] = 90
    Therefore △ABC is a right triangle.

15. Quadrilateral APHR is a parallelogram (Definition 7-1)
    m∠ HAB + m∠ NAB = 90 (Theorem 2-6.5)
    m∠ HAR + m∠ NAC = 90 (Subtraction property)
    m∠ NAB = m∠ NAC (Definition 1-29)
    m∠ HAB = m∠ HAR (Subtraction property)
    Parallelogram APHR is a rhombus (Theorem 7-4.6)

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Exercises

1. Rectangle, square.
2. All parallelograms.
3. Rhombus, square.
4. Rectangle, square.
5. All parallelograms.
6. Square.

7. m∠ HIJ = 90, m∠ LIJ = 60 and m∠ LIH = 150
   m∠ IHL = m∠ ILH (Theorem 3-4.2)
   m∠ IHL + m∠ ILH + m∠ LIH = 180 (Theorem 6-4.2)
   $y + x + 150 = 180$
   $y = 15$.

8. See Theorem 7-3.2 and Theorem 7-4.3.
9. See Theorem 7-4.2.
10. Use Theorem 7-1.5, and Theorem 7-4.3, HL. Also, each acute angle has measure 45.

11. Since a square is a rhombus we may use the result of Exercise 14 on page 274 and since a square is also a rectangle use the result of Exercise 12 on page 274. Other equally simple methods may be used involving congruent triangles.

12. m∠ BAC = (½) m∠ BAD = 45 (Theorem 7-4.2)
    △AQP is an isosceles right triangle (Definition 3-12, Definition 1-32)
    AQ = PQ (Definition 3-12, Definition 1-32)
    m∠ PQC = m∠ B = 90 (Theorem 2-6.5)
    △PQC ≅ △PBC (HL)
    PQ = PB (Definition 3-3).

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Exercises continued

13. m∠ TSN = ($\frac{1}{2}$)(90) = 45 (Theorem 7-4.2)
m∠ STN = m∠ SNT = ($\frac{1}{2}$) (180 - m∠ TSN) = ($\frac{1}{2}$) (180 - 45)
= 67 $\frac{1}{2}$ (Theorem 3-4.2, Postulate 2-1, Multiplication property)
$\overline{PR} \perp \overline{SQ}$ (Theorem 7-4.3)
m∠ STR = 90 (Theorem 2-6.5)
m∠ NTR = m∠ STR - m∠ STN = 90 - 67 $\frac{1}{2}$ = 22 $\frac{1}{2}$ (Postulate 2-11)
m∠ STN = 3m∠ NTR (Closure for multiplication).

14. m∠ ADP + m∠ RDC = 90 (Corollary 6-4.2b)
m∠ DCQ + m∠ RDC = 90 (Corollary 6-4.2b)
∠ ADP ≅ ∠ DCQ (Theorem 3-1.3)
AD = DC (Theorem 7-4.1)
Similarly, m∠ A = m∠ QDC = 90
△APD ≅ △DQC (ASA)
$\overline{PD} \cong \overline{QC}$ (Definition 3-3).

15. m∠ CBQ = m∠ DCQ = m∠ ADS = m∠ BAS (Definition 7-7, Subtraction property)
AB = BC = CD = DA (Definition 7-7, Subtraction property)
△APB ≅ △BQC ≅ △CRD ≅ △DSA (ASA)
PB = QC = RD = SA (Definition 3-3)
AP = BQ = CR = DS (Definition 3-3)
PQ = QR = RS = PS (Subtraction property)
Quadrilateral PQRS is a parallelogram (Theorem 7-2.1)
m∠ BAS + m∠ 4 = 90 (Definition 1-27)
m∠ BAS + m∠ 1 = 90 (Postulate 2-1)
m∠ APB = 90 (Theorem 6-4.2)
m∠ BPS = 90 (Theorem 3-1.2)
Parallelogram PQRS is a square (Definition 7-7).

16. Quadrilateral PQRS is a rectangle (Theorem 7-3.3)
∠ 2 ≅ ∠ 3 (Theorem 3-1.3)
Similarly ∠ 1 ≅ ∠ 2
∠ 1 ≅ ∠ 4
∠ 4 ≅ ∠ 3
△APB ≅ △BQC ≅ △CRD ≅ △DSA (AAS)
PB = QC = RD = SA (Definition 3-3)
AP = BQ = CR = DS (Definition 3-3)
PQ = QR = RS = PS (Subtraction property)
Rectangle PQRS is a square (Definition 7-7).

17. PQ = QR = SR = PS (Theorem 7-4.1)
PB = QC = RD = SA (Addition property)
m∠ APB = m∠ BQC = m∠ CRD = m∠ ASD (Theorem 7-3.1)
△APB ≅ △BQC ≅ △CRD = △DSA (SAS)
AB = BC = CD = DA (Definition 3-3)
Quadrilateral ABCD is a rhombus (Theorem 7-4.4)
m∠ 4 + m∠ SDA = 90 (Corollary 6-4.2b)
m∠ 3 + m∠ SDA = 90 (Postulate 2-1)
m∠ ADC = 90 (Postulate 2-10)
Rhombus ABCD is a square (Definition 7-7).

18. ∠ E ≅ ∠ F (Theorem 3-4.2)
△QEC ≅ △QFC (ASA)
$\overline{EQ} \cong \overline{FQ}$ (Definition 3-3)
$\overline{CQ} \perp \overline{EF}$ (Corollary 4-4.2a)
$\overline{AQC} \perp \overline{BD}$ (Theorem 2-6.6)
$\overline{EF} \parallel \overline{BD}$ (Theorem 6-1.1)

19. m∠ PQB + m∠ CPQ = 90 (Corollary 6-4.2b)
m∠ SPA + m∠ CPQ = 90 (Postulate 2-10)
m∠ PQB = m∠ SPA (Postulate 2-1)
$\overline{PQ} \cong \overline{SP}$ (Theorem 7-4.1)
m∠ BPQ = 45 = m∠ PSA (Theorem 7-4.2)
△PBQ ≅ △SAP (ASA)
$\overline{AS} \cong \overline{BP}$ (Definition 3-3)

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Class Exercises

1. Exactly 1, Postulate 6-1, Theorem 6-1.1.
2. $\overline{DB}$. Definition 1-15.
3. $\overline{EC}$. Theorem 7-6.1.
4. Midpoint.
5. Midline.
6. DE = $\frac{1}{2}$BC, Theorem 7-6.3.

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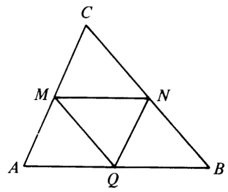
Exercises

Use Theorem 7-6.3 for Exercises 1-4.

1. 7 2. 9 3. $8\frac{1}{2}$ 4. 7-3/4

5.



Perimeter of △ABC = AB + BC + AC = 15
MN = $\frac{1}{2}$(AB) (Theorem 7-6.3)
MQ = $\frac{1}{2}$(BC) (Theorem 7-6.3)
NQ = $\frac{1}{2}$(AC) (Theorem 7-6.3)
Perimeter of △MNQ = MN + MQ + NQ
Perimeter of △MNQ = $\frac{1}{2}$(AB) + $\frac{1}{2}$(BC) + $\frac{1}{2}$(AC)
Perimeter of △MNQ = $\frac{1}{2}$(AB + BC + AC)
Perimeter of △MNQ = $\frac{1}{2}$(15)
Perimeter of △MNQ = $7\frac{1}{2}$.

6. Since DE = $\frac{1}{2}$BC (Theorem 7-6.3)
$7x - 1 = \frac{1}{2}(3x + 20)$
$x = 2$
Therefore DE = 13 and BC = 26.

7. $\overline{DE}$ is a midline (Theorem 7-6.4 and Definition 7-9)
Therefore D is midpoint of $\overline{AB}$
AD = $\frac{1}{2}$(AB) = DB
$18x - 31 = \frac{1}{2}(7x + 35)$
$x = 3$
AD = DB = $18x - 31$ = 23

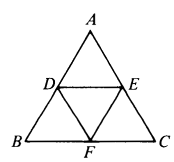
8. In △ABC, QP = $\frac{1}{2}$(BC) (Theorem 7-6.3)
5 = $\frac{1}{2}$(BC); Therefore BC = 10
In △DBC, MN = $\frac{1}{2}$(BC) = $\frac{1}{2}$(10) = 5. (Theorem 7-6.3)

9. 3/5

10. $\frac{2}{3}$

11. Since DF is 2/5 of BF, 2/5 x 15 = 6.

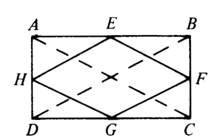
12.



(1) Draw equilateral △ABC with midpoints D, E, F of $\overline{AB}$, $\overline{AC}$ and BC, respectively.
DE = ($\frac{1}{2}$)BC (Theorem 7-6.3)
DF = ($\frac{1}{2}$)AC (Theorem 7-6.3)
EF = ($\frac{1}{2}$)AB (Theorem 7-6.3)
DE = DF = EF (Division property)
△DEF is equilateral.

(2) Use Theorem 7-6.2 to prove that the measure of each of the angles of △DEF equals 60.

13.



(1) Draw rectangle ABCD with midpoints E, F, G, H.
Each side of quadrilateral EFGH is ($\frac{1}{2}$)AC or ($\frac{1}{2}$)BD (Theorem 7-6.3)
AC = BD (Theorem 7-3.2)
Quadrilateral EFGH is a rhombus (Theorem 7-4.1).

(2) $\overline{EF} \parallel \overline{AC}$ (Theorem 7-6.2)
$\overline{HG} \parallel \overline{AC}$ (Theorem 7-6.2)
$\overline{EF} \parallel \overline{HG}$ (Corollary 6-1.1c)
Similarly $\overline{EH} \parallel \overline{BD}$

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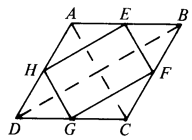
13. *(2) continued*

$\overline{FG} \parallel \overline{BD}$
$\overline{EH} \parallel \overline{FG}$
Quadrilateral EFGH is a parallelogram (Definition 7-1)
Parallelogram EFGH is a rhombus (Theorem 7-4.3)

14.

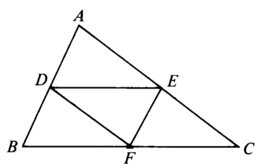


(1) Draw rhombus ABCD with midpoints E, F, G, H.
$\overline{EF} \parallel \overline{AC}$ (Theorem 7-6.2)
$\overline{HG} \parallel \overline{AC}$ (Theorem 7-6.2)
$\overline{EF} \parallel \overline{HG}$ (Corollary 6-1.1c)
Similarly, $\overline{EH} \parallel \overline{BD}$, $\overline{FG} \parallel \overline{BD}$, $\overline{EH} \parallel \overline{FG}$
$\overline{AC} \perp \overline{BD}$ (Theorem 7-4.3)
$\overline{EF} \perp \overline{EH}$ (Corollary 6-1.1b)
EFGH is a rectangle (Definition 7-5).

(2) The proof using Theorem 7-6.3 is similar.

15. Each side of the smaller triangle has half the length of the side of the larger triangle which is parallel to it. Therefore, the smaller triangle has half the perimiter of the larger triangle.

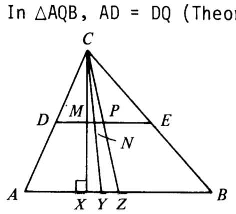
16.



Draw $\triangle ABC$ with D, E, F as midpoints of sides $\overline{AB}$, $\overline{AC}$, $\overline{CB}$, respectively.
Quadrilateral ADFE is a parallelogram (Theorem 7-6.2, Definition 7-1)
$\triangle AED \cong \triangle FDE$ (Theorem 7-1.1)
Similarly, $\triangle FDE \cong \triangle ECF$ and $\triangle FDE \cong \triangle DFB$.

17. In $\triangle AQB$, AD = DQ (Theorem 7-6.4)

18.



$\overline{DE}$ is a midline (Given)
$\overline{CX}$ is an altitude (Given)
$\overline{CY}$ bisects $\angle$ ACB (Given)
$\overline{CZ}$ is a median. (Given)
$\overline{DE} \parallel \overline{AB}$ (Theorem 7-6.2)
M, N, and P are midpoints of $\overline{CX}$, $\overline{CY}$, and $\overline{CZ}$ respectively (Theorem 7-6.4).
CM = MX (Definition 1-15)
CN = NY (Definition 1-15)
CP = PZ (Definition 1-15)

19. ML = $(\frac{1}{2})$PS (Theorem 7-6.3)
NK = $(\frac{1}{2})$PS (Theorem 7-6.3)
MN = $(\frac{1}{2})$RQ (Theorem 7-6.3)
LK = $(\frac{1}{2})$RQ (Theorem 7-6.3)
2ML + 2LK = PS + RQ, which is the perimeter of quadrilateral MNKL (Addition property).

20. $\overline{DE} \parallel \overline{BC}$ (Theorem 7-6.2)
DE = $(\frac{1}{2})$BC (Theorem 7-6.3)
Similarly, $\overline{MN} \parallel \overline{BC}$
MN = $(\frac{1}{2})$BC
$\overline{DE}$ = $\overline{MN}$ (Transitive property)
$\overline{DE} \parallel \overline{MN}$ (Corollary 6-1.1c)
Therefore, DENM is a parallelogram (Theorem 7-2.2).

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21. PD = $(\frac{1}{2})$MC (Theorem 7-6.3)
Similarly RE = $(\frac{1}{2})$MC.
$\overline{PD} \cong \overline{RE}$ (Transitive property)

22. Let BF = $(\frac{1}{2})$BC (Definition 1-15)
$\overline{DE}$ = $\overline{BF}$ (Transitive property)
$\overline{DE} \parallel \overline{BF}$ (Corollary 6-1.1c)
Quadrilateral DEFB is a parallelogram, EF = BD (Definition 7-1)
EF = $(\frac{1}{2})$AB (Theorem 7-6.3)
BD = AD (Transitive property)
AE = EC (Theorem 7-6.4, Definition 1-15).

23. $\overline{ML}$ is a midline of $\triangle$ABK.
Therefore BL = LK (Theorem 7-6.4)
$\overline{NK}$ is a midline of $\triangle$BCM.
Therefore KC = LK (Theorem 7-6.4)
Thus BL = LK = KC (Transitive property).

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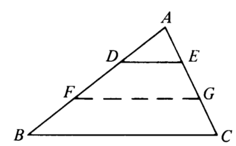
24. In $\triangle$ABD, $\overline{PQ} \parallel \overline{BD}$ (Theorem 7-6.2) and PQ = $\frac{1}{2}$ (BD) (Theorem 7-6.3).
In $\triangle$CBD, $\overline{SR} \parallel \overline{BD}$ (Theorem 7-6.2), and SR = $\frac{1}{2}$(BD) (Theorem 7-6.3)
Therefore, $\overline{PQ} \parallel \overline{SR}$ and PQ = SR.
Thus PQRS is a parallelogram (Theorem 7-2.2)
Also in $\triangle$ADC, $\overline{PS} \parallel \overline{AC}$ (Theorem 7-6.2)
Since $\overline{AC} \perp \overline{BD}$, $\overline{PS} \perp \overline{PQ}$.

25.



AE = $(\frac{1}{2})$EC = EG (Theorem 7-6.4)
D is the midpoint of $\overline{AF}$ (Theorem 7-6.4)
AD = $(\frac{1}{3})$AB (Multiplication property.
Use Theorem 7-6.1 for an alternate proof.

26. Since $\overline{PD}$ = $\frac{1}{2}$(AD) = $\frac{1}{2}$(BC) = BQ and $\overline{PD} \parallel \overline{BQ}$, PBQD is a parallelogram (Theorem 7-2.2)
Since $\overline{PM} \parallel \overline{DN}$, in $\triangle$ADN, AM = MN (Theorem 7-6.4)
Similarly since QN $\parallel$ BM, in $\triangle$CBM, CN = MN (Theorem 7-6.4)
Therefore AM = MN = CN.

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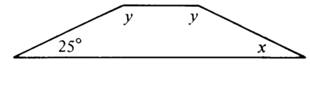
Exercises

1. Length of median = $\frac{1}{2}(3 + 7) = 5$.
2. $18\frac{1}{2}$.
3. 33.9
4. $17\frac{2}{3}$
5. $14 = \frac{1}{2}(11 + b)$
$b = 17$
6. 45.
7. 8
8. 38
9.



$x = 25$ (Theorem 7-7.3)
$y + 25 = 180$ (Theorem 7-7.5)
$y = 155$.

10. 58, 122, and 122 (see solution for Exercise 9).
11. 100, 80, and 80 (see solution for Exercise 9).
12. 170, 10, and 10 (see solution for Exercise 9).

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Exercises continued

13. AB = ½(PQ + SR) (Theorem 7-7.2)
$5x + 23 = \frac{1}{2}(16x - 15 + 13x + 4)$
$x = 3$
AB = $5x + 23 = 38$
PQ = $16x - 15 = 33$
SR = $13x + 4 = 43$

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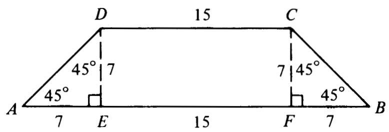
14. Trapezoid ABCD is isosceles (Theorem 7-7.8)
m∠ ADC = m∠ BCD (Theorem 7-7.3)
$23x - 3 = 5x + 33$
$x = 2$
m∠ ADC = m∠ BCD = 43
∠ DAB is supplementary to ∠ ADC (Theorem 7-7.5)
m∠ DAB = 137 = m∠ CBA

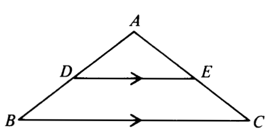
15.



Since △ABF and △DCE are *isosceles* right triangles,
AF = BF = 7, and DE = CE = 7.
Since BCEF is a rectangle, FE = BC = 15.
Therefore AD = AF + FE + DE = 7 + 15 + 7 = 29.

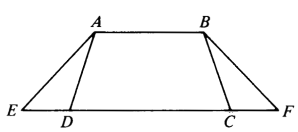
16. Draw trapezoid ABCD with median $\overline{MN}$.
$\overline{MN}$ ∩ $\overline{BD}$ at P, and $\overline{MN}$ ∩ $\overline{AC}$ at Q.
P and Q are midpoints of $\overline{BD}$ and $\overline{AC}$, respectively.
(Theorem 7-6.)
You may also wish to apply Theorem 7-6.4
to △ABD and △ABC.

17.



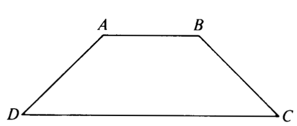
∠ B ≅ ∠ C (Theorem 3-4.2)
Quadrilateral DECB is an isosceles trapezoid
(Theorem 7-7.6).

18.



∠ ADC ≅ ∠ BCD (Theorem 7-7.3)
∠ ADE ≅ ∠ BCF (Theorem 3-1.4)
△ ADE ≅ △BCE (SAS)
∠ E ≅ ∠ F (Definition 3-3)
Trapezoid ABFE is an isosceles trapezoid (Theorem 7-7.6).

19.



Draw isosceles trapezoid ABCD with $\overline{AD}$ ≅ $\overline{BC}$
m∠ A + m∠ D = 180 (Corollary 6-3.1b)
∠ D ≅ ∠ C (Theorem 7-7.3)
m∠ A + m∠ C = 180 (Postulate 2-1)
Similarly, m∠ B + m∠ D = 180.

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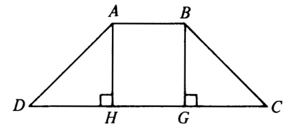
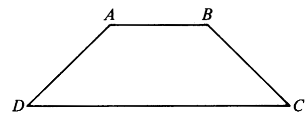
20.



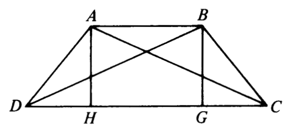
Draw trapezoid ABCD with ∠ D ≅ ∠ C,
$\overline{AH}$ ⊥ $\overline{DC}$
$\overline{BG}$ ⊥ $\overline{DC}$.
AH = DG (Theorem 7-1.6)
∠ AHD ≅ ∠ BGC (Theorem 7-3.1)
△AHD ≅ △ BGC (AAS)
AD = BC (Definition 3-3)
Trapezoid ABCD is isosceles (Definition 7-13).

21.



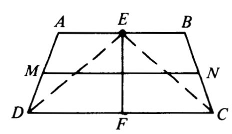
Draw trapezoid ABCD with m∠ BAD + m∠ C = 180.
$\overline{AB}$ ∥ $\overline{DC}$ (Definition 7-10)
m∠ BAD + m∠ D = 180 (Corollary 6-3.1b)
∠ C ≅ ∠ D (Theorem 3-1.4)
Trapezoid ABCD is isosceles (Theorem 7-7.6).

22.



Draw trapezoid ABCD with $\overline{AC}$ ≅ $\overline{BD}$
$\overline{AH}$ ⊥ $\overline{DC}$
$\overline{BG}$ ⊥ $\overline{DC}$.
AH = DG (Theorem 7-1.6)
△AHC ≅ △BGD (HL)
∠BDC ≅ ∠ACD (Definition 3-3)
△ADC ≅ △BCD (SAS)
∠ ADC ≅ ∠ BCD (Definition 3-3)
Trapezoid ABCD is isosceles (Theorem 7-7.6).

23.



Draw isosceles trapezoid ABCD where $\overline{AD}$ ≅ $\overline{BC}$.
E and F are midpoints of $\overline{AB}$ and $\overline{DC}$, respectively,
and $\overline{MN}$ is the median.
$\overline{AE}$ ≅ $\overline{BE}$ (Theorem 7-7.3)
∠ A ≅ ∠ B (Theorem 7-7.3)
△EAD ≅ △EBC (SAS)
$\overline{ED}$ = $\overline{EC}$ (Definition 3-3)
$\overline{EF}$ ⊥ $\overline{DC}$ (Theorem 4-4.2)
$\overline{MN}$ ∥ $\overline{DC}$ (Theorem 7-7.1)
$\overline{EF}$ ⊥ $\overline{MN}$ (Corollary 6-1.1b)

24. ∠ BAC ≅ ∠ ECA (Theorem 6-6.1)
∠ ABE ≅ ∠ CEB (Theorem 6-6.1)
$\overline{AQ}$ ≅ $\overline{CQ}$ (Definition 1-15)
△ABQ ≅ △CEQ (AAS)
$\overline{BQ}$ ≅ $\overline{QE}$ (Definition 3-3)
$\overleftrightarrow{PQ}$ ∥ $\overleftrightarrow{DE}$ (Theorem 7-6.2)
$\overleftrightarrow{PQ}$ ∥ $\overleftrightarrow{AB}$ (Corollary 6-1.1c)
AB = CE (Definition 3-3)
DE = DC - CE = DC - AB (Subtraction property)
PQ = (½)DE (Theorem 7-6.3)
PQ = (½)(DC - AB)(Postulate 2-1)
E is the midpoint of $\overline{DC}$ (Theorem 7-6.4).

25. MN = $x + y$ (Postulate 2-1)
AB + DC = $2x + 2y$ (Postulate 2-1)
MN = (½)(AB + DC) (Division property, Postulate 2-1).

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Review Exercises

1. $\triangle DCB$; Theorem 7-1.1.
2. BC (Theorem 7-1.2).
3. $\angle ABC$ (Theorem 7-1.3)
4. $\overline{EC}$ (Theorem 7-1.5)
5. $\triangle CEB$; Theorem 7-1.5; Theorem 7-1.2, SSS Postulate (other methods also possible). 6. $\overline{DC}$ (Def. 7-1)
7. Use Theorem 7-1.4 to get the angle measures 47, 133, and 133.
8. BE = DE (Theorem 7-1.5).
   $7x - 20 = 12x - 35$
   $x = 3$
   BD = 2(BE) = $2(7x - 20)$ = 2.

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9. $\overline{PQ} \cong \overline{QT}$ (Theorem 7-1.2)
   $\overline{RS} \cong \overline{PQ}$ (Theorem 7-1.2)
   $\overline{QT} \cong \overline{RS}$ (Transitive property)
   $\angle TQN \cong \angle SRN$ (Theorem 6-3.1)
   $\angle T \cong \angle RSN$ (Theorem 6-3.1)
   $\triangle TQN \cong \triangle SRN$ (ASA)
   $\overline{QN} \cong \overline{RN}$ (Definition 3-3)
   $\overline{TN} \cong \overline{SN}$ (Definition 3-3)

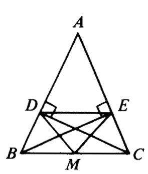
10. True.
11. True.
12. False (could be trapezoid).

13. $\overline{BM} \cong \overline{CM}$ (Definition 3-9)
    $m\angle BEM = 90 = m\angle CDM$ (Theorem 3-1.5, Theorem 3-1.2)
    $\triangle BEM \cong \triangle CDM$ (AAS)
    EM = DM (Definition 3-3)
    Quadrilateral BECD is a parallelogram (Theorem 7-2.6).

14. 14; Theorem 7-3.5.

15. $\triangle ADC \cong \triangle BCD$ (HL)
    AD = BC (Definition 3-3)
    Quadrilateral ABCD is a parallelogram (Theorem 7-2.2).
    Parallelogram ABCD is a rectangle (Theorem 7-3.4).

16. AC = BD (Theorem 7-3.2)
    ED = EC (Transitive property)
    $\triangle EFD \cong \triangle EFC$ (HL)
    $\overline{DF} \cong \overline{CF}$ (Definition 3-3)

17. 

    DM = $(\frac{1}{2})$BC (Theorem 7-3.5)
    EM = $(\frac{1}{2})$BC (Theorem 7-3.5)
    DM = EM (Transitive property)
    $\triangle DME$ is isosceles (Definition 3-12).

18. To prove that a quadrilateral is a rhombus, prove that:
    1. it has four congruent sides.
    2. it is a parallelogram with consecutive sides congruent.
    3. it is a parallelogram in which a diagonal bisects an angle of the parallelogram.
    4. it is a parallelogram with perpendicular diagonals.

19. Perpendicular.

20. Since AB = AD (Theorem 7-4.1) $4x - 7 = 2x + 17$, and $x = 12$.
    BC = AD = 2(12) + 17 = 41.

21. Since $\triangle DEC$ is a right triangle (Theorem 7-4.3), $\angle BDC$ is supplementary to $\angle ACD$ (Corollary 6-4.2b)
    Therefore $m\angle ACD = 67$.

22. $m\angle DAC = m\angle BAC$ (Theorem 7-4.2)
    Therefore $m\angle DAC = 32$.

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23. Since $\triangle AED$ is a right triangle (Theorem 7-4.3),
    $m\angle ADB + m\angle DAC = 90$.
    Therefore $(7x - 11) + (2x - 7) = 90$, and $x = 12$.
    $m\angle DBC = m\angle ADB$ (Theorem 6-3.1)
    Thus $m\angle DBC = 7x - 11 = 73$.

24. DE = $(\frac{1}{2})$AB = EB (Theorem 7-3.5).
    $\angle EDB \cong \angle EBD$ (Theorem 3-4.2)
    Similarly, $\angle BDF \cong \angle DBF$.
    $\angle EBD \cong \angle BDF$ (Theorem 6-3.1)
    $\angle EDB \cong \angle DBF$ (Transitive property)
    $\overline{DE} \parallel \overline{BF}$ (Theorem 6-2.1)
    Quadrilateral DEBF is a parallelogram (Theorem 7-2.4).
    Parallelogram DEBF is a rhombus (Definition 7-6).

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25. Congruent sides, all angles, right angles, congruent diagonals, perpendicular diagonals, diagonals bisect opposite angles, diagonals bisect each other, and opposite sides parallel.

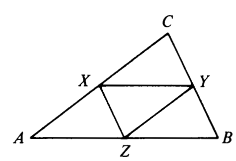
26. To prove that a quadrilateral is a square, prove that:
    1. it is a rectangle with consecutive sides congruent.
    2. it is a rectangle with a diagonal bisecting one of its angles.
    3. it is a rectangle with perpendicular diagonals.
    4. it is a rhombus with one right angle.
    5. it is a rhombus with congruent diagonals.

27. Yes, it has all the properties of both quadrilaterals.

28. 45.

29. AP = BQ = CR = DS (Subtraction property)
    $\angle A \cong \angle B \cong \angle C \cong \angle D$ (Theorem 3-1.1)
    $\triangle SAP \cong \triangle PBQ \cong \triangle QCR \cong \triangle RDS$ (SAS)
    $\overline{SP} \cong \overline{PQ} \cong \overline{QR} \cong \overline{RS}$ (Definition 3-3)
    $m\angle ASP + m\angle APS = 90$ (Corollary 6-4.2b)
    $m\angle APS = m\angle DSR$ (Definition 3-3)
    $m\angle ASP + m\angle DSR = 90$ (Postulate 2-1)
    Quadrilateral PQRS is a square (Definition 7-7).

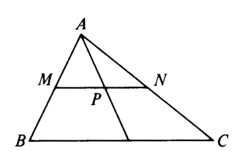
30. $\frac{1}{2}(24) = 12$ (Theorem 7-6.3)

31. 

    P_1 of $\triangle XYZ$ = XY + XZ + YZ = 15
    P_2 of $\triangle ABC$ = AB + BC + AC
    XY = $\frac{1}{2}$(AB) (Theorem 7-6.3)
    XZ = $\frac{1}{2}$(BC) (Theorem 7-6.3)
    YZ = $\frac{1}{2}$(AC) (Theorem 7-6.3)
    XY + XZ + YZ = $\frac{1}{2}$(AB + BC + AC)
    15 = $\frac{1}{2} P_2$ of $\triangle ABC$
    P_2 of $\triangle ABC$ = 30.

32. MN = $\frac{1}{2}$(BC) (Theorem 7-6.3)
    $7x - \frac{1}{2} = \frac{1}{2}(3x + 21)$ and $x = 2$.
    Therefore MN = $\frac{1}{2}(3x + 21) = 13\frac{1}{2}$.

33. Draw $\triangle ABC$ with $\overline{MN}$ a midline ($\overline{AMB}$, $\overline{ANC}$), and $\overline{AD}$ a median intersecting $\overline{MN}$ at P.
    $\overline{MPN} \parallel \overline{BC}$ (Theorem 7-6.2)
    P is the midpoint of $\overline{AD}$ (Theorem 7-6.4)
    $\overline{MN}$ bisects $\overline{AD}$ (Definition 1-15).



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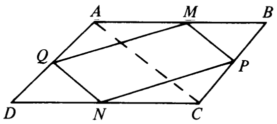
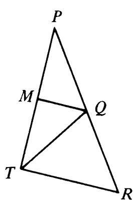
Review Exercises continued

34.



$\overline{MP} \parallel \overline{NQ}$ (Corollary 6-1.1c)
P is the midpoint of $\overline{BE}$ (Theorem 7-6.4)
MP = (½)AE (Theorem 7-6.3)
Similarly, NQ = (½)CE (Theorem 7-1.5)
AE = CE(Theorem 7-1.5)
MP = NQ (Postulate 2-1)
Quadrilateral MPNO is a parallelogram (Theorem 7-2.2).

35.



Q is the midpoint of $\overline{PR}$ (Theorem 7-6.4)
QT = QR (Transitive property)
∠ QTR ≅ ∠ R (Theorem 3-4.2)

36. Definition 7-10, Theorem 7-7.1, Theorem 7-7.2.

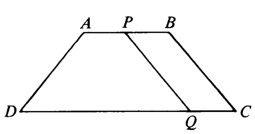
37. Congruent legs, congruent diagonals, congruent base angles, and opposite angles supplementary.

38. To prove that a trapezoid is isosceles, prove that:
   1. its nonparallel sides are congruent.
   2. the base angles of one pair are congruent.
   3. the opposite angles of one pair are supplementary.
   4. its diagonals are congruent.

39. 18 = ½(32 + x) (Theorem 7-7.2)
   x = 4.

40. Use Theorem 7-6.1 or Theorem 7-6.4.

41.



Quadrilateral PBCQ is a parallelogram (Theorem 7-2.2).
$\overline{BC} \cong \overline{PQ}$ (Theorem 7-1.2)
$\overline{PQ} \cong \overline{AD}$ (Transitive property)
Trapezoid APQD is isosceles (Definition 7-13).

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Chapter Test

1. 34; Theorem 7-3.2.

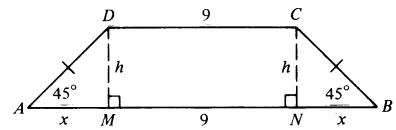
2. △ADB is isosceles (Definition 7-6).
   Therefore m∠ ADB = 50 (Theorem 3-4.2)

3. 10 = ½(x + 6); x = 14

4. If $\overline{AB} \parallel \overline{PQ} \parallel \overline{RS} \parallel \overline{DC}$
   AP = PR = RD = x (Theorem 7-6.1)
   BQ = QS = SC = y (Theorem 7-6.1)
   $3x$ = AD = 21
   x = 7
   AD = BC = 21 (Theorem 7-1.2)
   $3y$ = BC = 21
   y = 7
   SB = $2y$ = 14

5. AD = DB = CD (Theorem 7-3.5)
   $15x$ - 17 = $8x$ - 3
   x = 2
   DB = $15x$ - 17 = 13
   AB = AD + DB = 26

6.



△ADM ≅ △BCN (HL)
AM = NB (Definition 3-3)
Since DCNM is a rectangle, MN = 9
x + 9 + x = 17
x = 4
x = h = 4 (Theorem 3-4.3)

7. $\overline{MF} \parallel \overline{CHF}$ (Theorem 7-6.2)
   $\overline{MN} \parallel \overline{AB}$ (Theorem 7-6.2)
   $\overline{MF} \perp \overline{MN}$ (Corollary 6-1.1b)

8. PQ = (½)BC (Theorem 7-6.3)
   RS = (½)BC (Theorem 7-6.3)
   $\overline{RS}$ = $\overline{PQ}$ (Transitive property)
   $\overline{PQ} \parallel \overline{BC}$ (Theorem 7-6.2)
   $\overline{RS} \parallel \overline{BC}$ (Theorem 7-6.2)
   $\overline{PQ} \parallel \overline{RS}$ (Corollary 6-1.1c)
   Quadrilateral PQSR is a parallelogram (Theorem 7-2.2).

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Exercises

1. $\frac{BC}{DC} = \frac{BE}{EC}$

2. $\frac{EC}{GF} = \frac{EH}{HF}$

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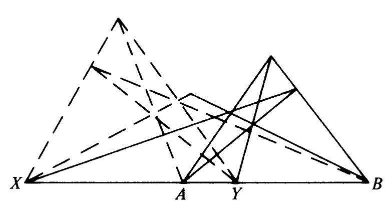
3.



The dashed lines depict the second quadrilateral.

4. Since A, B, C, D is an harmonic range, O-A, B, C, D is an harmonic pencil. Since O-E, G, F, H is the same pencil as O-A, B,C, D, we have that E, G, F, H is an harmonic range.
   No. They are equal if and only if $\overline{ACBD} \parallel \overline{EGFH}$.

5. Refer to Exercise 4.

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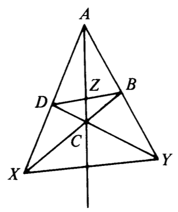
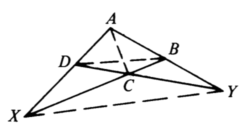
Exercises continued

6. Quadrangle
   Vertices A, B, C, D
   Sides $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{CD}$, $\overleftrightarrow{DA}$, $\overleftrightarrow{AC}$, $\overleftrightarrow{DB}$
   Diagonal points X, Y, Z.



   Quadrilateral
   Vertices X, D, A, B, Y, C
   Sides $\overleftrightarrow{XA}$, $\overleftrightarrow{XB}$, $\overleftrightarrow{YD}$, $\overleftrightarrow{YA}$
   Diagonals $\overleftrightarrow{AC}$, $\overleftrightarrow{DB}$, $\overleftrightarrow{XY}$.



7. The harmonic conjugate of C is the ideal point on $\overleftrightarrow{AB}$.

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Class Exercises

For example, $\frac{2}{3} = \frac{4}{6} = \frac{10}{15}$.

1. 16 2. 24 3. $\frac{16}{24} = \frac{2}{3}$ 4. Equal.

5. $\frac{2}{5} = \frac{6}{15} = \frac{8}{20} = \frac{10}{25} = \frac{26}{65} = \frac{2}{5}$

6. It would equal any of the seven ratios.

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Exercises

1. $\frac{1}{4}$ 2. $\frac{1}{3}$ 3. $\frac{3}{4}$ 4. $\frac{4}{1}$

5. 21 6. $7b$ 7. $5\frac{3}{5}$ 8, $\frac{15}{17}$

9. $\frac{pr}{q}$ 10. $\sqrt{AB \cdot CD}$

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11. $\frac{x}{y} = \frac{3}{5}$ and $\frac{5}{y} = \frac{3}{x}$.

12. $\frac{RT}{PQ} = \frac{PS}{PR}$ and $\frac{PS}{RT} = \frac{PR}{PQ}$.

13. $\frac{3}{5} = \frac{13}{x}$ and $x = 21\frac{2}{3}$

14. 12 15. $15\frac{2}{5}$. 16. 24

17. $\frac{2}{x} = \frac{x}{8}$ and $x = 4$

18. 18. 19. $\sqrt{15}$. 20. 2.

21. $\frac{a}{b} = \frac{c}{d}$ if and only if $a \cdot d = b \cdot c$ (Theorem 8-1.1)
    Equivalently, $a \cdot d = b \cdot c$ if and only if $\frac{b}{a} = \frac{d}{c}$

    $\frac{a}{b} = \frac{c}{d}$ if and only if $\frac{b}{a} = \frac{d}{c}$ (Transitive property).

    Note: many variations of this proof are possible.

22. $\frac{a}{b} = \frac{c}{d}$ if and only if $a \cdot d = b \cdot c$ (Theorem 8-1.1).
    Equivalently, $a \cdot d = b \cdot c$ if and only if $\frac{a}{c} = \frac{b}{d}$.

    $\frac{a}{b} = \frac{c}{d}$ if and only if $\frac{a}{c} = \frac{b}{d}$ (Transitive property).

23. See proof outline on Page 301.

24. See proof outline on Page 301.

25. $\frac{70}{49}$ 26. $\frac{11}{22}$

27. $\frac{25}{15}$ 28. $p = z$, $q = w$.

29. EC (Corollary 8-1.1b).

30. DE (Corollary 8-1.1b).

31. BC (Theorem 8-1.1).

32. AB (Theorem 8-1.1).

33.

    $3x + 5x = 40$, and $x = 5$. Therefore $AP = 3x = 13$.

34.

    $3x = 28$, and $x = \frac{28}{3}$. Therefore $CD = 7x = \frac{196}{3}$, and
    $QD = \frac{112}{3}$.

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Exercises

1. $\frac{NP}{MP} = \frac{NQ}{KQ}$ (Theorem 8-2.1

   $\frac{15}{12} = \frac{10}{x - 10}$

   $15x - 150 = 120$

   $15x = 270$

   $x = 18$

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Exercises continued

2. $\frac{NQ}{KQ} = \frac{NP}{PM}$ (Theorem 8-2.1)

   $\frac{4}{5} = \frac{x}{36 - x}$

   $5x = 144 - 4x$

   $x = 16$

3. $\frac{NQ}{KQ} = \frac{NP}{PM}$ (Theorem 8-2.1)

   $\frac{5}{40} = \frac{x}{8}$

   $x = 1$

4. $\frac{NP}{PM} = \frac{NQ}{KQ}$ (Theorem 8-2.1)

   $\frac{6}{4} = \frac{12 - x}{x}$

   $6x = 48 = 4x$

   $x = 4\frac{4}{5}$.

5. $\frac{28}{21} = \frac{8}{6}$

   $\frac{4}{3} = \frac{4}{3}$

   Therefore $\frac{NQ}{KQ} = \frac{NP}{PM}$ and $\overline{PQ} \parallel \overline{MK}$ (Theorem 8-2.2).

6. $\frac{2}{3} \neq \frac{4}{3}$

   Therefore $\frac{NP}{PM} \neq \frac{NQ}{KQ}$ and $\overline{PQ} \parallel \overline{MK}$ (Theorem 8-2.2)

7. $\frac{8}{2} \neq \frac{15}{5}$

   Therefore $\frac{NP}{PM} \neq \frac{NQ}{KQ}$ and $\overline{PQ} \parallel \overline{MK}$ (Theorem 8-2.2)

8. $\frac{4\frac{1}{2}}{3\frac{3}{4}} = \frac{18}{15}$

   $\frac{\frac{9}{2}}{\frac{15}{4}} = \frac{6}{5}$

   $\frac{6}{5} = \frac{6}{5}$; Therefore $\frac{NP}{PM} = \frac{NQ}{KQ}$ and $\overline{PQ} \parallel \overline{MK}$ (Theorem 8-8.2).

9. $\frac{JK}{NK}$; Corollary 8-2.1a

10. $\frac{KM}{MH}$; Theorem 8-2.1

11. $\frac{NJ}{KJ}$; Corollary 8-2.1a

12. $\frac{HK}{MH}$; Corollary 8-2.1a

13. $\frac{7}{9} = \frac{5}{x}$

    $x = 6\frac{3}{7}$

14. $\frac{5}{13} = \frac{x}{39}$

    $x = 15$

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15. $\frac{1}{7} = \frac{3}{x}$ (Postulate 8-1)

    $x = 21$

16. $\frac{5}{3} = \frac{16 - x}{x}$ (Postulate 8-1)

    $x = 6$

17. $\frac{15}{x - 15} = \frac{5}{4}$ (Postulate 8-1)

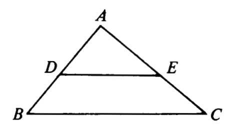
    $x = 27$

18. $\frac{3}{x} = \frac{4}{5}$ (Theorem 8-2.1)

    $x = 3\frac{3}{4}$

    $\frac{x}{4} = \frac{5}{y}$ (Postulate 8-1)

    $y = 5\frac{1}{3}$

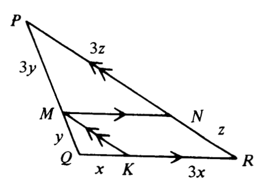
19. 

    Given: AB/AD = AC/AE

    Prove: $\overline{DE} \parallel \overline{BC}$

    From Theorem 8-1.3, (AB - AD)/AD = (AC - AE)/AE.
    Therefore BD/AD = EC/AE.
    Thus, $\overline{DE} \parallel \overline{BC}$ (Theorem 8-2.2).

20. $\overleftrightarrow{MP} \parallel \overline{AB}$ (Corollary 8-2.2a)
    $\overline{MPN} \parallel \overline{DC}$ or $\overline{PN} \parallel \overline{DC}$ (Corollary 6-1.1c)
    DP/BD = NC/BC (Corollary 8-2.1a).

21. 

    QK/KR = QM/MP (Theorem 8-2.1)
    1/3 = QM/MP
    3/1 = MP/QM (Corollary 8-1.1a)

22. NR/PN = QM/MP (Theorem 8-1.2)
    (NR + PN)/PN = (QM + MP)/MP
    PR/PN = PQ/MP
    PN/PR = MP/PQ (Corollary 8-1.1a)
    PN/PR = $3y/4y$ = 3/4

23. MQ/PQ = $1y/4y$ = 1/4

24. NR/PN = $1z/3z$ = 1/3

25. AE/EC = BP/PC (Theorem 8-2.1)
    $\overline{EP} \parallel \overline{CD}$ (Corollary 6-1.1c)
    BE/ED = BP/PC (Theorem 8-2.1)
    AE/EC = BE/ED (Transitive property).

26. AF/FD = AE/EC (Theorem 8-2.1)
    8/12 = AE/EC
    AD/DB = AE/EC (Theorem 8-2.1)
    20/DB = 8/12
    DB = 30.

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27. BE/CE = DE/PE (Corollary 8-2.1a)
DE/PE = BD/BQ (Corollary 8-2.1a)
BE/CE = BD/BQ (Transitive property)
$\angle$ADF $\cong$ $\angle$ AFD (Theorem 3-4.2)
$\angle$ ADF $\cong$ $\angle$ CPD (Theorem 6-3.1)
$\angle$ AFD $\cong$ $\angle$ CFP (Theorem 3-1.5)
$\angle$ CPD $\cong$ $\angle$ CFP (Transitive property)
CF = CP (Theorem 3-4.3)
BQ = CP (Theorem 7-1.2)
CF = BQ (Transitive property)
BE/CE = BD/CF (Transitive property)
BE • CF = BD • CE (Theorem 8-1.1).

28. BP/PC = BN/NA (Theorem 8-2.1)
BP/PC = BQ/QR (Theorem 8-2.1)
BN/NA = BQ/QR (Transitive property)
$\overline{NQ} \parallel \overline{AR}$ (Theorem 8-2.2).

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1. Write each statement and reason of proof of Theorem 8-3.1 (pages 310-311), but replace $\triangle$ECB in statement 8 with $\triangle$ACD.

2. Have students realize the analogy between an interal (Theorem 8-3.1) and external (Corollary 8-3.1a) partitioning of a line segment ($\overline{BC}$).

3. *Corollary 8-3.1a* An exterior angle bisector of a triangle determines with each of the other vertices segments along the line containing the opposite side of the triangle which are proportional to the two remaining sides.

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1. BT/TC = AB/AC (Theorem 8-3.1)
4/TC = 16/24
TC = 6

2. BT/TC = AB/AC (Theorem 8-3.1)
BT/6 = 20/15
BD = 8

3. BT/TC = AB/AC (Theorem 8-3.1)
BT/(18 - BT) = 12/24
BT = 6 and TC = 12

4. BT/TC = AB/AC (Theorem 8-3.1)
3/4 = 6/AC
AC = 8

5. BT/TC = AB/AC (Theorem 8-3.1)
3/TC = 5/AC
TC/AC = 3/5

6. BT/TC = AB/AC (Theorem 8-3.1)
(BT + TC)/TC = (AB + AC)/AC (Theorem 8-1.2)

BC/TC = (AB + AC)/AC

BC/4 = (AB + AC)/7
BC/(AB + AC) = 4/7

7. BT/TC = AB/AC (Theorem 8-1.2)
BT/TC = 11/11 = 1/1

8. BT/TC = AB/AC (Theorem 8-1.2)
BT/(7 - BT) = 5/6
BT = $3\frac{2}{11}$, and TC = $3\frac{9}{11}$.

9. BT/TC = AB/AC (Theorem 8-3.1)
BT/5 = 13/14
BT = 65/14
BC = BT + TC
BC = 65/14 + 5
BC = 135/14

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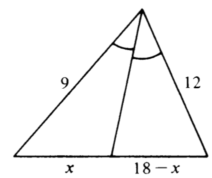
10. BT/TC = AB/AC (Theorem 8-3.1)
$4/2x = 3x/6$
$x = 2$
AB = $3x$ = 6
TC = $2x$ = 4

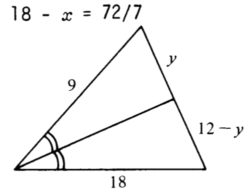
11. BT/TC = AB/AC (Theorem 8-3.1)
$3b/4d = 2a/5d$
$8ad = 15bd$
$8a = 15b$
$a/b = 15/8$

12.

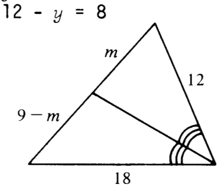


$x/(18 - x) = 9/12$ (Theorem 8-3.1)
$x = 7\frac{5}{7}$

$18 - x = 72/7$



$y/(12 - y) = 9/18$ (Theorem 8-3.1)
$y = 4$
$12 - y = 8$



$m/(9 - m) = 12/18$ (Theorem 8-3.1)

$m = 18/5$

$9 - m = 27/5$

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13. RT/QT = RP/PQ (Corollary 8-3.1a)
RT/9 = 10/6
RT = 15

14. RT/QT = RP/PQ (Corollary 8-3.1a)
15/QT = 11/8
QT = 120/11

15. RT/QT = RP/PQ (Corollary 8-3.1a)
$(x + 18)/18 = 24/16$
x = 9 = RQ

16. RT/QT = RP/PQ (Corollary 8-3.1a)
RT/(RT - 40) = 32/24
RT = 160, and QT = 120
x - 40 = 120 = QT

17. RT/QT = RP/PQ (Corollary 8-3.1a)
17/10 = RP/4
RP = 34/5

18. No, $\overline{SP} \parallel \overline{TQ}$ and R does not exist.

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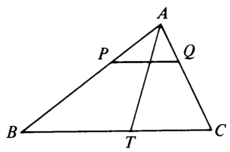
Exercises

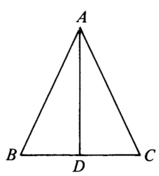
19. AD/DB = AF/BF (Theorem 8-3.1)
AE/EC = AF/CF (Theorem 8-3.1)
BF = CF (Definition 1-15)
AD/DB = AE/EC (Transitive property)
$\overline{DE} \parallel \overline{BC}$ (Theorem 8-2.2).

20.



AB/BT = AC/CT (Definition 1-29)
AB/BP = AC/CQ (Postulate 2-1)
$\overline{PQ} \parallel \overline{BC}$ (Corollary 8-2.2a)

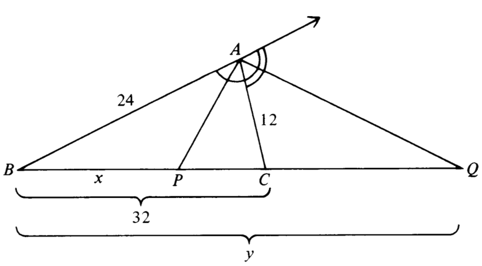
21



Draw △ABC with AD with $\overline{AD}$ an angle bisector and D the midpoint of $\overline{BC}$.
AB/AC = BD/DC (Theorem 8-3.1)
BD = DC (Definition (1-15)
AB/AC = 1 (Postulate 2-1)
AB = AC (Multiplication property)
△ABC is isosceles (Definition 3-12).

22. Use the diagram accompanying Theorem 8-3.1.
CD/DB = CA/AB (Given)
Draw $\overline{BE} \parallel \overline{AD}$.
CD/DB = CA/AE (Theorem 8-2.1)
AB = AE (Theorem 8-1.5)
∠ 4 ≅ ∠ 2 (Theorem 3-4.3)
∠ 3 ≅ ∠ 4 (Corollary 6-3.1a)
∠ 1 ≅ ∠ 2 (Theorem 6-3.1)
∠ 1 ≅ ∠ 3 (Transitive property)
$\overline{AD}$ bisects ∠ BAC (Definition 1-29).

23.



Let BP = x, and BQ = y.
From Theorem 8-3.1:

$$\frac{x}{32 - x} = \frac{24}{12}$$

and $x = 21\frac{1}{3}$
From Corollary 8-3.1a:

$$\frac{y}{y - 32} = \frac{24}{12}$$

and $y = 64$.

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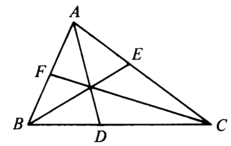
24.



Using angle bisector $\overline{AD}$: BD/DC = AB/AC (Theorem 8-3.1)
Using angle bisector $\overline{BE}$: EC/AE = BC/AB (Theorem 8-3.1)
Using angle bisector $\overline{CF}$: AF/BF = AC/BC (Theorem 8-3.1)

By the multiplication property:

$$\frac{BD}{DC} \cdot \frac{EC}{AE} \cdot \frac{AF}{BF} = \frac{AB}{AC} \cdot \frac{BC}{AB} \cdot \frac{AC}{BC} = 1$$

Therefore, AF • BD • EC = BF • DC • AE.

25. DP/PB = AD/AB (Theorem 8-3.1)
AQ/QC = AD/DC (Theorem 8-3.1)
AB = DC (Theorem 7-1.2)
DP/PB = AQ/QC (Postulate 2-1, Transitive property)
DP/(PM + MB) = AQ/(QM + MC) (Postulate 2-1)
DP/(PM + DM) = AQ/(QM + AM) (Theorem 7-1.5)
DP/(2PM + DP) = AQ/(2QM + AQ) (Postulate 2-1)
DP/2PM = AQ/2QM (Theorem 8-1.2)
DP/PM = AQ/QM (Multiplication property)
$\overline{PQ} \parallel \overline{AD}$ (Theorem 8-2.2).

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26. The proof of the converse of Corollary 8-3.1a is similar to the proof of the converse of Theorem 8-3.1 (see Exercise 22). Using this converse, BC/AC = BD/DA. $\overline{DC}$ is an exterior angle bisector of △ABC (Corollary 8-3.1a We can prove that $\overline{CE}$ bisects ∠ ACB. BE/EA = BC/AC = ½ (Transitive property). BE/AB = BE/(BE + EA) = $\frac{1}{3}$ (Postulate 2-1).

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Exercises

1. K 2. RK 3. RP/PQ.

4. RQ/JK = RP/KH (Definition 8-6)
6/15 = 4/KH
HK = 10.

5. RQ/JK = PQ/JH (Definition 8-6)
6/15 = PQ/12
PQ = 24/5

6. From Exercise 4 or Exercise 5: RQ/JK = 6/15 = 2/5.

7. True (Definition 3-3, Definition 8-6).

8. False.

9. True (Theorem 8-4.1).

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10. HM/AS = HB/AP = MB/SP (Definition 8-6)
8/AS = 9/3
AS = 8/3

11. 10/AP = 9/3
AP = $3\frac{1}{3}$

12. MB/SP = 9/3 = 3/1 (Definition 8-5).

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Exercises continued

13. $\wp\triangle HMB/\wp\triangle ASP = 27/9 = 3/1$, or
$\wp\triangle HMB/\wp\triangle ASP = MB/SP = 9/3 = 3/1$

14. Consider two similar n-gons. If their ratio of similitude is R, then the ratio of any pair of corresponding sides is R. Use Theorem 8-1.4 to complete the proof.

15. 5/7

16.

13	14	15
x		21

$13/x = 15/21$ (Definition 8-6)
$x = 91/5$

17.

13	14	15	$\wp_1 = 42$
		21	$\wp_2 =$

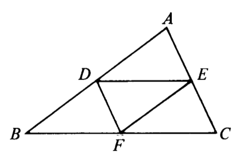
$15/21 = 42/\wp_2$ (From Exercise 14, page 318)

$\wp_2 = 294/5$

18. Corresponding sides are in proportion, corresponding angles are congruent (Transitive property). The triangles are similar (Definition 8-6).

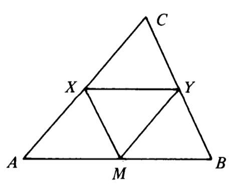
19. Use Postulate 2-1 and Definition 8-6.

20.



Draw $\triangle ABC$ with D, E, and F as midpoints of $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$, respectively.
$DE = (\frac{1}{2})CB$ (Theorem 7-6.3)
$DE/CB = \frac{1}{2}$ (Division property)
Similarly, $EF/AB = \frac{1}{2}$ and $DF/AC = \frac{1}{2}$
$DE/CB = EF/AB = DF/AC$ (Transitive property)
Quadrilaterals DECF, DEFB, and AEFD are parallelograms (Definition 7-1)
$\angle EDF \cong \angle C$ (Theorem 7-1.3)
$\angle DEF \cong \angle B$ (Theorem 7-1.3)
$\angle DFE \cong \angle A$ (Theorem 7-1.3)
$\triangle ABC \sim \triangle FED$ (Definition 8-6)

21.



$\triangle XYM \sim \triangle ABC$ (from Exercise 20, page 318)
$XY = \frac{1}{2}(AB)$ (Theorem 7-6.3)
$XY/AB = \frac{1}{2}$
$\wp\triangle XYZ/\wp\triangle ABC = XY/AB = \frac{1}{2}$ (from Exercise 14, page 318)

22. All the angles are congruent to one another (Corollary 3-4.2a). The sides of each triangle are congruent (Definition 3-12). The ratios of the corresponding sides are equal (Definition 8-1). Equilateral triangles are similar (Definition 8-6).

23. $\angle CEF \cong \angle A$ (Definition 3-3)
$\overline{EF} \parallel \overline{AB}$ (Corollary 6-3.1a)
Similarly $\overline{DE} \parallel \overline{BC}$
Quadrilateral DEFB is a parallelogram (Definition 7-1)
$\angle B \cong \angle DEF$ (Theorem 7-1.3).

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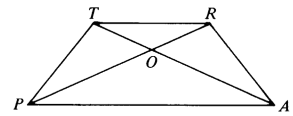
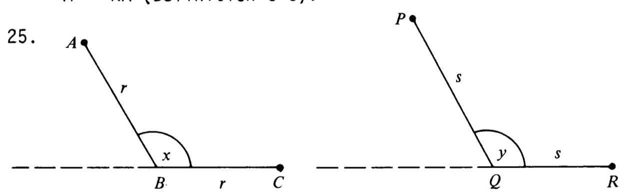
24.



The ratio of similitude of similar triangles TAP and RPA is 1:1 (Definition 8-1)
$\triangle TAP \cong \triangle RPA$ (SSS)
$\overline{TP} \cong \overline{RA}$ (Definition 3-3).

25.



The figures above show a part of two regular polygons with the same number of sides.
$AB/PQ = BC/QR = \ldots = r/s$

$m\angle x = \dfrac{180(n-2)}{n}$; $m\angle y = \dfrac{180(n-2)}{n}$ (Corollary 6-5.1a)

Therefore $m\angle x = m\angle y$ (Transitive property)

Therefore two regular polygons with same number of sides are similar (Definition 8-4).

26. $\overline{DE} \parallel \overline{BC}$ (Corollary 8-2.2a)
$\angle ADE \cong \angle B$ (Corollary 6-3.1a)
$\angle AED \cong \angle C$ (Corollary 6-3.1a)
$\triangle ADE \sim \triangle ABC$ (Definition 8-6).

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Class Exercises
(For exercises 1-5 there are alternate answers)

1. Corollary 8-5.1c
2. Corollary 8-5.1b
3. No, corresponding parts not similar.
4. Corollary 8-5.1b.
5. Corollary 8-5.1a.

Exercises

1. No, corresponding angles not congruent.
2. Yes, Theorem 8-5.1.
3. Yes, Theorem 8-5.1.

4. $\angle B \cong \angle C$ (Theorem 3-4.2)
$m\angle BMP = 90 = m\angle CNP$ (Theorem 2-6.5)
$\triangle MPB \sim \triangle NPC$ (Corollary 8-5.1a)
The resulting proportion is: $MB/NC = MP/NP = BP/CP$.

5. $\angle BAE \cong \angle E$ (Theorem 6-3.1)
$\angle B \cong \angle BCE$ (Theorem 6-3.1)
$\triangle FAB \sim \triangle FEC$ (Corollary 8-5.1a)
$FE/AF = FC/FB$ (Definition 8-6)
$FE/(AF + FE) = FC/(FB + FC)$ (Theorem 8-1.2)
$FE/AE = FC/BC$ (Postulate 2-4)
$FE \cdot BC = AE \cdot FC$ (Theorem 8-1.1).

6. $\angle PRQ \cong \angle TSU$ (Theorem 6-3.1
$\triangle PQR \sim \triangle TUS$ (Corollary 8-5.1b)
$PQ/TU = RQ/SU$ (Definition 8-6)
$PQ \cdot SU = RQ \cdot TU$ (Theorem 8-1.1).

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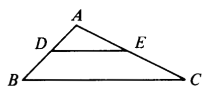
Exercises continued

7. The third pair of corresponding angles of the two triangles are congruent (Corollary 6-4.2a). Therefore by Theorem 8-5.1 the two triangles are similar.

8. Use the same procedure as in exercise 7.

9.

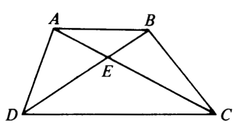


Since $\overline{DE} \parallel \overline{BC}$ (Given),
∠ ADE ≅ ∠ B and ∠ AED ≅ ∠ C (Corollary 6-3.1a)
△ADE ~ △ABC (Corollary 8-5.1a)

10. Base angles of both triangles are congruent to one another (Theorem 3-4.2).
The two triangles are similar (Corollary 8-5.1a).

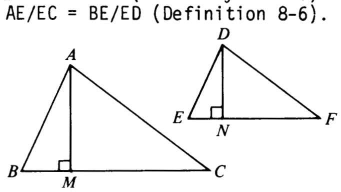
11. The measure of each angle of any equilateral triangle is 60 (Corollary 3-4.2a, Theorem 6-4.2).
Any two equilateral triangles can be proved similar (Theorem 8-5.1).

12.



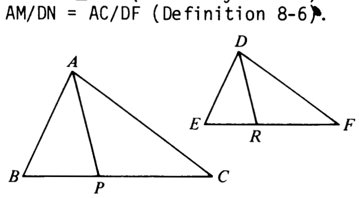
Draw trapezoid ABCD with diagonals intersecting at E.
∠ BAC ≅ ∠ ACD (Theorem 6-3.1)
∠ ABD ≅ ∠ BDC (Theorem 6-3.1)
△ABE ~ △CDE (Corollary 8-5.1a)
AE/EC = BE/ED (Definition 8-6).

13.



Draw △ABC ~ △DEF such that $\overline{AM}$ is an altitude of △ABC.
$\overline{DN}$ is an altitude of △DEF
∠ C ≅ ∠ F (Definition 8-6)
△AMC ~ △DNF (Corollary 8-5.1b)
AM/DN = AC/DF (Definition 8-6).

14.



Draw △ABC ~ △DEF such that $\overline{AP}$ is an angle bisector of △ABC.
$\overline{DR}$ is an angle bisector of △DEF.
∠ C ≅ ∠ F (Definition 8-6)
∠ BAC ≅ ∠ EDF (Definition 8-6)
m∠ PAC = (½)m∠ BAC (Definition 1-29)
m∠ RDF = (½)m∠ EDF (Definition 1-29)
m∠ PAC = m∠ RDF (Postulate 2-1)
△PAC ~ △RDF (Corollary 8-5.1a)
AP/DR = AC/DF (Definition 8-6)

15. △BDE ~ △BCA (Corollary 8-5.1b)
EB/AB = BD/BC (Definition 8-6).

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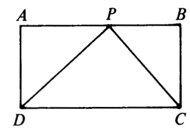
16. ∠ A ≅ ∠ C (Theorem 7-1.3)
$\overline{AB} \parallel \overline{DC}$ (Definition 7-1)
∠ APS ≅ ∠ CSP (Theorem 6-3.1)
Similarly, ∠ QPS ≅ ∠ RSP
∠ APQ ≅ ∠ CSR (Subtraction property)
△APQ ~ △CSR (Corollary 8-5.1a)
AQ/RC = QP/SR (Definition 8-6).

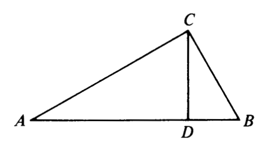
17.



m∠ ADP + m∠ APD = 90 (Corollary 6-4.2b)
m∠ BPC + m∠ APD = 90 (Corollary 6-4.2b)
∠ ADP ≅ ∠ BPC (Theorem 3-1.3)
△DAP ~ △PBC (Corollary 8-5.1b)
AP/BC = AD/PB (Definition 8-6)
AP • PB = AD • BC (Theorem 8-1.1).

18. m∠ APD + m∠ ADP = 90 (Corollary 6-4.2b)
m∠ PDC + m∠ ADP = 90 (Corollary 6-4.2b)
∠ APD ≅ ∠ PDC (Theorem 3-1.3)
△APD ~ △PDC (Corollary 8-5.1b)
PD/AP = DC/PD (Definition 8-6)
$(PD)^2$ = AP • DC (Theorem 8-1.1)

19.



△ADC ~ △ACB (Corollary 8-5.1b)
AC/AB = CD/CB (Definition 8-6)
AC • CB = CD • AB (Theorem 8-1.1).

20. △PSR ~ △PTQ (Corollary 8-5.1b)
QT/RS = PQ/RP (Definition 8-6)
QT • RP = RS • PQ (Theorem 8-1.1).

21. ∠ A ≅ ∠ C (Theorem 7-1.3)
∠ E ≅ ∠ ABE (Theorem 6-3.1)
△ABF ~ △CEB (Corollary 8-5.1a)
AF/BC = BF/BE (Definition 8-6)
AF • BE = BF • BC (Theorem 8-1.1).

22. △ADG ~ △BCG (Corollary 8-5.1b)
∠ DAG ≅ ∠ GBC (Definition 8-6)
△AFD ~ △BED (Corollary 8-5.1b)
AF/BE = AD/BD (Definition 8-6)
AF • BD = BE • AD (Theorem 8-1.1)
AC = AF + FC (Theorem 7-1.2)
AC = DE + AF (Theorem 7-1.2)
AC=DE +(BE • AD)/BD (Postulate 2-1)
AC/AB = DE/AB + (BE • AD)/(BD • AB) (Multiplication property)
AC/AB = (DE • BD)/(AB • BD) + (BE • AB) (Multiplication property).

23. ∠ BAE ≅ ∠ DEA (Theorem 6-3.1)
∠ ABD ≅ ∠ EDB (Theorem 6-3.1)
△EDG ~ △EBG (Corollary 8-5.1a)
GE/AG = GD/GB (Definition 8-6)
Similarly, △BGF ~ △DGA
GD/GB = AG/GF
GE/AG = AG/GF (Transitive property).

24. △ACZ ~ △AYB (Corollary 8-5.1c)
△BCZ ~ △BXA (Corollary 8-5.1c)
AZ/CZ = AB/YB (Definition 8-6)
BZ/CZ = AB/AX (Definition 8-6)
AZ/CZ + BZ/CZ = AB/YB + AB/AX (Addition property)
AZ + BZ = AB (Postulate 2-4)
AB/CZ = AB/BY + AB/AX (Postulate 2-1)
1/CZ = 1/BY + 1/AX (Division property)

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Exercises continued

25. AB/AC = BD/DC (Theorem 8-3.1)
    AB/AC = DE/DC (Postulate 2-1)
    FE/AC = DE/DC (Theorem 8-3.1)
    AB/AC = FE/AC (Transitive property)
    $\overline{AB} \cong \overline{FE}$ (Postulate 2-1).

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Class Exercises

1. AE/BE = EC/ED (Theorem 8-1.1).
2. $\angle$ AEB $\cong$ $\angle$ CED (Theorem 3-1.5).
3. $\triangle$ABC $\sim$ $\triangle$CDE (Theorem 8-6.1).
4. $\angle$ B $\cong$ $\angle$ ACD.
5. CD/AC = BC/AB.
6. AB/AC = BC/CD = AC/AD.
7. AC is the mean proportional between AB and AD.

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8. $\triangle$ABC $\sim$ $\triangle$AED
9. $\angle$ BAC $\cong$ $\angle$ DAC
10. AE/AB = AD/AC.
11. $\triangle$AED $\sim$ $\triangle$ABC.
12. $\angle$AED = $\angle$ ABC.

Exercises

1. $\triangle$TRP $\sim$ $\triangle$NRM (Theorem 8-6.1)
2. Yes, Theorem 8-6.2.
3. Yes, Corollary 8-5.1a.
4. Not similar.
5. Yes, Theorem 8-6.1.

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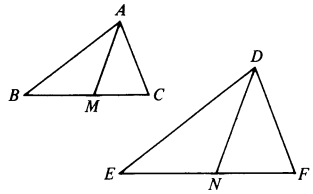
6. Yes, Theorem 8-6.1.
7. Yes, Theorem 8-6.2.
8. $\triangle$AED $\sim$ $\triangle$ABC (Theorem 8-6.1)
   $\angle$ ADE $\cong$ $\angle$ C (Definition 8-6)
   m$\angle$ ADE + m$\angle$ BDE = 180 (Definition 1-28).
   m$\angle$ C + m$\angle$ BDE = 180 (Postulate 2-1).

9. BC/QR = AC/PR (Definition 8-6)
   DC/SR = AC/PR (Definition 8-6)
   m$\angle$ ACB = m$\angle$ PRQ (Definition 8-6)
   m$\angle$ ACD = m$\angle$ PRS (Definition 8-6)
   BC/QR = DC/SR (Transitive property)
   m$\angle$ BCD = m$\angle$ QRS (Addition property)
   $\triangle$BDC $\sim$ $\triangle$QSR (Theorem 8-6.1).

10. Since the included angles of the two pairs of proportional legs are right angles, Theorem 8-6.1 proves the triangles similar.

11. 

    Draw $\triangle$ABC $\sim$ $\triangle$DEF such that $\overline{AM}$ is a median of $\triangle$ABC and $\overline{DN}$ is a median of $\triangle$DEF.
    $\angle$ B $\cong$ $\angle$ E (Definition 8-6)
    AB/DE = BC/EF (Definition 8-6)
    BM/BC = 1/2 = EN/EF (Definition 1-15)
    BC/EF = BM/EN (Postulate 2-1)
    AB/DE = BM/EN (Transitive property)
    $\triangle$ABM $\sim$ $\triangle$DEN (Theorem 8-6.1)
    AM/DN = AB/DE (Definition 8-6).

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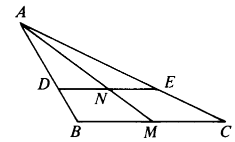
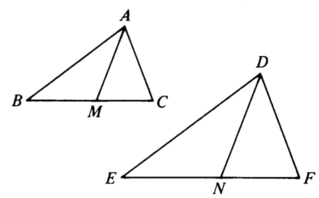
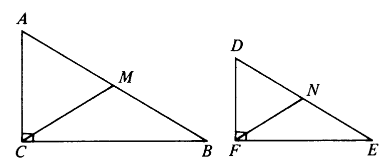
12. 

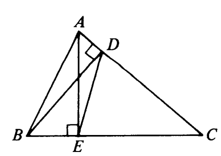
    $\triangle$ADN $\sim$ $\triangle$ABM (Corollary 8-5.1c)
    BM/DN = AM/AN (Definition 8-6)
    Similarly, $\triangle$ANE $\sim$ $\triangle$AMC
    CM/EN = AM/AN
    BM/DN = CM/EN (Transitive property)
    DN = EN (Division property)
    $\overline{AN}$ is a median of $\triangle$ADE (Definition 1-15).

13. 

    Draw $\triangle$ABC and $\triangle$DEF such that $\overline{AM}$ is a median of $\triangle$ABC, $\overline{DN}$ is a median of $\triangle$DEF.
    AB/DE = BC/EF = AM/DN
    BC/BM = 2/1 = EF/EN (Addition property)
    BC/EF = BM/EN (Corollary 8-1.1b)
    $\triangle$ABM $\sim$ $\triangle$DEN (Theorem 8-6.2)
    $\angle$ B $\cong$ $\angle$ E (Definition 8-6)
    $\triangle$ABC $\sim$ $\triangle$DEF (Theorem 8-6.1).

14. 

    Draw right $\triangle$ABC and the right $\triangle$DEF such that m$\angle$ ACB = m$\angle$ DFE = 90
    AC/DF = AB/DE
    AM/AB = 1/2 = DN/DE (Definition 1-15)
    AM/DN = AB/DE (Transitive property)
    CM = AM (Theorem 7-3.5)
    FN = DN (Theorem 7-3.5)
    CM/FN = AB/DE (Postulate 2-1)
    $\triangle$AMC $\sim$ $\triangle$DNF (Theorem 8-6.2)
    $\angle$ A $\cong$ $\angle$ D (Definition 8-6)
    $\triangle$ABC $\sim$ $\triangle$DEF (Theorem 8-6.1).

15. 

    Since $\angle$ AEC $\cong$ $\angle$ BDC (Theorem 3-1.1)
    $\triangle$BDC $\sim$ $\triangle$AEC (Corollary 8-5.1a), then DC/EC = BC/AC.
    Therefore $\triangle$ABC $\sim$ $\triangle$EDC (Theorem 8-6.1).

16. $\overline{AD}$ bisects $\angle$ BAC (Converse of Theorem 8-3.1)
    $\triangle$ADC $\sim$ $\triangle$ABE (Theorem 8-5.1)
    $\angle$AEB $\cong$ $\angle$ ACD (Definition 8-6)
    $\triangle$BED $\sim$ $\triangle$ACD (Corollary 8-5.1a)
    $\angle$ EBC = $\angle$ DAC (Definition 8-6)
    BD/AD = DE/DC (Definition 8-6)
    $\triangle$ADB $\sim$ $\triangle$CDE (Theorem 8-6.1)
    $\angle$BAD $\cong$ $\angle$ ECD (Definition 8-6)
    $\angle$ EBC $\cong$ $\angle$ ECB (Transitive property).

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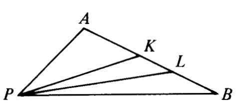
Exercises continued

17. PC = (¼)BC = (¼)2QC = (½)QC (Multiplication property)
PC/QC = 1/2 (Division property)
DQ/AD = 1/2 (Transitive property)
PC/QC = DQ/AD (Transitive property)
m∠ D = 90 = m∠ C (Theorem 7-3.1)
△PCQ ~ △QDA (Theorem 8-6.1)
PQ/QA = QC/AD = 1/2 (Definition 8-6, Postulate 2-1)
PQ = (½)QA (Multiplication property)
Show m∠ AQP = 90
△APQ ~ △QPC (Theorem 8-6.1)
∠ APQ ≅ ∠ QPC (Definition 8-6)
$\overline{PQ}$ bisects ∠ APC (Definition 1-29).

18. PR/PQ = PQ/PS (Mean proportional)
m∠PSB + m∠ PQB = 180 (Theorem 2-6.5, Addition property).
m∠ SPQ + m∠ B = 180 (Theorem 6-5.1, Subtraction property)
Similarly, m∠ RPQ + m∠ C = 180
∠ B ≅ ∠ C (Theorem 3-4.2)
∠ SPQ ≅ ∠ RPQ (Theorem 3-1.4)
△SPQ ~ △QPR (Theorem 8-6.1).

19. m∠ A = 90 = m∠ BCD (Given, Definition 7-5)
△MAB ~ △DCB (Theorem 8-6.1)
BD/BM = BC/AB (Definition 8-6)
∠ ABM ≅ ∠ CBD (Theorem 7-3.1)
∠ ABC ≅ ∠ MBD (Addition property)
△ABC ~ △MBD (Theorem 8-6.1)
m∠ BMD = m∠ A = 90 (Definition 8-6, Transitive property).

20.



△KAP ~ △PAB (Theorem 8-6.1)
∠ PKA ≅ ∠ BPA (Definition 8-6)
∠ KPA ≅ ∠ PBA (Definition 8-6)
m∠ PKA = m∠ KPB + m∠ PBK (Theorem 6-4.1)
m∠ PKA = m∠ BPL + m∠ KPL + m∠ APK (Theorem 3-4.2)
m∠ ALP = m∠ KPL + m∠ APK (Theorem 3-4.2)
m∠ PKA = m∠ ALP + m∠ KPL (Theorem 6-4.1)
m∠ BPL + m∠ KPL + m∠ APK = m∠ ALP + m∠ KPL (Transitive property)
m∠ BPL + m∠ APK = m∠ ALP (Subtraction property)
m∠ KPL + m∠ APK = m∠ BPL + m∠ APK (Transitive property)
m∠ KPL = m∠ BPL (Subtraction property)
$\overline{PL}$ bisects ∠ KPB (Definition 1-29)

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Class Exercises

1. Since △ADC ~ △ACB, AD/AC = AC/AB (Definition 8-6).
2. AC is the mean proportional between AD and AB.
3. Since △CDB ~ △ACB, BD/BC = BC/AB (Definition 8-6).
4. BC is the mean proportional between BD and AB.
5. Since △ADC ~ △CDB, AD/CD = CD/BD (Definition 8-6).
6. CD is the mean proportional between AD and BD.

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Exercises

1. △MNP, △PMQ
△MNR, △QPR, △PMR

2. $\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$

3. $4\sqrt{2}$ 4. $4\sqrt{6}$ 5. $2y\sqrt{2}$ 6. $6xy\sqrt{3x}$

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7. $\frac{3}{x} = \frac{x}{6}$

$x^2 = 18$

$x = \sqrt{18} = \sqrt{9 \cdot 2} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$

8. $2\sqrt{3}$ 9. 9 10. $8\sqrt{3}$ 11. $6n\sqrt{3}$

12. $\frac{6}{x} = \frac{x}{8}$

$x^2 = 48$

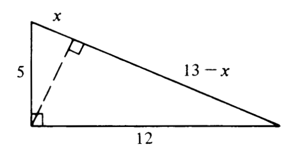
$x\sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$

13. $\sqrt{21}$ 14. 18 15. $\sqrt[4]{21}$ or $\sqrt{\sqrt{21}}$

16. See class exercises page 331, questions 1-4.

17. See class exercises page 331, questions 5-6.

18.



The side length 13 is the hypotenuse (Theorem 5-3.2)

13/5 = 5/x (Corollary 8-7.1a)
x = 25/13
$x/y = y/(13-x)$ (Corollary 8-7.1b)
$(25/13)/y = y/(144/13)$
$y^2 = 25/13 \cdot 144/13$
y = 60/13, 5 and 12 are also altitudes.

19. AD/DC = DC/DB (Corollary 8-7.1b)
$AD \cdot DB = DC^2$
AB/AC = AC/AD (Corollary 8-7.1a)
$AB \cdot AD = AC^2$
$AC^2 - DC^2 = AB \cdot AD - AD \cdot DB$
Now, $AB \cdot AD - AD \cdot DB = AD(AB - DB)$
and AB - DB = AD; so
$AC^2 - DC^2 = AD \cdot AD = AD^2$ (Postulate 2-1).

Could use the proof of Exercise 19 to solve Exercises 20-25, but should use Corollaries 8-7.1a and 8-7.1b to enable students to remember their corollaries.

20. Let AD = x, and BD = y

AB/AC = AC/AD
$(x = y)/2 = 2/x$
$x^2 + xy = 4$
$x^2 + 3 = 4$
$x = 1$ = AD
AB = $x + y$ = 4

AB/DC = BC/DB
$(x + y)$/BC = BC/y
4/BC = BC/3
BC = $2\sqrt{3}$

AD/CD = CD/DB

$\frac{x}{\sqrt{3}} = \frac{\sqrt{3}}{y}$

$xy = 3$

But. $x = 1$
Therefore $y = 3$ = DB

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Exercises continued

21. Let AD = x, and BD = y

AB/DC = BC/DB
$(x + y)/3 = 3/y$
$xy + y^2 = 9$

AD/CD = CD/DB
$\frac{x}{2\sqrt{2}} = \frac{2\sqrt{2}}{y}$
$xy = 8$

Substituting yields:

$8 + y^2 = 9$
$y^2 = 1$
$y = 1$ = BD
Since $xy = 8$, and $y = 1$
$x = 8$ = AD
Therefore AB = $x + y = 8 + 1 = 9$

AB/AC = AC/AD
$(x + y)$/AC = AC/x
$(AC)^2 = x^2 + xy$
$(AC)^2 = 64 + 8 = 72$
AC = $\sqrt{72} = 6\sqrt{2}$

22. AB/AC = AC/AD
$\frac{3}{\sqrt{3}} = \frac{\sqrt{3}}{x}$
$x = 1$ = AD
AB = $x + y$
$3 = 1 + y$
$y = 2$ = DB

AD/CD = CD/DB
x/CD = CD/y
1/CD = CD/2
CD = $\sqrt{2}$

AB/BC = BC/DB
3/BC = BC/2
BC = $\sqrt{6}$

23. x/CD = CD/y
$x/2 = 2/y$
$xy = 4$
AB = $x = y$
$5 = x + y$, $y = 5 - x$

$x(5 - x) = 4$
$5x - x^2 = 4$
$x^2 - 5x + 4 = 0$
$(x - 4)(x - 1) = 0$
$x = 4$ | $x = 1$
$y = 1$ | $y = 4$

$x = 4$ = AD, or $x = 1$ = AD
$y = 1$ = DB, or $y = 4$ = DB

AB/AC = AC/AD
5/AC = AC/x
$(AC)^2 = 5x$
If $x = 1$, AC = $\sqrt{5x} = \sqrt{5}$
If $x = 4$, AC = $\sqrt{5x} = \sqrt{20} = 2\sqrt{5}$

AB/BC = BC/DB
5/BC = BC/y
$(BC)^2 = 5y$
If $y = 1$, BC = $\sqrt{5y} = \sqrt{5}$
If $y = 4$, BC = $\sqrt{5y} = \sqrt{20} = 2\sqrt{5}$

24. AB = $x + y$
$x + y = 2$
$x = 1$ = AD

AB/BC = BC/DB
$\frac{2}{\sqrt{2}} = \frac{\sqrt{2}}{y}$
$y = 1$ = DB

AB/AC = AC/AD
2/AC = AC/1
AC = $\sqrt{2}$

AD/CD = CD/BD
x/CD = CD/y
$(CD)^2 = xy$
$(CD)^2 = 1$
CD = 1

25. AD/CD = CD/DB
$(x - 2)/(x + 3) = (x + 3)$/DB
DB = $(x + 3)^2/(x - 2)$
AB = AD + DB
AB = $(x - 2) + (x + 3)^2/(x - 2)$
AB = $(x - 2)^2 + (x + 3)^2/(x - 2)$
AB = $2x^2 + 2x + 13/(x - 2)$
AB/AC = AC/AD
$(AC)^2$ = (AB) $\cdot$ (AD)
$(AC)^2 = [(2x^2 + 2x + 13)/(x - 2)] \cdot (x - 2)$
$(AC)^2 = 2x^2 + 2x + 13$
AC = $\sqrt{2x^2 + 2x + 13}$
AB/BC = BC/DB
$(BC)^2$ = (AB) $\cdot$ (DB)
$(BC)^2 = [(2x^2 + 2x + 13)/(x - 2)] \cdot [(x + 3)^2/(x - 2)]$

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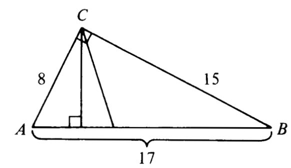
25. *continued*

BC = $\left(\frac{x + 3}{x - 2}\right) \cdot \sqrt{2x^2 + 2x + 13}$

26.



AB/AC = AC/AD
17/8 = 8/AD
AD = 64/17

Let x = AE
$17 - x$ = EB
AE/EB = AC/BC
$x/(17 - x) = 8/15$
$x = 136/23$ = AE

AE = AD + DE
136/23 = 64/17 + DE
DE = 136/23 - 64/17
DE = 5.91 - 3.76
DE = 2.15

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27. AC^2 = AD•AB (Corollary 8-7.1a)
BC^2 = DB•AB (Corollary 8-7.1a)
So AC^2/BC^2 = AD/DB = AE^2/EB^2 (Theorem 8-3.1)

Class Exercises

1. AC is the mean proportional between AB and AD (Corollary 8-7.1a).
2. Therefore $c/b = b/m$, or $b^2 = cm$. (Theorem 8-1.1).
3. BC is the mean proportional between AB and BD (Corollary 8-7.1a).
4. Therefore $c/a = a/n$; or $a^2 = cn$. (Theorem 8-1.1)
5. Adding the results of Exercises 2 and 4, we get $a^2 + b^2 = cm + cn = c(m + n)$. (Addition property, Distributive property)
6. But $m + n = c$.
7. Therefore $a^2 + b^2 = c^2$ (Postulate 2-1).

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Exercises

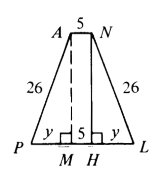
1. $(3\sqrt{5})^2 = x^2 + (\sqrt{3})^2$ (Theorem 8-8.1)
$x = \sqrt{42}$

2. $(4\sqrt{2})^2 = x^2 + x^2$ (Theorem 8-8.1)
$x = 4$

3. $(15)^2 = y^2 + (9)^2$ (Theorem 8-8.1)
$y = 12$
$x = 2y = 24$

4. $(17)^2 = x^2 + (15)^2$ (Theorem 8-8.1)
$x = 8$

5.



$y + 5 + y = 25$
$y = 10$
$x^2 + y^2 = (26)^2$ (Theorem 8-8.1)
$x = 24$

6. $(8)^2 = x^2 + (4)^2$ (Theorem 8-8.1)
$x = 4\sqrt{3}$ (Theorem 8-8.1)

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Exercises continued

7. $(\sqrt{13})^2 \neq (\sqrt{12})^2 + (\sqrt{5})^2$
$13 \neq 17$
Not the sides of a right triangle, (Theorem 8-8.2)

Exercises 8-12 are done the same way as Exercise 7.

8. No. 9. No. 10. Yes. 11. Yes. 12. Yes.

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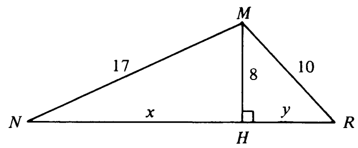
13.

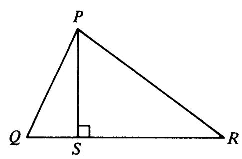


Use the Pythagorean Theorem:

$(17)^2 = x^2 + (8)^2$ || $(10)^2 = (8)^2 + y^2$
$x = 15$ || $y = 6$

$NR = x + y = 21$

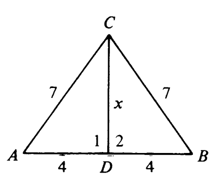
14.



$PS^2 = RP^2 - SR^2$ (Theorem 8-8.1)
$PS^2 = PQ^2 - QS^2$ (Theorem 8-8.1)
$RP^2 - SR^2 = PQ^2 - QS^2$ (Transitive property)
$PQ^2 - RP^2 = QS^2 - SR^2$ (Subtraction property)

15. $NM^2 = AM^2 - AN^2$ (Theorem 8-8.1)
$NM^2 = MC^2 - NC^2$ (Theorem 8-8.1)
$BM = MC$ (Definition 1-15)
$NM^2 = BM^2 - NC^2$ (Postulate 2-1)
$AM^2 - AN^2 = BM^2 - NC^2$ (Transitive property)
$AM^2 + NC^2 = BM^2 + AN^2$ (Addition property).

16.



$\triangle ACD \cong \triangle BCD$ (SSS)
$\angle 1 \cong \angle 2$ (Definition 3-3)
Therefore $\angle 1$ and $\angle 2$ are right triangles
$(7)^2 = x^2 + (4)^2$
$x = \sqrt{33}$

17. In right $\triangle PAB$
$(17)^2 = (8)^2 + (PB)^2$ (Theorem 8-8.1)
$PB = 15$

In right $\triangle PBC$
$(PB)^2 = (BC)^2 + (PC)^2$ (Theorem 8-8.1)
$(15)^2 = (9)^2 + (PC)^2$ (Theorem 8-8.1)
$PC = 12$

In right $\triangle PCD$
$(PD)^2 = (DC)^2 + (PC)^2$ (Theorem 8-8.1)
$(PD)^2 = 49/4 + 144$
$PD = 25/2$.

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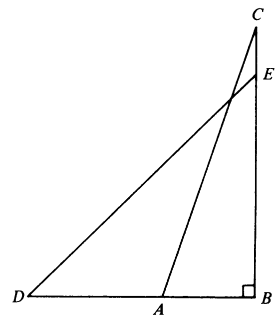
18. $D^2 = l^2 + w^2 + h^2$ (See Example 4, page 337)
$D^2 = 9 + 16 + 25$
$D^2 = 50$
$D = \sqrt{2 \cdot 25} = 5\sqrt{2}$

19. $D^2 = l^2 + w^2 + h^2$ (See Example 4, page 337)
$169 = 16 + 144 + h^2$
$h = 3$

20. $e = l = w = h$
$D^2 = l^2 + w^2 + h^2$ (See Example 4, page 337)
$D^2 = 3e^2$
$D = e\sqrt{3}$

21.

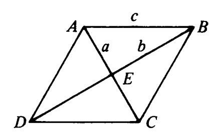


$AC = 25$
$DE = 25$
$CE = 4$
$BE = 20$
$BC = 24$

$(AC)^2 = (AB)^2 + (BC)^2$ || $(DE)^2 = (DB)^2 + (BE)^2$
$625 = (AB)^2 + 576$ || $625 = (DB)^2 + 400$
$AB = 7$ || $DB = 15$

$DA = DB - AB = 8$

22.



Draw rhombus ABCD with diagonals meeting at E.
Let $AE = a$, $BE = b$, and $AB = c$.
$\triangle AEB$ is a right triangle (Theorem 7-4.3)
$a^2 + b^2 = c^2$ (Theorem 8-8.1)
$4a^2 + 4b^2 = 4c^2$ (Multiplication property)
$AC = 2a$ (Theorem 7-4.3)
$BD = 2b$ (Theorem 7-4.3)
$AC^2 + BD^2 = 4(AB)^2$ (Postulate 2-1).

23. $AB^2 = AC^2 + CB^2 = AC^2 + (3CM)^2 = AC^2 + 9(CM)^2$ (Theorem 8-8.1)
$AN^2 = AC^2 + CN^2 = AC^2 + (2CM)^2 = AC^2 + 4(CM)^2$ (Theorem 8-8.1)
$AM^2 = AC^2 + CM^2$ (Theorem 8-8.1)
$3(AB)^2 = 3(AC)^2 + 27(CM)^2$ (Multiplication property)
$5(AM)^2 = 5(AC)^2 + 5(CM)^2$ (Multiplication property)
$8(AN)^2 = 8(AC)^2 + 32(CM)^2$ (Multiplication property)
$3(AB)^2 + 5(AM)^2 = 8(AN)^2$ (Addition property).

24. $BS^2 + SP^2 = BP^2 = BR^2 + RP^2$ (Theorem 8-8.1)
$CR^2 + RP^2 = CP^2 = CQ^2 + QP^2$ (Theorem 8-8.1)
$AQ^2 + PQ^2 = AP^2 = AS^2 + SP^2$ (Theorem 8-8.1)
$AQ^2 + BS^2 + CR^2 = AS^2 + BR^2 + CQ^2$ (Addition property).

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25. $ML^2 = MP^2 + LP^2$ (Theorem 8-8.1)
$NK^2 = NP^2 + KP^2$ (Theorem 8-8.1)
$KL^2 = KP^2 + LP^2$ (Theorem 8-8.1)
$MN^2 = NR^2 + MP^2$ (Theorem 8-8.1)
$ML^2 + NK^2 = MP^2 + LP^2 + NP^2 + KP^2 = KL^2 + MN^2$ (Addition property, Transitive property).

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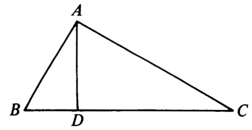
Exercises continued

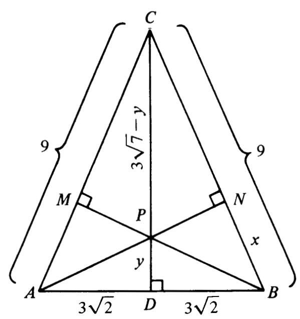
26.



$AC^2 + AB^2 = 2(AD)^2 + DC^2 + BD^2$ (Theorem 8-8.1)
(Addition property)
$(DC + BD)^2 = BC^2$ (Postulate 2-1, Distributive property)
m∠ BAC = 90 (Theorem 8-8.2).

27. $D^2 = l^2 + w^2 + h^2$ (See Example 4, page 337)
$D^2 = n^2 + (n + 1)^2 + (n^2 + n)^2$
$D^2 = (n^2 + n)^2 + n^2 + n^2 + 2n + 1$
$D^2 = (n^2 + n)^2 + 2n^2 + 2n + 1$
$D^2 = (n^2 + n)^2 + 2(n^2 + n) + 1$
$D^2 = [(n^2 + n) + 1]^2$
$D = n^2 + n + 1$

28.



In right △BCD
$(BC)^2 = (BD)^2 + (CD)^2$
$81 = 18 + (CD)^2$
$CD = 3\sqrt{7}$

In right △ACN
$(AC)^2 = (CN)^2 + (AN)^2$
$81 = (9 - x)^2 + (AN)^2$
$*81 - (9 - x)^2 = (AN)^2$

In right △ANB
$(AB)^2 = (AN)^2 + (NB)^2$
$72 = (AN)^2 + x^2$
$*72 - x^2 = (AN)^2$

Now equate the two (*) equations:

$81 - (9 - x)^2 = 72 - x^2$
$x = 4 = BN$
$9 - x = 5 = CN$

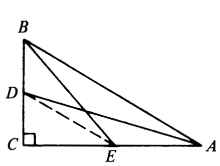
In △BCD and △PNC
∠ BDC ≅ ∠ PNC
∠ BCD ≅ ∠ NCP
∠ CBD ≅ ∠ NPC

Therefore △BCD ~ △PNC (Theorem 8-5.1)
Thus BC/CP = CD/CN = DB/PN

$$\frac{9}{3\sqrt{7} - y} = \frac{3\sqrt{7}}{5}$$

$$y = \frac{6\sqrt{7}}{7} = PD$$

29.



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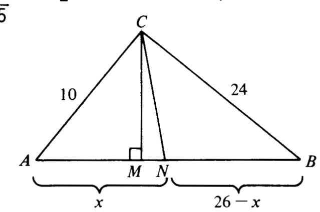
29. *continued*

AD = 15 ; BE = 10
Use Theorem 8-8.1:
$(BC)^2 + (CE)^2 = (BE)^2 = (10)^2 = 100$
$(AC)^2 + (DC)^2 = (AD)^2 = (15)^2 = 225$
By the addition property:
$(BE)^2 + (AD)^2 = [(BC)^2 + (AC)^2] + [(CE)^2 + (DC)^2]$ *(I)*

For △ABC:
$(AB)^2 = (BC)^2 + (AC)^2$ (Theorem 8-8.1) *(II)*
For △DEC:
$(DE)^2 = (CE)^2 + (DC)^2$ (Theorem 8-8.1) *(III)*

Substituting *(II)* and *(III)* in *(I)*:
$(BE)^2 + (AD)^2 = (AB)^2 + (DE)^2$
$100 + 225 = (AB)^2 + (DE)^2$
$325 = (AB)^2 + (DE)^2$
However, DE = ½(AB), (Theorem 7-6.3)
$325 = (AB)^2 + \left(\frac{AB}{2}\right)^2 = (AB)^2 + \frac{AB^2}{4}$
$AB = 2\sqrt{65}$

30.



$(26)^2 = (10)^2 + (24)^2$
Therefore △ABC is a right triangle (Theorem 8-8.2)
m∠ ACB = 90
26/10 = 10/**AM**,
AM = 50/13
$x/(26 - x) = 10/24$
$x = 130/17 = AN$
AN = AM + MN
130/17 = 150/13 + MN
MN = 840/221

31. Let BD = x
and let AD = y.
Use Theorem 8-8.1:
For △ ABD:
$x^2 + y^2 = (13)^2$.
$y^2 = 169 - x^2$

For △ADC:
$(14 - x)^2 + y^2 = (15)^2$
$y^2 = 225 - (14 - x)^2$

Therefore $225 - (14 - x)^2 = 169 - x^2$ (Transitive property)
$225 - 196 + 28x + x^2 = 169 - x^2$
$x = 5$, and $y = 12$ = AD

32. $AB^2 = (AE + ED)^2 + BD^2 = AE^2 + 2AE \cdot ED + ED^2 + BD^2$
(Theorem 8-8.1)
$BD^2 = BE^2 - ED^2$ (Theorem 8-8.1)
$AB^2 = AE^2 + BE^2 + 2AE \cdot ED$ (Postulate 2-1)
Similarly, $BC^2 = EC^2 + BE^2 - 2EC \cdot ED$
$AB^2 + BC^2 = AE^2 + BE^2 + 2AE \cdot ED + EC^2 + BE^2 - 2EC \cdot ED$
(Addition property)
AE = EC (Definition 1-15)
$AB^2 + BC^2 = 2(AE)^2 + 2(BE)^2$ (Postulate 2-1)
(Multiplication property).

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Class Exercises

1. Median of a right triangle
2. 60
3. 60
4. Equilateral, (Definition 3-12).
5. AC = AM = ½AB (Definition 1-15).
6. c^2
7. c
8. $a^2 + (\frac{1}{2} \cdot c)^2 = c^2$.
9. $a + \frac{1}{4}c^2 = c^2$.

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Class Exercises continued

10. 3/4

11. $a = \sqrt{(3/4)c^2} = \frac{\sqrt{3}}{2}(c)$

12. Theorem 8-9.2 and Theorem 8-9.3.

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For Exercises 1-6 use Theorem 8-9.3:

1. $6\sqrt{3}$
2. $15\sqrt{3}$
3. $\frac{17}{2}\sqrt{3}$
4. $\frac{47}{8}\sqrt{3}$
5. 9
6. $\frac{5}{2}\sqrt{6}$

For Exercises 7-12 use Corollary 8-9.3a:

7. $10\sqrt{3}$
8. $24\sqrt{3}$
9. $\frac{50\sqrt{3}}{3}$
10. $\frac{9\sqrt{3}}{2}$
11. 24
12. $\frac{20\sqrt{6}}{3}$

For Exercises 13-18 use Theorem 8-9.3:

13. $8\sqrt{3}$
14. $10\sqrt{3}$
15. $\frac{27\sqrt{3}}{2}$
16. $\frac{23\sqrt{3}}{4}$
17. 12
18. $\frac{5}{2}\sqrt{15}$

For Exercises 19-24 use Theorem 8-9.1:

19. $5\sqrt{2}$
20. $6\sqrt{2}$
21. $11\sqrt{2}$
22. $\frac{71\sqrt{2}}{2}$
23. 18
24. 28

For Exercises 25-30 use Theorem 8-9.2 and Corollary 8-9.3b:

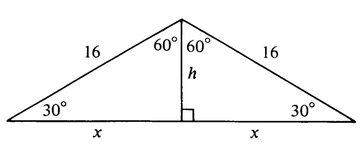
25. $6, 3\sqrt{3}$
26. $2, \sqrt{3}$
27. $10, 5\sqrt{3}$
28. $15, \frac{15\sqrt{3}}{2}$
29. $22\sqrt{3}, 33$
30. $6\sqrt{5}, 3\sqrt{15}$

For Exercises 31-36 use Corollary 8-9.1a:

31. $\frac{\sqrt{2}}{2}$
32. $3\sqrt{2}$
33. $\frac{5\sqrt{2}}{2}$
34. $\frac{25\sqrt{2}}{4}$
35. 8
36. $\frac{5\sqrt{6}}{2}$

37. By Corollary 8-9.3a, the side of the equilateral triangle is $(5)\left(\frac{2\sqrt{3}}{3}\right) = \frac{10\sqrt{3}}{3}$. Therefore the perimeter of the triangle is $(3)\left(\frac{10\sqrt{3}}{3}\right) = 10\sqrt{3}$

38.



As shown above two 30-60-90 triangles are formed by the altitude. From Theorem 8-9.3, $x = (16)\frac{\sqrt{3}}{2} = 8\sqrt{3}$. The base has length $(2)(8\sqrt{3}) = 16\sqrt{3}$.

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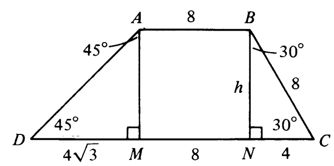
39. Use Theorem 8-9.2 to find the length of the altitude, 30. Use Theorem 8-9.3 to find the length of one-half the base, $5\sqrt{3}$. The length of the base is then $10\sqrt{3}$.

40. Use Corollary 8-9.1a to find the length of the altitude, $5\sqrt{2}$. Use Corollary 8-9.1a to find the length of one-half of the base, $5\sqrt{2}$. The length of the base is then $10\sqrt{2}$.

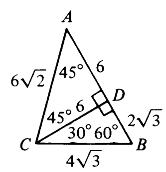
41. Use Theorem 8-9.3 to find the length of the altitude, $5\sqrt{3}$. Use Theorem 8-9.2 to find the length of one-half of the base, 5. The length of the base is then 10.

42.



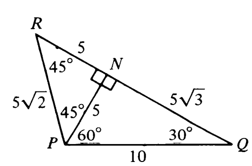
By drawing rectangle ABNM, we find that $\triangle AMD$ is an isosceles right triangle, and $\triangle BNC$ is a 30-60-90 triangle. Using Theorem 8-9.2 and Theorem 8-9.3 we obtain the lengths of $\overline{NC}$ and $\overline{BN}$, respectively, 4 and $4\sqrt{3}$. Therefore AM = $4\sqrt{3}$ also, and DM = $4\sqrt{3}$. Thus the altitude is $4\sqrt{3}$ and base $\overline{DC}$ haslength $12 + 4\sqrt{3}$.

43.



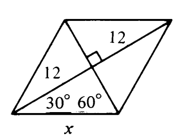
Notice the two special right triangles in the figure above. In isosceles right $\triangle ADC$, CD = 6 and AC = $6\sqrt{2}$ (Theorem 8-9.1). In 30-60-90 $\triangle ADB$, AB = $6(\frac{2}{3}\sqrt{3}) = 4\sqrt{3}$ (Corollary 8-9.3a) and BD = $2\sqrt{3}$ (Theorem 8-9.2). $p = 6\sqrt{2} + (6 + 2\sqrt{3}) + 4\sqrt{3} = 6(\sqrt{3} + \sqrt{2} + 1)$.

44.



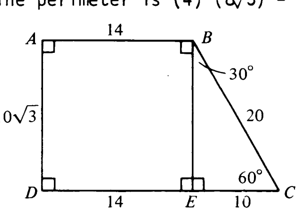
Notice the two special right triangles in the figure above. In 30-60-90 $\triangle PNQ$, NQ = $5\sqrt{3}$ (Theorem 8-9.3) and PN = 5 (Theorem 8-9.2). Therefore in isosceles right $\triangle PNR$, RN = PN = 5, and PR = $5\sqrt{2}$ (Theorem 8-9.1) $p = 5\sqrt{2} + 10 + (5 + 5\sqrt{3}) = 5(3 + \sqrt{3} + \sqrt{2})$.

45.



In the figure above, $x = 12\left(\frac{2\sqrt{3}}{3}\right) = 8\sqrt{3}$. The perimeter is $(4)(8\sqrt{3}) = 32\sqrt{3}$.

46.



After considering rectangle ABED a 30-60-90 triangle is realized. EC = 10 (Theorem 8-9.2) and

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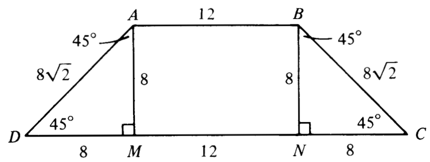
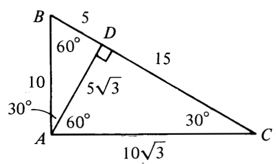
46. *continued*
AD = BE = $10\sqrt{3}$ (Theorem 8-9.3).
$\wp = 14 + 20 + 24 + 10\sqrt{3} = 58 + 10\sqrt{3}$.

47.



In the figure above ABNM is a rectangle, therefore
MN = AB = 12.
DM = NC = 8 (Corollary 8-9.1a).
$\wp = 12 + 8\sqrt{2} + 28 + 8\sqrt{2} = 8(5 + 2\sqrt{2})$.

48.



Since there are two 30-60-90 triangles formed,
BD = 5 (Corollary 8-9.3b) and AB = 10 (Corollary 8-9.3a).
Also AC = $10\sqrt{3}$ (Theorem 8-9.2) and DC = 15 (Theorem 8-9.3)
$\wp = 10 + 20 + 10\sqrt{3} = 10(3 + \sqrt{3})$.

49. $AC^2 + BC^2 = AB^2$ (Theorem 8-8.1)
$AC^2 + BC^2 = (AC\sqrt{2})^2 = 2(AC)^2$ (Postulate 2-1)
$BC^2 = AC^2$ (Division property).
BC = AC (Division property).

50. $AC^2 = (\frac{1}{4})(AB)^2$ (Multiplication property)
$BC^2 = 3(AC)^2 = (3/4)(AB)^2$ (Postulate 2-1)
$AC^2 + BC^2 = (\frac{1}{4})(AB)^2 + (3/4)(AB)^2 = AB^2$ (Addition prop.)
m∠ C = 90 (Theorem 8-8.2).

51. $\overline{EG}$, $\overline{EC}$ and $\overline{CG}$ are diagonals of three congruent square faces,
△ECG is equilateral
EG = EC = CG = $8\sqrt{2}$ (Theorem 8-9.1)
CN = $(EN)\sqrt{3}$ (Corollary 8-9.3b)
EN = $\frac{1}{2}$EG = $\frac{1}{2}(8\sqrt{3}) = 4\sqrt{3}$
CN = $(4\sqrt{2})(\sqrt{3}) = 4\sqrt{6}$.

52. $\overline{ME} \parallel \overline{AB}$ (Theorem 6-1.1)
$\overline{ME} = (\frac{1}{2})AB$ (Theorem 7-6.4, Theorem 7-6.3)
$\overline{PM} \perp \overline{MB}$ (Theorem 4-5.2)
PM = MB (Definition 1-32, Definition 3-12)
PM = MB = $AB(\sqrt{2}/2)$ (Corollary 8-9.1a)
$PE^2 = (\frac{1}{2})(AB)^2 + (\frac{1}{4})(AB)^2 = (3/4)(AB)^2$ (Postulate 2-1)
PE = $\frac{1}{2}(AB\sqrt{3})$ (Postulate 2-1).

53. In △CBD:
BD = $2\sqrt{3}$ (Corollary 8-9.3b)
BC = 2(BD) = $4\sqrt{3}$ (Theorem 8-9.2)

In △ABD:
AD = $(DB)\sqrt{2} = 2\sqrt{6}$ (Theorem 8-9.1)

In △ABC:
$(AC)^2 = (AB)^2 + (BC)^2$ (Theorem 8-8.1)
AC = $2\sqrt{15}$.

In △ADC:
$(2\sqrt{15})^2 = (6)^2 + (2\sqrt{6})^2$
60 = 60
Therefore △ADC is a right triangle (Theorem 8-8.2)
and $\overline{AD} \perp \overline{CD}$.

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Class Exercises

1. $y = \sqrt{2}$; $x = 2$ (Theorem 8-9.1)
2. $y = \sqrt{3}$ (Theorem 8-9.3); $x = 2$ (Theorem 8-9.2)

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3.

	$b = 30$	$b = 45$	$b = 60$
sin $b°$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos $b°$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan $b°$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

4. $\frac{\sin 45° \cdot \cos 45°}{\sin 30°} = \frac{\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2}}{\frac{1}{2}} = 1$

5. $(\sin 60°)^2 + (\cos 60°)^2 = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = 1$

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Exercises

1. .8387
2. .1908
3. .1763
4. .9511
5. 1.0000
6. 37°
7. 69°
8. 35°
9. 70°
10. 3/5
11. 3/4
12. 3/5
13. 4/3
14. 4/5
15. 4/5
16. $\tan \angle N \cdot \tan \angle M = \frac{12}{16} \cdot \frac{16}{12} = 1$
17. $\tan \angle M - \frac{\sin \angle M}{\cos \angle M} = \frac{16}{12} - \frac{16/20}{12/20} = 0$
18. $(\sin \angle N)^2 + (\cos \angle N)^2 = \left(\frac{12}{20}\right)^2 + \left(\frac{16}{20}\right)^2 = 1$
19. $1 - (\sin \angle M)^2 = 1 - \left(\frac{16}{20}\right)^2 = \frac{9}{25}$.

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20. $\tan 25° = \frac{x}{20}$
$x = 20 \cdot \tan 25° \approx 9.3$

21. $\cos 37° = \frac{x}{75}$
$x = 75 \cdot \cos 37° \approx 59.9$

22. $\sin 31° = \frac{24}{x}$
$x = \frac{24}{\sin 31°} = 46.6$

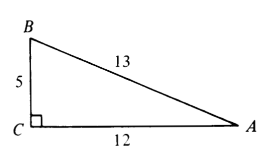
23. $\tan \theta = \frac{5}{6} = .8333$
$\theta = 40$

*.8391 > 58
.8333 > 235
.8098

24. $\sin \theta = \frac{24}{25} = .9600$
$\theta = 74$

*.9613 > 13
.9600 > 37
.9563

25.



$(13)^2 = (5)^2 + (12)^2$
Therefore △ABC is right triangle (Theorem 8-8.2)
$\tan \angle A = \frac{5}{12} = .4167$
m∠ A = 23
m∠ A + m∠ B = 90
m∠ B = 67

*.4245 > .0078
.4167 > .0107
.4060

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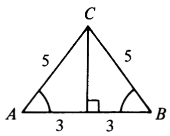
Exercises continued

26.



$\cos \angle A = \frac{3}{5} = .6000$
$m\angle A = 53 = m\angle B$
$m\angle A + m\angle B + m\angle ACB = 180$
$m\angle ACB = 74$

27. $\sin 28^\circ = \frac{5}{x}$
$x = \frac{5}{.4695} \approx 10.65$
$\tan 28^\circ = \frac{5}{y}$
$y = \frac{5}{.5317} = 9.404$

28. Apply Theorem 8-8.1. This check is not entirely accurate. The values for x and y are approximate (rounded off).

29. $BE = h$
$\sin 40^\circ = \frac{h}{30}$
$h = 30(.6428)$
$h = 19.2840 \approx 19$

$\sin \angle C = \frac{h}{50} = \frac{19}{50}$
$\sin \angle C = .3800$
$m\angle C = 22$

30. $\tan 85^\circ = AB/10$
$AB = 10(11.4361)$
$AB = 114.361 \approx 114$

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31. $\tan 15^\circ = \frac{200}{x}$
or $\tan 75^\circ = \frac{x}{200}$
$x = 200 \cdot \tan 75^\circ = 746.42 \approx 746$ ft.

32. $\tan 86^\circ = \frac{x}{24000}$
$x = 24000(14.3007)$
$x = 343216.8$ ft

$\frac{343216.8}{5280} = 65.0$ miles

33. $PR = 100$
$\tan 41^\circ = \frac{RB}{100}$
$RB = 100(.8693) \approx 87$
$\tan 32 = \frac{AR}{100}$
$AR = 100(.6249) \approx 62$
$AB = RB + AR = 149$

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Review Exercises

1. $\frac{1}{3}, \frac{2}{3}, \frac{2}{1}$
2. 28
3. 6, $2\sqrt{15}$, 3
4. $\frac{a-b}{b}$
5. $\frac{m}{r}$
6. $\frac{2}{3}$
7. $\frac{15}{22}$.

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8. $FH = x - 7$
$EG/GD = EH/FH$
$\frac{5}{12} = \frac{7}{x-7}$
$x = \frac{119}{5}$

9. $\frac{x}{12} = \frac{30}{10}$
$x = 36$

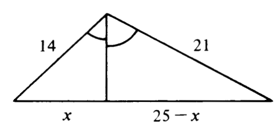
10. In $\triangle ADC$
$2/3 = AF/FC$

In $\triangle ABC$
$AF/FC = AG/GB$
$2/3 = AG/GB$
$2/3 = (10-x)/x$
$x = 6$

11. In $\triangle BCM$
$CR/RM = CN/NB$ (Theorem 8-2.1)
$CR/RM = 14/21 = 2/3$
Let $2x = CR$
$3x = RM$
$AM = CR + RM = 5x = MC$

In $\triangle ANR$
$AP/PN = AM/MR = 5x/3x = 5/3$ (Theorem 8-2.1)

12.



$x/(25 - x) = 14/21$ (Theorem 8-3.1)
$x = 10$
$25 - x = 15$

13. $8/12 = x/y$ (Theorem 8-3.1)

$y = \frac{3}{2}x$
However we are given:
$x + y + 20 = 50$
$x + y = 30$
$x + \frac{3}{2}y = 30$
$x = 12$
$y = 18$

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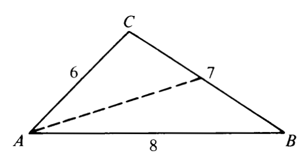
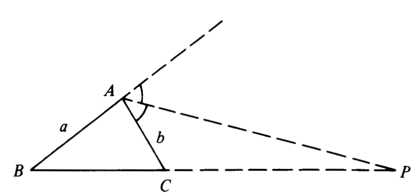
14.



$\frac{x}{7 - x} = \frac{6}{8}$ (Theorem 8-3.1)
$x = 3 = CD$

15.



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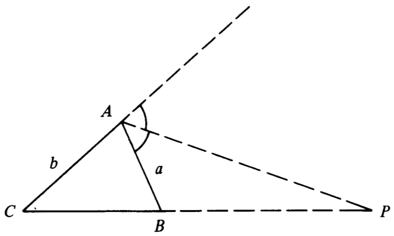
15. *continued*

If $a > b$:
BP/CP = AB/AC (Corollary 8-3.1a)
BP/CP = a/b, PC/BC = b/a
(BP-CP)/CP = $(a-b)/b$
BC/CP = $(a-b)/b$
PC/BC = $b/(a-b)$



If $b > a$:
CP/BP = AC/AB (Corollary 8-3.1a)
CP/BP = b/a
(CP-BP)/BP = $(b-a)/a$
BC/PB = $(b-a)/a$, PB/BC = $a/(b-a)$

16.

10	12	14	$\wp_1 = 36$
x		35	$\wp_2 =$ —

$10/x = 14/35$
$x = 25$

$\frac{\wp_1}{\wp_2} = \frac{14}{35}$

$\frac{36}{\wp_2} = \frac{2}{5}$

$\wp_2 = 90$

17.

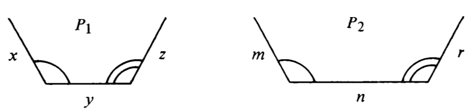
9	12	18	$\wp_1 = 39$
x	y	z	$\wp_2 = 52$

$\frac{9}{x} = \frac{12}{y} = \frac{18}{z} = \frac{\wp_1}{\wp_2}$

$\frac{9}{x} = \frac{39}{52}$ $\quad \frac{12}{y} = \frac{39}{52}$ $\quad \frac{18}{z} = \frac{39}{52}$

$x = 12$ $\quad y = 16$ $\quad z = 18$

18.



$\frac{x}{m} = \frac{y}{n} = \frac{z}{r}$ = ratio of similitude

Let $k = \frac{x}{m} = \frac{y}{n} = \frac{z}{r} = \ldots$

$k = \frac{x}{m}$, $\quad k = \frac{y}{n}$, $\quad k = \frac{z}{r}$

$x = nk$, $\quad y = kn$, $\quad z = kr$
$x + y + z + \ldots = k(m + n + r + \ldots)$
$\frac{x + y + z + \ldots}{m + n + r + \ldots} = k$
$\wp_1 = x + y + z + \ldots$
$\wp_2 = m + n + r + \ldots$
$\frac{\wp_1}{\wp_2} = \frac{x + y + z + \ldots}{m + n + r + \ldots} = k = \frac{x}{m} = \frac{y}{n} = \frac{z}{r} = \ldots$
(Theorem 8-1.4)

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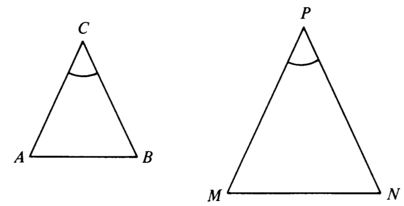
19. DE = ½(AB) (Theorem 7-6.3)
EF = ½(BC) (Theorem 7-6.3)
DF = ½(AC) (Theorem 7-6.3)
DE/AB = ½
EF/BC = ½
DF/AC = ½
△ABC ~ △DEF (Theorem 8-6.2)

20. ∠ D ≅ ∠ FCE
∠ DAF ≅ ∠ CEF (Theorem 6-3.1)
△ADF ~ △ECF (Corollary 8-5.1a)

21. $\overline{DE} \parallel \overline{CB}$ (Theorem 6-1.1)
△ADE ~ △ACB (Corollary 8-5.1c)

22.



m∠ C = m∠ P
m∠ A = m∠ B, m∠ M = m∠ N
m∠ A + m∠ B + m∠ C = 180
m∠ M + m∠ N + m∠ P = 180
m∠ A + m∠ B + m∠ C = m∠ M + m∠ N + m∠ P (Transitive prop.)
m∠ A + m∠ B = m∠ M + m∠ N
2m∠ A = 2m∠ M
m∠ A = m∠ M
Therefore m∠ A = m∠ M = m∠ B = m∠ N
Thus, △ABC ~ △MNP (Theorem 8-5.1)

23. △ACB ~ △CHB (Corollary 8-5.1b)
CB/AB = HB/CB
△HDB ~ △CEB (Corollary 8-5.1a)
HD/CE = HB/CB
CB/AB = HD/CE (Transitive property)

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24. *1.* If at least two angles of one triangle are congruent to the corresponding angles of the other triangle (AA).
2. If two sides of one triangle are proportional to two sides of the other triangle, and the angles included by those sides are congruent (SAS).
3. If the corresponding sides of the two triangles are proportional (SSS).

25. Theorem 8-6.1
26. Theorem 8-6.2
27. Theorem 8-6.1
28. AE/BE = EC/ED (Theorem 8-1.1),
△AEB ~ △CED (Theorem 8-6.1),
∠ B ≅ ∠ D (Definition 8-6).

29. △ABC is similar to:
△ADE, △DBF, △DCE, △CDF, △ACD, △CBD.

30. $2/x = x/8$ (Corollary 8-7.1b)
$x = 4$

31. $27/x = x/3$ (Corollary 8-7.1a)
$x = 9$

32. $x/16 = 16/4x$ (Corollary 8-7.1b)
$x = 8$

33. △ABC ~ △BEC (Theorem 8-5.1)
CA/BC = BC/CE (Definition 8-6)
or simply apply Corollary 8-7.1a to △ABC.

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Review Exercises continued

34. $\triangle ADC \sim \triangle ACB$ (Corollary 8-5.1b)
AB/AC = BC/CD (Definition 8-6)
CD • AB = AC • BC (Theorem 8-1.1)

35. $x^2 = (21)^2 + (28)^2$ (Theorem 8-8.1)
$x = 35$

36. $(13)^2 = x^2 + (5)^2$ (Theorem 8-8.1)
$x = 12$

37. $x^2 + x^2 = (10)^2$ (Theorem 8-8.1)
$x = 5\sqrt{2}$

38. $(7\sqrt{2})\left(\frac{\sqrt{3}}{2}\right) = \frac{7\sqrt{6}}{2}$ (Theorem 8-9.3)

39. $\left(\frac{2\sqrt{3}}{3}\right)(1) = \frac{2\sqrt{3}}{3}$ (Corollary 8-9.3a)

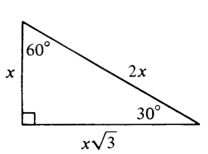
40. The other leg has length $3\sqrt{3}$ (Corollary 8-9.3b)
The other hypotenuse has length 6 (Theorem 8-9.2)

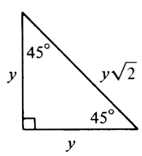
41. $(3\sqrt{2})\left(\frac{\sqrt{2}}{2}\right) = 3$ (Corollary 8-9.1a)

42. $(\sqrt{3})(\sqrt{2}) = \sqrt{6}$ (Theorem 8-9.1)

43. A side of the equilateral triangle has length
$(3\sqrt{3})\left(\frac{2\sqrt{3}}{3}\right) = 6.$
$p = 3 \cdot 6 = 18.$

44.





$\sin 30° = x/2x = \frac{1}{2}$

45. $\cos 45° = \frac{y}{y\sqrt{2}} = \frac{\sqrt{2}}{2}$

46. $\tan 60° = \frac{x\sqrt{3}}{x} = \sqrt{3}$

47. $\cos 60° = x/2x = \frac{1}{2}$

48. $\sin 60° = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2}$

49. $\tan 45° = \frac{x}{x} = 1$

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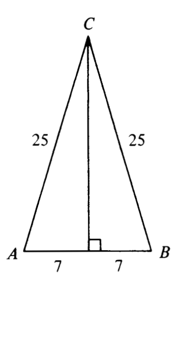
50. $\tan 25 = \frac{x}{5}$
$x = 5(.4663)$
$x = 2.3315$
$x \approx 2.3$

51. $\sin 79 = \frac{x}{15}$
$x = 15(.9816)$
$x \approx 14.7$

52.



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52. *continued*

$m\angle A = m\angle B$
$\cos \angle A = 7/25 = .2800$
$m\angle A = 74$
$m\angle B = 74$
$m\angle ACB = 32$

Chapter Test

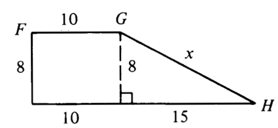
1. $\tan 25° = 35/QP$
$QP = 75.058$
$\sin 25° = x/QP$
$x = (QP)(\sin 25°)$
$x = 31.7208$

2. $(20)^2 = (12)^2 + y^2$ [FG = y; GE = z] (Theorem 8-8.1)
$y = 16$
$(15)^2 = (12)^2 + z^2$ (Theorem 8-8.1)
$z = 9$
$x = y + z = 25$

3. $12 = x\sqrt{2}$
$x = 6\sqrt{2}$

4. $BD = y$
$6 = y\sqrt{3}$
$y = 2\sqrt{3}$
$x = 2y = 4\sqrt{3}$

5.



$x^2 = (8)^2 + (15)^2$ (Theorem 8-8.1)
$x = 17$

6. $\triangle ABC \sim \triangle BDE$
$\angle C \longleftrightarrow \angle BDE$
$\angle B \longleftrightarrow \angle B$
$\angle A \longleftrightarrow \angle BED$
AB/EB = AC/DE = BC/BD
$5/2 = 3/x.$ and $x = 6/5$
$\frac{3}{\frac{6}{5}} = \frac{4}{y}$, and $y = \frac{8}{5}$

7. $x/(7 - x) = 9/12$
$x = 3$ = DC = HC
AD = AG = 4
$\triangle ABC \sim \triangle GBH$ (Theorem 8-6.1)
Therefore AB/BG = BC/BH = AC/GH
12/8 = 9/6 = 7/GH
9/6 = 7/GH
GH = 14/3

8. $\triangle MNP \sim \triangle ABP$ (Corollary 8-5.1c)
MN/AB = PN/PB (Definition 8-6)
$m\angle MNP = m\angle ABP$ (Definition 8-6)
$\triangle PNL \sim \triangle PBC$ (Corollary 8-5.1c)
NL/BC = PN/PB (Definition 8-6)
$m\angle LNP = m\angle CBP$ (Definition 8-6)
MN/AB = NL/BC (Transitive property)
$m\angle MNL = m\angle ABC$ (Postulate 2-10)
$\triangle MNL \sim \triangle ABC$ (Theorem 8-6.1).

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Exercises

1. False. Point P is a point in the interior of the circle.
2. False. $\overleftrightarrow{AB}$ is a secant of ⊙ P.
3. True.
4. False. $\overline{BD}$ is a secant segment of ⊙ P.
5. True.
6. True.
7. True
8. True.
9. True.
10. False.

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Exercises continued

11. False. The diameter of a sphere is a chord of the sphere.
12. False. A chord of a circle contains as many points of a circle than a secant of the same circle.
13. False. A sphere has many segments that can be called a radius.
14. True. 15. True 16. True. 17. True.

18. A————•————B
P

AB = 2(BP)
$5x + 6 = 2(x + 12)$
$5x + 6 = 2x + 24$
$3x = 18$
$x = 6$
AP = BP = $x + 12 = 18$

19. MN = 2(NQ)
$7x - 5 = 2(5x - 13)$
$7x - 5 = 10x - 26$
$x = 7$
NQ = MQ = $5x - 13 = 22$

20. Simply draw any two diameters of a circle and prove by Definition 9-4 that they are congruent.

21. Simply draw any two diameters of a sphere and prove by Definition 9-4 that they are congruent.

22. Apply Definition 9-4 and definition of a radius.
23. Apply Definition 9-4 and definition of a radius.

24. $\overline{AP} \cong \overline{BP}$ (radii)
$\triangle DAP \cong \triangle CBP$ (SAS)
$\overline{DP} \cong \overline{CP}$ (Definition 3-3)

25. The diagonals are congruent and bisect each other, so quadrilateral ABCD is a rectangle (Theorem 7-2.6, Theorem 7-3.4).

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26. $m\angle NMP = (\frac{1}{2})m\angle AMP$ (Definition 1-29)
$m\angle P = (\frac{1}{2})(m\angle B + m\angle P)$ (Definition 3-12)
$m\angle AMP = m\angle B + m\angle P$ (Theorem 6-4.1)
$m\angle NMP = m\angle P$ (Transitive property)
$\overline{MN} \parallel \overline{BP}$ (Theorem 6-2.1).

27. CQ = DQ (Radii)
AQ = BQ (Radii)
$\overline{GQ} \cong \overline{HQ}$ (Subtraction property)
$\triangle CQG \cong DQH$ (SAS)
$\overline{CG} \cong \overline{DH}$ (Definition 3-3)

28. If r is the radius of ⊙P, NP < r and MP > r (Definitions of interior, exterior regions)
NP < MP (Transitive property
$m\angle N > m\angle M$ (Theorem 5-3.1).

29. Draw $\overline{DP}$ and $\overline{CP}$.
$\overline{BP} \cong \overline{AP}$ (Radii)
$\overline{DP} \cong \overline{CP}$ (Radii)
$\triangle BPD \cong \triangle APC$ (SSS)
$\angle B \cong \angle A$ (Definition 3-3)
$\overline{AC} \parallel \overline{BD}$ (Theorem 6-2.1)

Class Exercises

1. $\overline{BM}$ 2. $\overline{BP}$. Radii of the same circle.
3. M and P are two points that are equidistant from the endpoints of $\overline{AB}$.
4. $\overleftrightarrow{MP}$ is the perpendicular bisector of $\overline{AB}$ (Corollary 4-4.2a).

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5. $\triangle AMP$ and $\triangle BMP$ are right triangles.
6. $\overline{AP} \cong \overline{BP}$.

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7. By the reflexive property, $\overline{MP} \cong \overline{MP}$.
8. BMP (HL)
9. $\overline{BM}$ (Definition 3-3)
10. Theorem 9-2.2.
11. Theorem 4-4.3.
12. Yes; radii of the same circle.
13. Theorem 4-4.2.
14. Theorem 9-2.1, Theorem 9-2.2, Theorem 9-2.3.

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Exercises

1. True.
2. False. If a line contains the center of a circle and the midpoint of one of the chords of the circle, and is perpendicular to another chord, then the two chords are parallel.

3. False. The perpendicular bisector of one chord of a circle is also the perpendicular bisector of any chord parallel to the given chord.

4. True.

5. False. The closer a chord is to the center of a circle, the longer it is.

6. $\overleftrightarrow{MQ} \perp \overline{AB}$ (Theorem 9-2.1)
$\triangle AMC \cong \triangle BMC$ (SAS)
$\overline{AC} \cong \overline{BC}$ (Definition 3-3),
$\triangle ABC$ is isosceles (Definition 3-12).

7. $\overline{AB} \cong \overline{AC}$ (Theorem 3-4.3)
$\overleftrightarrow{AM}$ is the perpendicular bisector of $\overline{BC}$ (Corollary 4-4.2a)
$\overleftrightarrow{AM}$ contains Q (Theorem 9-2.3).

8. The midpoint of a chord is contained in the perpendicular bisector of the chord (Theorem 9-2.1). The distances of these chords from the center of the circle are equal (Theorem 9-2.4). These midpoints determine the required circle (Definition 9-1).

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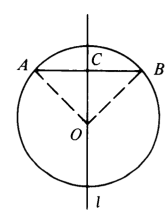
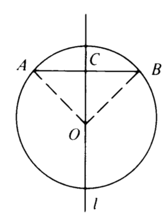
9.



AC = CB (Definition 1-15)
AO = BO (Definition 9-1)
OC = OC (Reflexive property)
$\triangle AOC \cong \triangle BOC$ (SSS)
$m\angle ACO = m\angle BCO$ (Definition 3-3)
Therefore $l \perp \overline{AB}$ (Definition 1-25)

10.



$l \perp \overline{AB}$ (Given)
OA = OB (Definition 9-1)
OC = OC (Reflexive property)
$\triangle AOC \cong \triangle BOC$ (HL)
AC = BC (Definition 3-3).

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Exercises continued

11. AC = CB (Definition 1-15)
$l \perp \overline{AB}$ (Given)
OC = OC (Reflexive property)
$m\angle ACO = m\angle BCO$ (Definition 1-25)
$\triangle ACO \cong \triangle BCO$ (SAS)
AO = BO (Definition 3-3)
Therefore l contains O (Theorem 4-4.3)

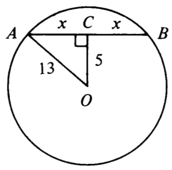
12. Use the diagram in Example 1.
$\overline{AB}$ and $\overline{CD}$ are congruent chords of $\odot P$, $PM \perp \overline{AB}$ at M
$\overline{PN} \perp \overline{CD}$ at N (Given)
$AM = (\frac{1}{2})AB$ (Theorem 9-2.2)
$DN = (\frac{1}{2})DC$ (Theorem 9-2.2)
$\overline{AM} \cong \overline{DN}$ (Multiplication property)
$\overline{AP} \cong \overline{DP}$ (Radii)
$\triangle AMP \cong \triangle DNP$ (HL)
$\overline{PM} \cong \overline{PN}$ (Definition 3-3).

13. See proof outline on page 365.

14. The distance of the diameter from the center of the circle is zero, so there is no chord closer to the center than the diameter. Thus, the diameter is the longest chord.

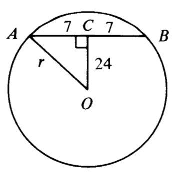
15. Refer to the diagram for Theorem 9-2.5.
Assume CD is not greater than AB.
Either CD = AB or CD < AB (Trichotomy property)
If CD = AB, then PN = PM (Theorem 9-2.4), contradicting the hypothesis.
If CD < AB, then PN > PM (Theorem 9-2.5), contradicting the hypothesis.
Our assumption is false.
CD > AB.

16.



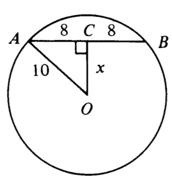
$(13)^2 = x^2 + (5)^2$
$x = 12$
$AB = 2x = 24$

17.



$r^2 = (7)^2 + (24)^2$
$r = 25$

18.



$(10)^2 = x^2 + (8)^2$
$x = 6$

19. Choose any three collinear points, A, B, C. From Theorem 9-2.3, the perpendicular bisector of $\overline{AB}$ and the perpendicular bisector of $\overline{BC}$ intersect at the center of the circle of which $\overline{AB}$ and $\overline{BC}$ are chords. However these two perpendicular bisectors are parallel (Theorem 6-1.1). Therefore the three points cannot be collinear. (Indirect proof).

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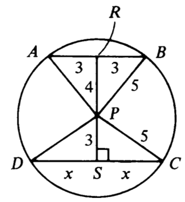
20. The point of intersection of the perpendicular bisectors of two nonparallel chords of a circle will determine the center of the circle.

21.

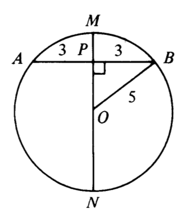


$RS = 7$, $(RP)^2 + (3)^2 = (5)^2$
$RP = 4$
$RS = RP + PS$
$7 = 4 + PS$
$PS = 3$
$(5)^2 = x^2 + (3)^2$
$x = 4$
$DC = 2x = 8$

22. Draw $\overline{AQ}$, $\overline{BQ}$, $\overline{DQ}$, and $\overline{CQ}$.
$\triangle QMP \cong \triangle QNP$ (AAS)
$\overline{MP} \cong \overline{NP}$ (Definition 3-3)
$\overline{QM} \cong \overline{QN}$ (Definition 3-3)
$\overline{AB} \cong \overline{CD}$ (Theorem 9-2.4)
$MB = (\frac{1}{2})AB$ (Theorem 9-2.2)
$ND = (\frac{1}{2})CD$ (Theorem 9-2.2)
MB = ND (Postulate 2-1)
BP = DP (Subtraction property)
QB = QD (Radii)
$\overleftrightarrow{QP} \perp \overleftrightarrow{BD}$ (Corollary 4-4.2a)

23. $\triangle AQP \cong \triangle CQP$ (SSS)
$\angle APF \cong \angle CPF$ (Definition 3-3)
$\triangle QMP \cong \triangle QNP$ (AAS)
$\overline{QM} \cong \overline{QN}$ (Definition 3-3)
$\overline{AB} \cong \overline{CD}$ (Theorem 9-2.4)

24.



The longest chord is a diameter (Theorem 9-2.6)
MN = 10
$\overline{AB} \perp \overline{MN}$ ($\overline{AB}$ is the shortest chord through P)
BO = MO = 5 = radius
$(PO)^2 + (3)^2 = (5)^2$
PO = 4

25. Draw $\overline{PM} \perp \overline{AMCB}$ and $\overline{QN} \perp \overline{ACNB}$
$\overline{PM} = \overline{NQ}$ (Theorem 7-1.6)
$\overline{PM} \parallel \overline{NQ}$ (Theorem 6-1.1)
Quadrilateral PMNQ is a parallelogram (Theorem 7-2.2)
Thus PQ = MN
However, $MC = (\frac{1}{2})AC$ and $NC = (\frac{1}{2})BC$ (Theorem 9-2.2)
Therefore, $MN = (\frac{1}{2})AB$.
Hence, $PQ = (\frac{1}{2})AB$.

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26. Draw $\overline{AC}$ intersecting $\overline{TQ}$ at M.
$\overleftrightarrow{PQ}$ is the perpendicular bisector of $\overline{AC}$ (Corollary 4-4.3a)
$\triangle AMT \cong \triangle CMT$ (SAS)
$\angle ATM \cong \angle CTM$ (Definition 3-3)
Draw $\overline{QR} \perp \overline{TB}$ and $\overline{QS} \perp \overline{TD}$ such that $\overline{TRB}$ and $\overline{TSD}$.
$\triangle RTQ \cong \triangle STQ$ (AAS)
$\overline{RQ} \cong \overline{SQ}$ (Definition 3-3)
$\overline{AB} \cong \overline{CD}$ (Theorem 9-2.4)

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Exercises continued

27. $\overline{PQ}$ is the perpendicular bisector of $\overline{AB}$ (Refer to Exercise 26)
$\overline{PQ}$ meets $\overline{AB}$ at M (Theorem 8-8.1)
PM = 12 (Theorem 8-8.1)
AP = PD = 13
MD = PD - PM = 13 - 12 = 1
AQ = DQ + C = DQ + 3
Letting DQ = x
$(x + 3)^2 = 25 + (x + 1)^2$ (Theorem 8-8.1)
$x = 17/4$
AQ = 3 + 17/4 = 29/4.

28. SP + PQ > SQ (Theorem 5-4.1)
SP = RP (Radii)
RP + PQ > SQ (Postulate 2-1)
RQ > SQ (Postulate 2-1)

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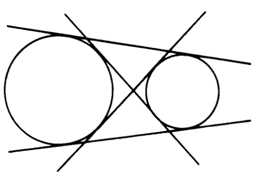
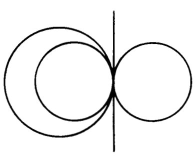
Class Exercises

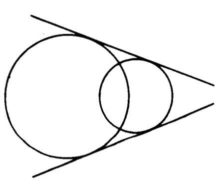
1. Yes.



2. Yes.



3. One, see figure above.
4. They are perpendicular (Theorem 9-3.2)
5. No.
6. Yes.



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Exercises

1. True.
2. False. A tangent is a line intersecting a circle in one point.
3. False. Two tangents can be drawn to a circle from an external point.
4. 11 + 7 = 18 (sum of radii).
5. 6 - 4 = 2 (difference of radii).
6. 0   7. 1   8. 2   9. 1   10. 0
11. 0, 1, 2, 3, 4.
12. Apply Theorem 9-3.2
13. Use Theorem 9-3.2 and Theorem 6-1.1.
14. $\overline{CMPT} \perp \overline{KL}$ (Theorem 9-2.1)
$\overline{CMPT} \perp \overleftrightarrow{ATB}$ (Theorem 9-3.2)
$\overline{KL} \parallel \overleftrightarrow{AB}$ (Theorem 6-1.1)

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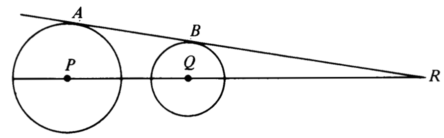
15. $\overline{PQ} \perp \overleftrightarrow{AT}$ (Theorem 9-3.3)
$\angle ATP \cong \angle ATQ$ (Theorem 3-1.1)
$\triangle ATP \cong \triangle ATQ$ (SAS)
$\overline{AP} \cong \overline{AQ}$ (Definition 3-3)

16. AP = BP (Radii)
$\triangle APB$ is isosceles (Definition 3-12)
$m\angle PAB = m\angle PBA$ (Theorem 3-4.2)
$\overline{PA} \perp \overline{TA}$ (Theorem 9-3.2)
$\overline{PB} \perp \overline{TB}$ (Theorem 9-3.2)
$m\angle PAT = m\angle PBT$ (Theorem 3-1.1)
$m\angle BAT = m\angle ABT$ (Subtraction property)
$\triangle ATB$ is isosceles (Theorem 3-4.2).

17. $\overline{PM} \perp \overline{CMD}$ (Theorem 9-3.2)
$\overline{AC} \parallel \overline{PM} \parallel \overline{BD}$ (Theorem 6-1.1)
AP = BP (Radii)
CM = DM (Theorem 7-6.1)
M is the midpoint of $\overline{CD}$ (Definition 1-15)

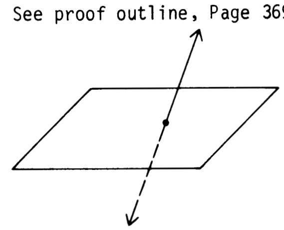
18.



Draw $\overline{AP}$ and $\overline{BQ}$.
$\overline{AP} \perp \overline{ARB}$ (Theorem 9-3.2)
$\overline{BQ} \perp \overline{ARB}$ (Theorem 9-3.2)
$m\angle PAB = 90 = m\angle QBA$ (Theorem 2-6.5)
$\angle PRA \cong \angle QRB$ (Theorem 3-1.5)
$\triangle PAR \sim \triangle QBR$ (Corollary 8-5.1a)
AR/BR = PR/QR (Definition 8-6)
AR • QR = BR • PR (Theorem 8-1.1)

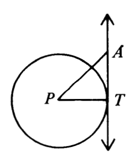
19. See proof outline, Page 369.

20.



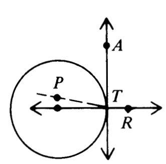
$\overleftrightarrow{AT}$ is tangent to ⊙P at T (Given)
PA > PT (Definition of exterior point)
$\overline{PT}$ is the shortest distance from P to $\overleftrightarrow{AT}$ (Definition 9-9)
$\overline{PT} \perp \overleftrightarrow{AT}$ (Theorem 5-4.2).

21.



Assume $\overline{PT}$ is not perpendicular to $\overleftrightarrow{AT}$.
There exists a point A on $\overleftrightarrow{AT}$ such that $\overline{PA} \perp \overleftrightarrow{AT}$
$\overline{PA}$ must be the shortest distance from P to $\overleftrightarrow{AT}$
(Theorem 5-4.2)
PA < PT since $\overline{PT}$ is a radius
A is in the exterior of ⊙P by the Given.
We have a contradiction.
$\overline{PT} \perp \overleftrightarrow{AT}$.

22.



$\overleftrightarrow{AT}$ is tangent to ⊙P at T.(Given)
$\overrightarrow{TR} \perp \overleftrightarrow{AT}$ (Given)
Assume $\overrightarrow{TR}$ does not contain P. There exists $\overleftrightarrow{PT}$ such that $\overleftrightarrow{PT} \perp \overleftrightarrow{AT}$ (Theorem 9-3.2)

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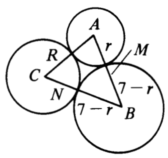
Exercises continued

22. *continued*

We have a contradiction.
$\overleftrightarrow{TR}$ contains P.

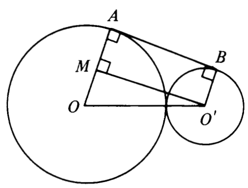
23.



Let r = AM.
MB = 7 - r = NB
CN = 9 - NB = 9 - (7 - r) = 2 + r = CR
AR = 5 - (2 + r) = 3 - r
AR = AM
3 - r = r
r = 1 ½
BM = 7 - 1 ½ = 5 ½
CN = 9 - 5 ½ = 3 ½.

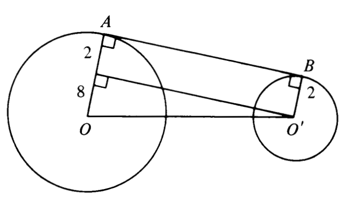
24. ∠ A, ∠ B, ∠ C, and ∠ D are right angles (Theorem 9-3.2)
Quadrilaterals ABQE and CDQF are rectangles
(Theorem 7-3.3, Theorem 6-5.1)
AE = BQ (Theorem 7-3.2)
CF = DQ (Theorem 7-3.2)
BQ = DQ (Radii)
AE = CF (Transitive property)
AP = CP (Radii)
EP = FP (Subtraction property)
△PEQ ≅ △PFQ (HL)
$\overline{EQ}$ = $\overline{FQ}$ (Definition 3-3)
$\overline{AB}$ ≅ $\overline{CD}$ (Transitive property)

25.



AO = 13½
BO' = 6 = AM (Since ABO'M is a rectangle)
MO = AM + MO
13½ = 6 + MO
MO = 7½ = 15/2
OO' = 13½ + 6 = 19½ = 39/2
$(OO')^2 = (MO)^2 + (MO')^2$ (Theorem 8-8.1)
$\left(\frac{39}{2}\right)^2 = \left(\frac{15}{2}\right)^2 + (MO')^2$
$1521/4 = 225/4 + (MO')^2$
$(MO')^2 = 324$
MO' = 18 = AB

26.



OO' = 17, AO = 10, AO' = 2
$(OO')^2 = (MO)^2 + (MO')^2$
$(17)^2 + (8)^2 + (MO')^2$
MO' = 15 = AB

27. ∠ MRP ≅ ∠ RPT (Theorem 6-3.1)
∠ RMP ≅ ∠ TPN (Corollary 6-3.1a)
$\overline{PR}$ ≅ $\overline{PM}$ (Radii)
∠ RMP ≅ ∠ MRP (Theorem 3-4.2)
∠RPT ≅ ∠ TPN (Transitive property)
△RTP ≅ △NTP (SAS)
∠ TRP ≅ ∠ TNP (Definition 3-3)
∠ TRP is a right angle (Theorem 9-3.2)

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27. *continued*

∠ TNP is also a right angle (Postulate 2-1)
$\overleftrightarrow{TN}$ is tangent to ⊙P (Theorem 9-3.1)

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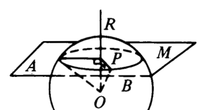
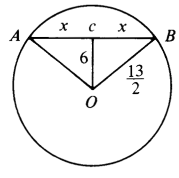
1. center
2. point; circle
3. tangent plane
4. circle
5. parallel
6. center of the circle of intersection
7. Sphere S and small circle P: A is on circle P (Given)
B is on circle S (Given)
$\overline{AP}$ < $\overline{AS}$ (Theorem 5-4.2)
$\overline{AS}$ ≅ $\overline{BS}$ (Radii)
$\overline{AP}$ < $\overline{BS}$ (Postulate 2-1)
8. They have the same radius as the sphere.
9. Since any two great circles of the same sphere must intersect on the sphere (Postulate 2-6), and both circles must share the same center (Theorem 9-4.1), their line of intersection is a chord of the sphere containing the center; that is, the diameter (Definition 9-4).
10. See solution for exercise 10 page 367.
11. Consider *any* two points on the intersection of the sphere and the plane (Do not choose points which are endpoints of a segment containing the point of intersection of the plane and a perpendicular to the plane from the center of the sphere).



Let A and B be *any* points on sphere O. $\overrightarrow{OR}$ ⊥ plane M.
Therefore $\overline{PB}$ ⊥ $\overrightarrow{OR}$ and $\overline{AP}$ ⊥ $\overrightarrow{OR}$ (Definition 4-7)
Also $\overline{OA}$ ≅ $\overline{OB}$ (radii of the sphere).
Therefore △APO ≅ △BPO (Thereom 6-6.2) and $\overline{AP}$ ≅ $\overline{BP}$.
Thus A and B lie on a circle, or the intersection of the sphere and the plane is a circle.

12. Consider a diameter of the circle of intersection and apply *Class Exercises 1-4 on page 362.*
13. Use the solution of *Exercises* 11 (Page 377).
14. Consider a diameter of the circle of intersection and apply *Class Exercises* 11-13 on page 363.
15. Follow the proof outline at the top of page 369.
16. See the solution for *Exercises* 22 on page 372.
17.



$\overline{AB}$ is diameter of the circle formed by a plane that intersects sphere O.
$\left(\frac{13}{2}\right)^2 = x^2 + (6)^2$

$x = \frac{5}{2}$

AB = $2x$ = 5

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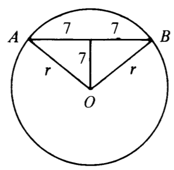
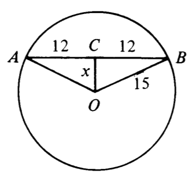
18.



$r^2 = (7)^2 + (7)^2$
$r = 7\sqrt{2}$

19.



$(15)^2 = x^2 + (12)^2$
$x = 9$

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20. Draw $\overline{ST}$.
$\overline{SA} \perp \overrightarrow{TA}$ (Theorem 9-4.7); $\overline{SB} \perp \overrightarrow{TB}$ (Theorem 9-4.7)
$m\angle SAT = 90 = m\angle SBT$ (Theorem 2-6.5)
$\overline{SA} \cong \overline{SB}$ (Radii)
$\triangle SAT \cong \triangle SBT$ (HL)
$\overline{TA} \cong \overline{TB}$ (Definition 3-3)

21. $\overline{SQ} \perp$ plane P (Theorem 9-4.3)
$\overline{SQ} \perp \overline{AQ}$ (Theorem 4-5.2)
$\overline{SQ} \perp \overline{BQ}$ (Theorem 4-5.2)
$\triangle SQB \cong \triangle SQA \cong \triangle AQB$ (SAS)
SB = SA = AB (Definition 3-3)
$\triangle ABS$ is equilateral (Definition 3-12)

22. Find the point of intersection of perpendicular lines of two circles of intersection formed by two nonparallel planes intersecting the sphere.

23. Consider the diameters of the two circles of intersection and then see the solution of *exercise* 15 on page 367.

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Class Exercises

1. semicircle
2. $m\overset{\frown}{BC}$
3. $m\overset{\frown}{DC}$
4. $\angle CPE$
5. $\overset{\frown}{CE}$
6. $\overset{\frown}{BCD}$
7. $\overset{\frown}{BE}$
8. $\overset{\frown}{BCD}$
9. $\overline{BD}$
10. $\triangle CPE$

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Exercises

1. $\overset{\frown}{ACB}$, $\overset{\frown}{AB}$
2. $\overset{\frown}{AC}$, $\overset{\frown}{CB}$
3. $\overset{\frown}{CAB}$, $\overset{\frown}{ABC}$
4. $\overset{\frown}{AC}$
5. $\angle BPC$
6. 50; since MP = RP
7. 80; since $m\angle MPR = 180 - (50 + 50) = 80$
8. 80; since $m\angle SPN = 80$
9. 100
10. 100
11. 180
12. Apply Theorem 9-5.2
13. First apply Theorem 9-5.2 to get $m\overset{\frown}{DE} = m\overset{\frown}{EF} = m\overset{\frown}{GF} = m\overset{\frown}{HG}$. Then apply Postulate 9-1 and the addition property to reach the desired conclusion.

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14. $\triangle MRP \cong \triangle MSP$ (HL)
$\angle APM \cong \angle BPM$ (Definition 3-3)
$\overset{\frown}{AM} \cong \overset{\frown}{MB}$ (Theorem 9-5.1)

15. $\overline{AQ} \cong \overline{BQ}$ (radii)
$\triangle AMQ \cong \triangle BMQ$ (HL)
$\angle AQM \cong \angle BQM$ (Definition 3-3)
$\overset{\frown}{AP} \cong \overset{\frown}{BP}$ (Theorem 9-5.1)

16. $\triangle AQM \cong \triangle BQM$ (SSS)
$\angle AQM \cong \angle BQM$ (Definition 3-3)
$\overset{\frown}{AP} \cong \overset{\frown}{BP}$ (Theorem 9-5.1)

17. $\overleftrightarrow{MP}$ contains Q (Theorem 9-2.3)
$\triangle AQM \cong \triangle BQM$ (SAS)
$\angle AQM \cong \angle BQM$ (Definition 3-3)
$\overset{\frown}{AP} \cong \overset{\frown}{BP}$ (Theorem 9-5.1)

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18. If two central angles of a circle are congruent then they have the same degree measure.
If the angles have the same degree measure then their arcs also have the same degree measure (Definition 9-17). By Definition 9-18 these arcs are then congruent. A reverse argument is used to prove the converse.

19. Refer to the diagram beside Theorem 9-5.2.
Given $\odot P \cong \odot Q$, $\overset{\frown}{AB} \cong \overset{\frown}{CD}$
Prove $\overline{AB} \cong \overline{CD}$.
$\angle P \cong \angle Q$ (Theorem 9-5.1)
$\triangle APB \cong \triangle CQD$ (SAS)
$\overline{AB} \cong \overline{CD}$ (Definition 3-3)

20. $\overset{\frown}{AB} \cong \overset{\frown}{AC}$ (Theorem 9-5.2)
$\overset{\frown}{BD} \cong \overset{\frown}{CD}$ (Postulate 9-1)
$\overline{BD} \cong \overline{CD}$ (Theorem 9-5.3)
$\triangle ABD \cong \triangle ACD$ (SSS)
$\overline{BD} \cong \overline{CD}$ (Definition 3-3)

21. $\angle AEC \cong \angle AED$ (Theorem 3-1.4)
$\triangle AEC \cong \triangle AED$ (SAS)
$\overline{AC} \cong \overline{AD}$ (Definition 3-3)
$\overset{\frown}{AC} \cong \overset{\frown}{AD}$ (Theorem 9-5.2)
$\triangle CEB \cong \triangle DEB$ (SAS)
$\overline{CB} \cong \overline{DB}$ (Definition 3-3)
$\overset{\frown}{CB} \cong \overset{\frown}{DB}$ (Theorem 9-5.2)
$m\overset{\frown}{AC} + m\overset{\frown}{CB} = m\overset{\frown}{AD} + m\overset{\frown}{DB}$ (Postulate 9-1)
$\overset{\frown}{ACB} \cong \overset{\frown}{ADB}$ (Postulate 9-1)
$\overline{AB}$ is a diameter of $\odot Q$ (Theorem 9-5.3)

22. $\triangle MPB$ and $\triangle NPB$ are isosceles (Radii, Definition 3-12)
$m\angle MPA = 2m\angle ABM$ (Theorem 6-4.1)
$m\angle NPA = 2m\angle ABN$ (Theorem 6-4.1)
$m\angle MPA = m\angle NPA$ (Transitive property)
$\overset{\frown}{MA} \cong \overset{\frown}{NA}$ (Theorem 9-5.1)

23. $\overset{\frown}{ACB} \cong \overset{\frown}{DBC}$ (Theorem 9-5.2)
$m\overset{\frown}{AC} + m\overset{\frown}{CB} = m\overset{\frown}{DB} + m\overset{\frown}{CB}$
$m\overset{\frown}{AC} = m\overset{\frown}{DB}$
$\overline{AC} \cong \overline{DB}$ (Theorem 9-5.3)
$\overline{PR} \perp \overline{AC}$ (Theorem 9-2.1)
$\overline{PS} \perp \overline{BD}$ (Theorem 9-2.1)
$\overline{PR} \cong \overline{PS}$ (Theorem 9-2.4)

24. Quadrilateral ABCD is a parallelogram (Theorem 7-2.1)
$\overline{BD}$ is a diameter since $m\overset{\frown}{AD} + m\overset{\frown}{AB} = m\overset{\frown}{BC} + m\overset{\frown}{DC}$
Similarly, $\overline{AC}$ is a diameter
Parallelogram ABCD is a rectangle (Theorem 7-3.4).

25. Draw $\odot Q$ such that M is the midpoint of $\overset{\frown}{AB}$
N is the midpoint of major arc $\overset{\frown}{AB}$, and
$\overline{MN}$ meets $\overline{AB}$ at P.
Prove $\overline{MN} \perp \overline{AB}$, $\overline{AP} \cong \overline{BP}$.
$\overset{\frown}{AM} \cong \overset{\frown}{BM}$ (Theorem 9-5.2)
$\overset{\frown}{AN} \cong \overset{\frown}{BN}$ (Theorem 9-5.2)
$\overset{\frown}{MAN} \cong \overset{\frown}{MBN}$ (Addition property)
$\overline{MN}$ is a diameter of $\odot Q$
$\angle AQM \cong \angle BQM$ (Theorem 9-5.1)
$\overline{AQ} \cong \overline{BQ}$ (Radii)
$\triangle APQ \cong \triangle BPQ$ (SAS)
$\overline{AP} \cong \overline{BP}$ (Definition 3-3)
$\overleftrightarrow{PQN} \perp \overline{AB}$ (Corollary 4-4.2a)

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Exercises continued

26. Draw $\overline{MQN} \perp \overline{AB}$, meeting $\overline{AB}$ at M and $\overline{CD}$ at N.
    Quadrilateral ACNM is a rectangle (Theorem 7-3.3)
    AM = CN (Theorem 7-2.2)
    M and N bisect $\overline{AB}$ and $\overline{CD}$, respectively (Theorem 9-2.2)
    Thus, AB = 2AM
    CD = 2CN
    $\overline{AB} \cong \overline{CD}$ (Transitive property)
    $\widehat{AB} \cong \widehat{CD}$ (Theorem 9-5.2)

27. Draw $\overline{QC}$ meeting $\overline{AB}$ at M.
    $\overline{QC} \perp \overline{CD}$ (Theorem 9-3.2)
    $\overline{QC} \perp \overline{AB}$ (Corollary 6-1.1b)
    $\widehat{AC} \cong \widehat{BC}$ (See Exercise 15).

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Class Exercises

1. $\widehat{ADC}$ 2. $\widehat{ABC}$ 3. $\widehat{ADC} + \widehat{ABC}$ 4. 360°
5. 180° 6. supplementary 7. 360
8. 360° 9. 180° 10. supplementary

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Exercises

1. $\widehat{BAE}$ 2. $\widehat{ACE}$ 3. $\widehat{CAE}$ 4. $\angle$ AEC 5. $\angle$ BAE

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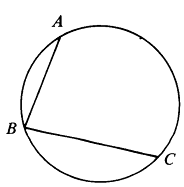
6. $\angle$ BCE and $\angle$ BDE 7. Congruent
8. Corollary 9-6.1a 9. $\angle$ ECD
10. 90 11. 180 12. $\widehat{DE}$

13.



$m\,\widehat{ABC} = 200$
$m\,\widehat{AC} = 360 - 200$
$m\,\widehat{AC} = 160$
$m\angle B = 80 = \frac{1}{2}(m\ AC)$

Exercises 14-17 are done in a similar way to Exercise 13

14. 150 15. 10
16. $(335-3x)/2$ 17. x

For Exercises 18-22 apply Theorem 9-6.1

18. 40 19. $37\frac{1}{2}$ 20. $72\frac{1}{2}$
21. $(5x-7)/2$ 22. $90-3x$
23. $x = 180 - 60$; $y = \frac{1}{2}x = \frac{1}{2}(120) = 60$
24. Use Theorem 9-6.2: $x = 108$; $y = 93$
25. $x = 180 - (116 + 24) = 40$; $y = \frac{1}{2}x = \frac{1}{2}(40) = 20$
26. $m\,\widehat{AB} = 70$; $y = 360 - (160 + 70) = 130$; $x = \frac{1}{2}(70) = 35$
27. $x = \frac{1}{2}(180) = 90$ (Corollary 9-6.1c); $m\,\widehat{CB} = 180 - 123 = 57$.
    $y = \frac{1}{2}(57) = 28\frac{1}{2}$.
28. $m\,\widehat{BC} = 70$ (Theorem 9-6.3); $x = y = \frac{1}{2}(70) = 35$
    (Corollary 9-6.1a)
29. *See Example 2 on page 386*
30. Apply Theorem **9-6.1**
31. Apply Theorem 9-6.1
32. See the solution for Exercise 27 on page 383.
33. See the solution for Exercise 24 on page 383.
34. $m\angle ACB = 90$ (Corollary 9-6.1c)
    CM = AM = BM (radii)
    CM = $\frac{1}{2}$AB (Transitive property).

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35. $m\angle T = \frac{1}{2}(m\,\widehat{AC}) = 53$
    $m\angle T = \frac{1}{2}(m\,\widehat{BD})$
    $53 = \frac{1}{2}(m\,\widehat{BD})$
    $m\,\widehat{BD} = 106$

36. $\angle$ ADB is a right angle (Corollary 9-6.1c)
    Therefore $\angle$ ADC is a right angle (Definition 1-26)
    AD = DC (Given)
    $\triangle$ADB and $\triangle$ADC are right triangles (Definition 1-32)
    $\triangle ADB \cong \triangle ADC$ (SAS)
    $\overline{AB} \cong \overline{DC}$ (Definition 3-3)

37. $\overline{AB} \cong \overline{AC}$ (Given)
    $\angle$ ADB is a right angle (Corollary 9-6.1c)
    Therefore $\angle$ ADC is a right angle (Definition 1-26)
    $\triangle$ADB and $\triangle$ADC are right triangles (Definition 1-32)
    $\triangle ADB \cong \triangle ADC$ (Theorem 6-6.2)
    $\overline{BD} \cong \overline{DC}$ (Definition 3-3)

38. $m\angle TRN = x$ (Given)
    $m\angle PMT = y$ (Given)
    $m\angle RMS = m\angle PMT = y$ (Theorem 2-6.3)
    $m\angle NPS = m\angle RTN$ (Corollary 9-6.1b)
    Therefore $m\angle PSN = m\angle TRN$
    $m\,\widehat{PT} = 140$
    $m\angle PRT = \frac{1}{2}(m\,\widehat{PT}) = 70$
    $m\angle PRT + m\angle TRN = 180$
    $70 + x = 180$
    $x = 110$
    In quadrilateral RMSN
    $m\angle N + m\angle NRM + m\angle RMS + m\angle MSN = 360$
    $45 + x + y + x = 360$, and
    $y = 95$.

39. $m\angle D = \frac{1}{2}(m\,\widehat{BC})$ (Theorem 9-6.1)
    $18 = \frac{1}{2}(m\,\widehat{BC})$
    $m\,\widehat{BC} = 36$
    $m\angle A = \frac{1}{2}(m\,\widehat{BC})$ (Theorem 9-6.1)
    $m\angle A = 18$
    $x = m\angle A + m\angle E$ (Theorem 5-2.5), and
    $x = 18 + 35 = 53$.
    $m\angle DFC = y$ (Theorem 2-6.2)
    Therefore $m\angle D + x + m\angle DFC = 180$
    $18 + 53 + y = 180$, and $y = 109$.

40. $m\angle PMT = 90$ (Corollary 9-6.1c)
    M is the midpoint of $\overline{TA}$ (Theorem 9-2.2).

41. $m\angle A + m\angle BCD = 180$ (Theorem 9-6.2)
    $m\angle BCD + m\angle BCP = 180$ (Theorem 9-6.2)
    $\angle A \cong \angle BCP$ (Theorem 3-1.4)
    $\triangle BCP \sim \triangle DAP$ (Theorem 8-5.1)
    BP/DP = CP/AP (Definition 8-6)

42. We proved in Exercise 41 that $\angle A \cong \angle BCP$.
    $\triangle BCP \sim \triangle BAD$ (Corollary 8-5.1a)
    AD/CP = BD/BP (Definition 8-6)
    AD • BP = CP • BD (Theorem 8-1.1)

43. Draw $\overline{CE}$.
    $m\angle ACE = 90$ (Corollary 9-6.1c)
    $\angle ABC \cong \angle AEC$ (Corollary 9-6.1a).
    $\triangle ADB \sim \triangle ACE$ (Corollary 8-5.1a)
    So AB • AC = AD • AE (Definition 8-6, Theorem 8-1.1).

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1. Corollary 6-3.1a
2. $m\angle BAF = \frac{1}{2}m\,\widehat{BF}$
3. $m\,\widehat{BN} = m\,\widehat{AM} = m\,\widehat{MF}$
4. $m\angle NMQ = m\angle BAF = \frac{1}{2}(m\,\widehat{BN} + m\,\widehat{NF}) = \frac{1}{2}(m\,\widehat{MF} + m\,\widehat{NF}) = \frac{1}{2}\,m\,\widehat{NM}$
5. $m\angle NEF = m\angle BAF = \frac{1}{2}(m\,\widehat{BN} + m\,\widehat{NF}) = \frac{1}{2}(m\,\widehat{AM} + m\,\widehat{NF})$
6. $m\angle GIF = m\angle BAF = \frac{1}{2}\,m\,\widehat{BF} = (m\,\widehat{BF} + m\,\widehat{BG} - m\,\widehat{BG})$
   $= \frac{1}{2}(m\,\widehat{BF} + m\,\widehat{BG} - m\,\widehat{HA}) = \frac{1}{2}(m\,\widehat{GBF} - m\,\widehat{HA})$
7. $m\angle JKF = m\angle BAF = \frac{1}{2}\,m\,\widehat{BF} = \frac{1}{2}(m\,\widehat{BF} + m\,\widehat{JB} - m\,\widehat{JB})$
   $= \frac{1}{2}(m\,\widehat{BF} + m\,\widehat{JB} - m\,\widehat{JA}) = \frac{1}{2}(m\,\widehat{JBF} - m\,\widehat{JA})$

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8. $m\angle JLM = m\angle NMQ = \frac{1}{2}m\,\widehat{NM} = \frac{1}{2}(m\,\widehat{NM} + m\,\widehat{JN} - m\,\widehat{JN})$
 $= \frac{1}{2}(m\,\widehat{NM} + m\,\widehat{JN} - m\,\widehat{JM}) = \frac{1}{2}(m\,\widehat{JBM} - m\,\widehat{JM})$

9. Theorem 9-8.1 through Theorem 9-8.5

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Exercises

1. $m\,\widehat{BD} = 180 - (90 + 64) = 26$
 $x = \frac{1}{2}(64 - 26) = 19$ (Theorem 9-8.3)

2. $x = \frac{1}{2}(46 + 42) = 44$ (Theorem 9-8.2)

3. $x = \frac{1}{2}(232) = 116$ (Theorem 9-8.1)
 Or $m\angle BCD = \frac{1}{2}(128) = 64$,
 and $m\angle ACD = 180 - m\angle BCD = 116$.

4. $m\,\widehat{BD} = 360 - 245 = 115$. $x = 180 - 115 = 65$
 (Corollary 9-8.5a)

5. $m\,\widehat{AD} = 70$; therefore $m\,\widehat{AB} = 360 - (70 + 70 + 157) = 63$
 $x = \frac{1}{2}(m\,\widehat{DAB} - m\,\widehat{BE}) = \frac{1}{2}(133 - 70) = 31\frac{1}{2}$.

6. $m\,\widehat{BD} = 2(62) = 124$ (Theorem 9-6.1)
 $x = 180 - 124 = 56$ (Corollary 9-8.5a)

7. The chord joining the parallel tangents is a diameter.

8. See *Proof Outline* on page 397.

9. See *Proof Outline* on page 397.

10. See *Proof Outline* on page 398.

11. See *Proof Outline* on page 398.

12. See *Proof Outline* on page 399.

13. $m\angle ABT = 105/2$, so $m\,\widehat{BC} = 2m\angle ABT = 105$.

14. $m\,\widehat{AB} = m\,\widehat{BC} = m\,\widehat{CD} = m\,\widehat{DE} = m\,\widehat{EA} = 360/5 = 72$
 (Theorem 9-5.2)
 $x = \frac{1}{2}(m\,\widehat{AB}) = 36$ (Theorem 9-8.4)
 $y = 180 - 2(72) = 36$ (Corollary 9-8.5a)
 $z = \frac{1}{2}(m\,\widehat{CDE})$ (Theorem 9-8.4)
 $z = \frac{1}{2}(144) = 72$

15. $m\,\widehat{AC} = 130$ (Theorem 9-6.1)
 $m\,\widehat{AD} + 152 + 130 = 360$
 $m\,\widehat{AD} = 78$
 $x = 180 - 130 = 50$ (Corollary 9-8.5a)
 $y = \frac{1}{2}(m\,\widehat{ACD})$ (Theorem 9-8.4)
 $y = \frac{1}{2}(282) = 141$
 $z = \frac{1}{2}(m\,\widehat{CAD})$ (Theorem 9-8.4)
 $z = \frac{1}{2}(208) = 104$.

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16. $11 = \frac{1}{2}(72 - x)$ (Theorem 9-8.4)
 $x = 50 = m\,\widehat{AD}$
 $m\,\widehat{BD} + 50 + 72 + 153 = 360$
 $m\,\widehat{BD} = 85$
 $y = \frac{1}{2}(m\,\widehat{BDA})$ (Theorem 9-8.1)
 $y = \frac{1}{2}(135) = 67\frac{1}{2}$
 $z = \frac{1}{2}(m\,\widehat{BE} - m\,\widehat{BD})$ (Theorem 9-8.4)
 $z = \frac{1}{2}(153 - 85) = 34$

17. $m\angle A + m\angle EDB = 180$ (Theorem 9-6.2)
 $96 + m\angle EDB = 180$
 $m\angle EDB = 84$
 $m\,\widehat{BAE} = 168$ (Theorem 9-6.1)
 $168 = m\,\widehat{AB} + 66$
 $m\,\widehat{AB} = 102$
 $m\,\widehat{BD} + 94 + 66 + 102 = 360$, and $m\,\widehat{BD} = 98$
 $x = \frac{1}{2}(m\,\widehat{BD}) = 49$ (Theorem 9-8.1)
 $m\angle ABD = \frac{1}{2}(m\,\widehat{AED}) = \frac{1}{2}(160) = 80$ (Theorem 9-6.1)
 $m\angle ABD + m\angle ABG = 180$
 $80 + y = 180$, and $y = 100$

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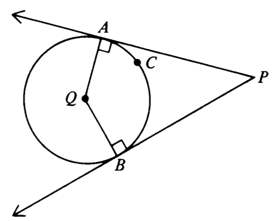
17. *continued*

 $z = \frac{1}{2}(m\,\widehat{AED} = m\,\widehat{BD})$ (Theorem 9-8.4)
 $z = \frac{1}{2}(160 - 98) = 31$

18. $m\angle ATC = (\frac{1}{2})m\,\widehat{TC}$ (Theorem 9-8.1)
 $m\angle BTD = (\frac{1}{2})\,m\,\widehat{TD}$ (Theorem 9-8.1)
 $m\angle ATC = m\angle BTD$ (Theorem 2-6.3)
 $m\,\widehat{TC} = m\,\widehat{TD}$ (Postulate 2-1)
 $m\angle TDB = (\frac{1}{2})m\,\widehat{TD}$ (Theorem 9-8.1)
 $m\angle TCA = (\frac{1}{2})m\,\widehat{TC}$ (Theorem 9-8.1)
 $m\angle TDB = m\angle TCA$ (Transitive property)
 $\overleftrightarrow{AC} \parallel \overleftrightarrow{BD}$ (Theorem 6-2.1)

19. $128 + x + y = 360$
 $x + y = 232$
 $40 = \frac{1}{2}(y - x)$
 $y - x = 80$
 $x = 76$
 $y = 156$

20. The angle adjacent to 115 is 65.
 $65 = \frac{1}{2}(x + y)$ which yields: $x + y = 130$
 $25 = \frac{1}{2}(x - y)$ which yields: $x - y = 50$
 Therefore: $x = 90$
 $y = 40$

21. 

 Draw $\overleftrightarrow{PA}$ and $\overleftrightarrow{PB}$ tangent to ⊙Q at points A and B, respectively
 $\widehat{ACB}$ is a minor arc.
 $\overline{QA} \perp \overleftrightarrow{PA}$ (Theorem 9-3.2)
 $\overline{QB} \perp \overleftrightarrow{PB}$ (Theorem 9-3.2)
 $m\angle QAP = 90 = m\angle QBP$ (Theorem 3-1.1)
 $m\angle P + m\angle AQB = 180$ (Theorem 6-5.1)
 $m\angle P + m\,\widehat{ACB} = 180$ (Postulate 2-1).

22. Let $m\angle PAB = m\angle ABP = x$ (Theorem 3-4.2)
 $m\angle APB = 180 - 2x$ (Theorem 6-4.2)
 $\overline{PB} \perp \overline{BC}$ (Theorem 9-3.2)
 $m\angle ABC = 90 - x$ (Postulate 2-11)
 $m\angle APB = m\,\widehat{AB}$ (Definition 9-17)
 $m\,\widehat{AB} = 180 - 2x$ (Transitive property)
 $m\angle ABC = (\frac{1}{2})(180 - 2x) = (\frac{1}{2})\,m\,\widehat{AB}$ (Postulate 2-1)

23. $m\angle BAC = (\frac{1}{2})m\,\widehat{AEC}$ (Theorem 9-8.1)
 $m\angle D = (\frac{1}{2})m\,\widehat{AEC}$ (Theorem 9-6.1)
 $m\angle BAC = m\angle D$ (Transitive property)
 Similarly, $m\angle DAC = m\angle B$
 $\triangle ABC \sim \triangle DAC$ (Corollary 8-5.1a)
 $BC/AC = AC/DC$ (Definition 8-6).

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24. We wish to prove that $\widehat{AC} \cong \widehat{BC}$.
 $\angle B \cong \angle BCE$ (Theorem 6-2.1)
 $m\angle B = (\frac{1}{2})\,m\,\widehat{AC}$ (Theorem 9-6.1)
 $m\angle BCE = (\frac{1}{2})\,m\,\widehat{BC}$ (Theorem 9-6.1)
 $m\,\widehat{AC} = m\,\widehat{BC}$ (Transitive property)

25. $\angle P \cong \angle Q$ (Corollary 9-6.1a)
 $m\angle P = (\frac{1}{2})(m\,\widehat{EC} - m\,\widehat{AB})$ (Theorem 9-8.3)
 $m\angle Q = (\frac{1}{2})\,(m\,\widehat{FD} - m\,\widehat{AB})$ (Theorem 9-8.3)
 $(\frac{1}{2})(m\,\widehat{EC} - m\,\widehat{AB}) = (\frac{1}{2})\,(m\,\widehat{FD} - m\,\widehat{AB})$ (Postulate 2-1)
 $m\,\widehat{EC} = m\,\widehat{FD}$ (Addition property).

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Class Exercises

1. $\widehat{AC}$
2. $\widehat{AC}$
3. Reflexive property
4. DPA
5. CP/AP = AP/DP
6. CP and DP
7. CP•DP
8. AP
9. $(AP)^2$
10. transitive; CP • DP

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Exercises

1. $x = 5$ (Theorem 9-9.1)
2. $(5 + 4)/x = x/4$ (Theorem 9-9.2)
3. (PA) • (PB) = (PD) • (PC) (Theorem 9-9.3)
   $(15) \cdot (2) = (x + 7) \cdot (x)$
   $x^2 + 7x - 30 = 0$
   $(x + 10) \quad (x - 3) = 0$
   $x = 10 \mid x = 3$

   (reject negative)

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4. AD = AE = 5 (Theorem 9-9.1)
   DL = LF = 1 (Theorem 9-9.1)
   x = AD + DL = 6
   SR = SE = 2 (Theorem 9-9.1)
   RI = IF = 3 (Theorem 9-9.1)
   y = SR + SI = 5
5. PB = PC (Theorem 9-9.1)
   PB = PA (Theorem 9-9.1)
   Therefore $x = y = 7$
6. PD = PC (Theorem 9-9.1)
   PA = PC (Theorem 9-9.1)
   PA = PD = PC (Transitive property)
   Therefore $y = 5$
   PB = PC (Theorem 9-9.1)
   $x = 5$
7. (AE) • (EB) = (DE) • (EC) (Theorem 9-9.4)
   (3) • (9) = (DE) • (4)
   DE = 27/4
   DC = DE + EC
   DC = 27/4 + 4 = 43/4
8. (AE) • (EB) = (DE) • (EC) (Theorem 9-9.4)
   (3) • (8) = (6) • (EC)
   EC = 4
9. (AE) • (EB) = (DE) • (EC) (Theorem 9-9.4)
   $(18 - x) \cdot (x) = (9) \cdot (5)$
   $18x - x^2 = 45$
   $x^2 - 18x + 45 = 0$
   $(x - 15) \cdot (x - 3) = 0$
   x = 15 = EB
   AE = $18 - x$ = 3
   or
   x = 3 = EB
   AE = $18 - x$ = 15
10. PB/AP = AP/PC
    9/AP = AP/4
    AP = 6.
11. x = BC
    $x + 3$ = PB
    (PB) • (PC) = (PD) • (PE)
    $(x + 3) \cdot (3) = (8) \cdot (6)$
    x = 13 = BC
12. x = PD
    $x - 7$ = PE
    PD/AP = AP/PE
    $x/12 = 12/(x - 7)$
    $x^2 - 7x - 144 = 0$
    $(x - 16) \cdot (x + 9) = 0$
    x = 16 = PD

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13. $\overline{AM} \cong \overline{DM}$ (Theorem 9-9.1)
    $\overline{MC} \cong \overline{MB}$ (Theorem 9-9.1)
    $\underline{AM} + \underline{MC}$ = DM + MB (Addition property)
    $\overline{AC} \cong \overline{DB}$
14. $(AP)^2$ = (PD) • (PC) (Corollary 9-9.2a)
    $(PB)^2$ = (PD) • (PC) (Corollary 9-9.2a)
    $(PB)^2 = (PB)^2$ (Transitive property)
    Therefore AP = PB
15. See *Class Exercises 1-6* on page 403.
16. See *Class Exercises 1-7* on page 403
17. See *Class Exercises 8-10* on page 403.

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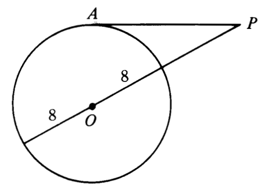
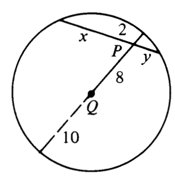
18.



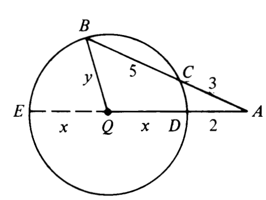
$(AP)^2 = 9(25) = 225$
AP = 15

19. AE = AH; EB = BF; CG = CF; GD = DH; (Theorem 9-9.1)
    (AE + EB) + (CG + GD) = AH + BF + CF + DH
    AB + CD = (AH + DH) + (BF + CF)
    AB + CD = AD + BC
20.



$xy = 2(18) = 36$ (Theorem 9-9.4)
Any other chord through P will have the same product for the two segments.

21. $x + y = 17$, $y = 17 - x$
    $xy = 6(10) = 60$ (Theorem 9-9.4)
    $x(17 - x) = 60$
    $x^2 - 17x + 60 = 0$
    $(x - 12)\ (x - 5) = 0$
    $x = 12 \mid x = 5$
    $y = 5 \mid y = 12$
22. $x = 8$ (Theorem 9-2.2)
    $8x = 4y$ (Theorem 9-9.4)
    $y = 2x = 16$
23.



$x = y$ (radii)
(AB) • (AC) = (AE) • (AD) (Theorem 9-9.3)
$(8) \cdot (3) = (2x + 2) \cdot (2)$
$x = 5 = y$

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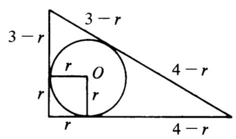
Exercises continued

24, BP = DP (Theorem 9-9.1)
AP = CP (Theorem 9-9.1)
AB = CD (Subtraction property)
2 MT = AB (Theorem 9-9.1, Transitive property).
2 NT = CD (Theorem 9-9.1, Transitive property).
$\overline{MT} = \overline{NT}$ (Postulate 2-1)
$\overline{MTN} \cong \overline{AB}$ (Postulate 2-4)

25. $m\angle D + m\angle FEC = 180$ (Theorem 9-6.2)
$m\angle PEC + m\angle FEC = 180$ (Theorem 9-6.2)
$\angle D \cong \angle PEC$ (Transitive property)
$\triangle EPC \sim \triangle DPF$ (Theorem 8-5.1)
EP/DP = CP/FP (Definition 8-6)
EP • FP = DP • CP (Theorem 8-1.1)

26.



$(5)^2 = (3)^2 + (4)^2$
Therefore the triangle is a right triangle (Theorem 8-8.2)
$4 - r + 3 - r = 5$
$r = 1$

27. Let E be the point of intersection of $\overleftrightarrow{DC}$ and the circumscribed circle of $\triangle ABC$
$\angle A \cong \angle E$ (Corollary 9-6.1a)
$\angle ADC \cong \angle BCE$ (Theorem 8-5.1)
$\triangle ADC \sim \triangle EBC$ (Theorem 8-5.1)
AC/DC = EC/BC (Definition 8-6)
AC • BC = DC • EC = DC (DC + DE) = DC^2 + (DC • DE) (Multiplication property)
DC • DE = AD • BD (Theorem 9-9.4)
AC • BC = DC^2 + (AD • BD) (Postulate 2-1)
DC^2 = (AC • BC) - (AD • DB) (Postulate 2-1)

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28. Draw $\overline{AD}$ and $\overline{CB}$.
$\triangle APD \sim \triangle CPB$ (Theorem 8-6.1)
$\angle DAB \cong \angle BCD$ (Definition 8-6)
Points A, B, C, and D are concyclic (Theorem 9-7.1).

29. $\angle BAC \cong \angle CDB$ (Definition 8-6)
Points A, B, C, and D are concyclic (Theorem 9-7.1).

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Class Exercises

1. 12π	2. $\frac{1}{4}$	3. $\frac{1}{4}$	4. 3π
5. 1/6	6. 2π	7. 5/12	8. 5π

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Exercises

1. - 5. 3 is worst approximation
$\frac{355}{113} \approx 3.1415929$ is the best approximation.

6. $c = 2\pi r$
$c = 2\pi\ (4) = 8\pi$

Exercises 7-10 are done in a way similar to Exercise 6

7. 14π 8. $4r\pi$ 9. $2x\pi$ 10. $\pi(6x - 10)$

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11. $c = 2\pi r$
$r = \frac{c}{2\pi}$
$r = \frac{6\pi}{2\pi} = 3$

Exercises 12-15 are done in a way similar to Exercise 11.

12. $6\frac{1}{2}$ 13. $\frac{9}{\pi}$ 14. $\frac{c}{2\pi}$ 15. $\frac{2x + 1}{2\pi}$

16. $c = \pi d$
$5\pi = \pi d$
$d = 5$

Exercises 17-20 are done in a way similar to Exercise 16.

17. 12 18. $\frac{20}{\pi}$ 19. $\frac{c}{\pi}$ 20. $\frac{4x + 7}{\pi}$

21. Length of semicircle = πr.

22. Length of semicircle = πr
Length of semicircle = (3.14)(1) = 3.14

Exercises 23-26 are done in a way similar to Exercise 22.

23. 12.56 24. 28.26 25. $6.28x$ 26. $(3x-2)(3.14)$

27. $r = 7$
Length of the arc = $\frac{n}{360} \cdot 2\pi r$ (Theorem 9-10.2)
$= \frac{90}{360} \cdot 14\pi = 11$ (when $\pi = 22/7$)

Exercises 28-31 are done in a way similar to Exercise 27.

28. $\frac{11}{12}$ 29. $\frac{385}{14}$ 30. $\frac{616x}{14}$ 31. $\frac{11(14x - 1)}{14}$

32. $r = 13$
70 Revolutions = 70(360) = n
Distance = $\frac{n}{360} \cdot 2\pi r$ (Theorem 9-10.2)
$= \frac{70(360)}{360} \cdot 26\pi$
= 5720 inches (when $\pi = 22/7$)

33. Length of $\widehat{AB} = \frac{n}{360} \cdot 2\pi r$
$= \frac{120}{360} \cdot 18\pi = 6\pi$

34. Length of $\widehat{AB} = \frac{n}{360} \cdot 2\pi r$
$= \frac{45}{360} \cdot 16\pi = 2\pi$

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35. $c = 2\pi r$ and $c' = 2\pi r'$ are the circumferences of two different circles.

$\frac{c}{c'} = \frac{2\pi r}{2\pi r'} = \frac{r}{r'}$

36. Use the *Class Exercises* on page 410 as a guide.

37. Length of $\widehat{PR} = \frac{n}{360} \cdot 2\pi r$
$2\pi r = \frac{x}{360} \cdot 12\pi = 60$

38. Length of $\widehat{PR} = \frac{n}{360} \cdot 2\pi r$
$\frac{4\pi}{5} = \frac{144}{360} \cdot 2\pi x = 1$

39. Length of $\widehat{MN} = \frac{n}{360} \cdot 2\pi r$
$2 = \frac{36}{360} \cdot 2\pi x$
$x = \frac{10}{\pi}$

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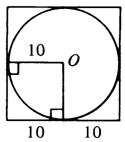
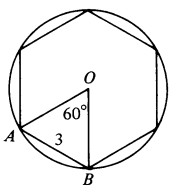
Exercises continued

40.



$c = 2\pi r, r = 10$
$c = 20\pi$

41.



P = 6(AB) = 18
AB = 3
$c = 2\pi r$
$c = 6\pi$
Since there are 6 chords of equal length, the 6 arcs must also have equal length.
$m\,\widehat{AB} = 360/6 = 60$
$m\angle AOB = 60$
$m\angle A = m\angle B = m\angle AOB = 60$
Therefore $\triangle AOB$ is equilateral and equiangular
AO = BO = AB = 3

42. The quotient of 2 rationals is rational (closure), $\frac{c}{d} = \pi$, an irrational number. Thus, not both c and d are rational.

43. A. $\frac{c_1}{c_2} = \frac{r_1}{r_2}$

$\frac{c_1}{c_2} = \frac{1x}{2x}$

$c_2 = 2c_1$

Doubled

B. $\frac{c_1}{c_2} = \frac{r_1}{r_2}$

$\frac{c_1}{c_2} = \frac{1x}{3x}$

$c_2 = 3c_1$

Tripled

44. A. $\frac{c_1}{c_2} = \frac{r_1}{r_2}$

$\frac{1x}{2x} = \frac{r_1}{r_2}$

$r_2 = 2r_1$

Doubled

B. $\frac{c_1}{c_2} = \frac{r_1}{r_2}$

$\frac{1x}{3x} = \frac{r_1}{r_2}$

$r_2 = 3r_1$

Tripled

45. $\frac{c_1}{c_2} = \frac{r_1}{r_2}$

$\frac{c_1}{c_2} = \frac{2}{3}$

Exercises 46–48 are done in a way similar to Exercise 45

46. $\frac{5}{6}$ 47. 8:1 48. 2:7

49. $\frac{c_1}{c_2} = \frac{r_1}{r_2}$

$\frac{5}{9} = \frac{r_1}{9}$

$r_1 = 5$

Exercises 50–52 are done in a way similar to Exercise 49

50. 15 51. 40/3 52. 45

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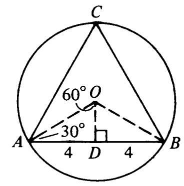
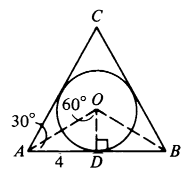
53.



AB = BC = AC = 8
$m\,\widehat{AB} = \frac{360}{3} = 120$
$m\angle AOB = 120$
$m\angle AOD = m\angle BOD = 60$
$AD = (OD)\sqrt{3}$
$4 = (OD)\sqrt{3}$
$OD = \frac{4\sqrt{3}}{3}$
$r = AO = 2(OD) = \frac{8\sqrt{3}}{3}$
$C = 2\pi r$
$C = \frac{16\pi\sqrt{3}}{3}$

54.



AB = BC = AC = 8
$m\angle AOB = \frac{360}{3} = 120$
$m\angle AOD = m\angle BOD = 60$
$AD = (OD)\sqrt{3}$
$4 = (OD)\sqrt{3}$
$r = OD = \frac{4\sqrt{3}}{3}$
$C = 2\pi r = \frac{8\pi\sqrt{3}}{3}$

55. $c_1 = 2\pi r_1$
$20 = 2\pi r_1$
$r_1 = \frac{10}{\pi}$

$c_2 = 2\pi r_2$
$25 = 2\pi r_2$
$r_2 = \frac{25}{2\pi}$

$r_1 - r_2 = \frac{5}{2\pi}$

56. In $\odot$P, PQ = PM = PN = 12
In $\odot$Q, QP = QN = QM = 12
Therefore $\triangle PQM$ and $\triangle PQN$ are equilateral
Length of $\widehat{MPN} = \frac{m\angle MQN}{360} \cdot 2\pi r$
$= \frac{120}{360} \cdot 24\pi = 8\pi$

Length of $\widehat{MQN} = \frac{m\angle MPN}{360}$

$= \frac{120}{360} \cdot 24\pi = 8\pi$

c_1 of $\odot$P = $2\pi r_1 = 24\pi$
c_2 of $\odot$Q = $2\pi r_2 = 24\pi$
$c_1 + c_2$ −(Length of $\widehat{MPN}$ + Length of $\widehat{MQN}$)
$24\pi + 24\pi - (8\pi + 8\pi) = 32\pi$

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57. BC = CA = BA = 8
BF = FC = CE = EA = AD = DB = 4
$m\angle A = m\angle B = m\angle C = 60$
Length of $\widehat{DF}$ = Length of $\widehat{FE}$ = Length of $\widehat{DE}$
Length of $\widehat{DF} = \frac{60}{360} \cdot 2\pi(4) = \frac{4\pi}{3}$
$3(4\pi/3) = 4\pi$

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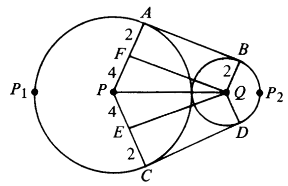

58. AB = FQ, CD = EQ
In right triangle PQF, PF = 4, PQ = 8
Therefore m∠ PQF = 30 (Theorem 8-9.2)
Therefore m∠ FPQ = 60
FQ = (PF)($\sqrt{3}$) = $4\sqrt{3}$ = AB
m∠ APC = 120
Therefore Reflex ∠ APC = 240
Length of $\widehat{AP_1C}$ = $\frac{240}{360} \cdot 2\pi(6)$
= 8π
Length of belt = AB + Length of $\widehat{BP_2D}$ + CD + Length of AP_1C
= $4\sqrt{3} + \frac{4\pi}{3} + 4\sqrt{3} + 8\pi = 8\sqrt{3} + \frac{28\pi}{3}$
In right triangle PQE, PE = 4, PQ = 8
Therefore m∠ PQE = 30 (Theorem 8-9.2)
Therefore m∠ PEQ = 60
EQ = (PE)$\sqrt{3}$ = $4\sqrt{3}$ = CD
m∠ BQD = 360 - (90 + 30 + 30 + 90) = 120
Length of $\widehat{BP_2D}$ = $\frac{120}{360} \cdot 2\pi(2) = \frac{4\pi}{3}$

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Review Exercises

1. False; A circle is the set of all points at a given distance from a given point in a plane.

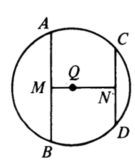
2. True.

3. True.

4. False; If P is a point of ⊙Q, then we may say that P is a point of ⊙Q.

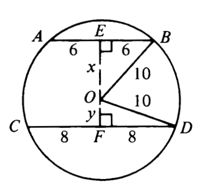
5. False; The plane containing the center of a sphere contains many diameters of the sphere.

6.



Draw parallel chords $\overline{AB}$ and $\overline{CD}$ of ⊙Q.
M and N are the midpoints of $\overline{AB}$ and $\overline{CD}$, respectively.
$\overline{MQ} \perp \overline{AB}$ (Theorem 9-2.1)
$\overline{NQ} \perp \overline{DC}$ (Theorem 9-2.1)
$\overline{MQ} \parallel \overline{NQ}$ (Corollary 6-1.1c)
This is impossible since they intersect at Q.
M, Q, and N are collinear.

7.



EF = $x + y$
$(10)^2 = x^2 + (6)^2$ (Theorem 8-8.1)
$x = 8$
$(10)^2 = y^2 + (8)^2$ (Theorem 8-8.1)

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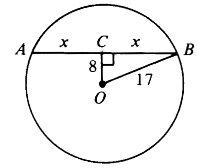
7. *continued*

$y = 6$
EF = 14

8.



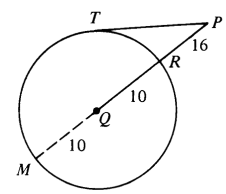
$(17)^2 = x^2 + (8)^2$ (Theorem 8-8.1)
$x = 15$
AB = $2x$ = 30

9. Draw $\overline{PE} \perp \overline{AB}$ at E,
$\overline{PF} \perp \overline{BC}$ at F.
△BEP ≅ △BFP (AAS)
$\overline{EP} \cong \overline{FP}$ (Definition 3-3)
$\overline{AB} \cong \overline{BC}$ (Theorem 9-2.4).

10. 2; 3; 0; 1.

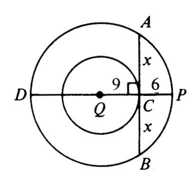
11. Tangent.

12.



$(PT)^2$ = (PM(PR)
$(PT)^2$ = (36)(16)
PT = 24

13.



(AC)(BC) = (PC)(CD)
$x \cdot x = 6 \cdot 24$
$x^2 = 6(24)$
$x = 12$
AB = $2x$ = 24

14. True.
15. False; It could be a point.

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16. True.
17. True.
18. Draw $\overline{ASD}$ and $\overline{BSC}$.
$\overline{SD} \cong \overline{SB}$ (radii)
△ABS ≅ △CDS (HL),
$\overline{AS} \cong \overline{CS}$ (Definition 3-3)
Thus, the planes are equidistant from S.

19. PA = PB (radii)
PN = PN (Reflexive property)
△APN ≅ △BPN (SSS)
m∠ APN ≅ m∠ BPN (Definition 3-3)
m $\widehat{AM}$ = m $\widehat{BM}$ (Theorem 9-5.1)
$\overline{AM} \cong \overline{BM}$ (Theorem 9-5.3)

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20. $\overset{\frown}{DAC} \cong \overset{\frown}{ACB}$ (Given)
$\overset{\frown}{AC} \cong \overset{\frown}{AC}$ (Reflexive property)
$\overset{\frown}{DAC} - \overset{\frown}{AC} \cong \overset{\frown}{ACB} - \overset{\frown}{AC}$
$\overset{\frown}{DA} \cong \overset{\frown}{CB}$
$\overline{DA} \cong \overline{CB}$ (Theorem 9-5.3)

21. $7x + 44 + 12x - 7 = 360$
$x = 17$
$7x + 44 = 163$
$12x - 7 = 197$

22. Apply the converse of Corollary 9-6.1b.

23. $\overset{\frown}{AC} \cong \overset{\frown}{BD}$ (Theorem 9-6.3)
$x = 20$ (Corollary 9-6.1b)

24. $m\angle D = 180 - (75 + 73) = 32$
$x = 32$ (Corollary 9-6.1a)

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25. Use Theorem 9-7.1 and Definition 1-15.

26. Square or rectangle by Theorem 9-7.2, Theorem 7-1.3.

27. $m\angle A + m\angle MND = 180$ (Theorem 9-6.2)
$m\angle MNC + m\angle MND = 180$ (Theorem 9-6.2)
$\angle A \cong \angle MNC$ (Theorem 3-1.4)
$m\angle B + m\angle A = 180$ (Corollary 6-3.1b)
$m\angle B + m\angle MNC = 180$ (Postulate 2-1)
Quadrilateral BMNC is a cyclic (Theorem 9-7.2).

28. $65 = \frac{1}{2}(67 + x)$ (Theorem 9-8.2)
$x = 63$

29. $m\overset{\frown}{CD} = 360 - 278$
$m\overset{\frown}{CD} = 82$
$x = \frac{1}{2}(m\overset{\frown}{CD}) = 41$ (Theorem 9-8.1)

30. Major $\overset{\frown}{AC} = 200$
$x = \frac{1}{2}(200) = 100$ (Theorem 9-8.1)

31. $m\angle ADM = (\frac{1}{2})(m\overset{\frown}{AM} + m\overset{\frown}{BN})$ (Theorem 9-8.2)
$m\angle AEN = (\frac{1}{2})(m\overset{\frown}{CM} + m\overset{\frown}{AN})$ (Theorem 9-8.2)
$m\overset{\frown}{AM} = m\overset{\frown}{CM}$ (Given)
$m\overset{\frown}{BN} = m\overset{\frown}{AN}$ (Given
$m\angle ADM = m\angle AEN$ (Transitive property)
$\overline{AD} \cong \overline{AE}$ (Theorem 3-4.3)

32. $2(x) = 5(3)$ (Theorem 9-9.4)
$x = 7\frac{1}{2}$

33. Let $x = AE = EB$
$(AE)(EB) = (CE)(ED)$ (Theorem 9-9.4)
$x \cdot x = (2)(8)$
$x^2 = 16$
$x = 4$

34. $(AP)^2 = (PC)(PB)$ (Theorem 9-9.2)
$x^2 = (12)(3)$
$x = 6$

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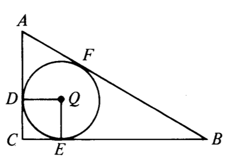
35.



Draw right $\triangle ABC$ with right $\angle C$.
Inscribe $\odot Q$ intersecting $\triangle ABC$ in F, E, and D with $\overline{AFB}$, $\overline{BEC}$ and $\overline{CDA}$.
Let r be the radius of $\odot Q$.
Quadrilateral DQEC is a square (use Theorem 9-3.2,

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35. *continued*

Theorem 6-5.1 and Theorem 7-3.3)
$AD = AF$ (Theorem 9-9.1)
$BE = BF$ (Theorem 9-9.1)
$CD = CE$ (Theorem 9-9.1)
$CD = CE = r$ (Radii)
Thus $AC + BC = AD + BE + CD + CE = AF + BF + 2r = AB + 2r$
(Postulate 2-1, Addition property)

36. $AP = BP$ and $DP = CP$ (Theorem 9-9.1)
$AD = BC$ (Subtraction property).

37. circumference diameter
38. 9
39. 18π
40. $\frac{30}{\pi}$

41. $x = \frac{150}{360} \cdot 2\pi r$

$x = \frac{150}{360} \cdot 6\pi$

$x = \frac{5\pi}{2}$

42. $2\pi = \frac{80}{360} \cdot 2\pi r$

$2\pi = \frac{80}{360} \cdot 2\pi x$

$x = \frac{9}{2}$

43. $2\pi = \frac{x}{360} \cdot 2\pi r$

$2\pi = \frac{x}{360} \cdot 16\pi$

$x = 45$

Chapter Test

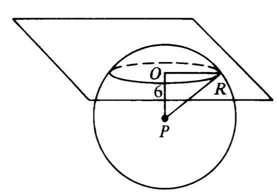
1. Tangent
2. 3
3. Greater
4. supplementary.
5. sum
6. congruent
7. π

8. $3(x) = 2(9)$ (Theorem 9-9.4)
$x = 6$

9. $m\angle DAC = \frac{1}{2}m\overset{\frown}{AC}$ (Theorem 9-8.1)
$x = \frac{1}{2}m\overset{\frown}{AC}$
Therefore $x = m\angle DAC = 47$

10. $34 = \frac{1}{2}(m\overset{\frown}{AE} - m\overset{\frown}{BD})$ (Theorem 9-8.3)
$34 = \frac{1}{2}(m\overset{\frown}{AE} - 59)$
$m\overset{\frown}{AE} = 127$
$x = \frac{1}{2}(m\overset{\frown}{AE} + m\overset{\frown}{BD})$ (Theorem 9-8.2)
$x = \frac{1}{2}(127 + 59) = 93$

11. The radius of the sphere, r, is found by
$c = 2\pi r$
$20\pi = 2\pi r$
$10 = r = PR$



$\overline{OP} \perp \overline{OR}$
$OP = 6$
By Theorem 8-8.1, $OR = 8$
Therefore the circumference of $\odot O = 2\pi(8) = 16\pi$.

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Class Exercises

1. A X B = {(a, - 1), (a, 0), (b,-1), (b, 0), (c, - 1),(c,0), (d,-1), (d, 0)}
2. B X A = {(- 1, a), (0, a), (- 1, b), (0, b), (-1,c) (0, c), (-1, d), (0, d)}
3. A X C = { (a, 1), (a, 2), (b, 1) (b, 2), (c, 1), (c, 2), (d,1) , (d,2)}
4. B X C = { (-1, 1), (-1, 2), (0, 1), (0, 2)}

5. 3 6. - 3 7. (-4,0) 8. (0, 2)
9. (3, 1) 10. (1, -2) 11. (-3,-3) 12. I, III

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Exercises

1. (0,0) 2. (2, 1) 3. (-4, 2) 4. (-3, -3)
5. (2, -2.25) 6. F 7. J 8. I
9. G 10. H

11. I. B, I; II. C; III. D; IV. E, G.

12. I 13. I or IV 14. III 15. III or IV
16. y-axis 17. x-axis 18. Positive x-axis
19. Negative y-axis.

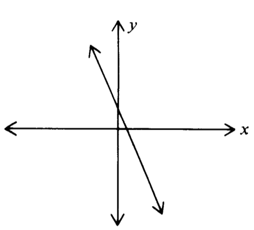
20. $y = 2x$

x	-3	-2	-1	0	1	2	3
y	-6	-4	-2	0	2	4	6

21. $2x - 3y = 6$

x	-3	-2	-1	0	1	2	3
y	-4	$-\frac{10}{3}$	$-\frac{8}{3}$	-2	$-\frac{4}{3}$	$-\frac{2}{3}$	0

22. {(-2, - 1), (- 1, $-\frac{1}{3}$),(0, $\frac{1}{3}$), (1, 1), (2, 5/3)}
23. {(-2, 3), (- 1, 6), (0, 9), (1, 12), (2, 15)}
24. {(- 2, - 2), (- 1, 1), (0, 4), (1, 7), (2, 10)}
25. {(- 2, 10), (- 1, 7), (0, 4), (1, 1), (2, -2)}
26. {(- 2, 10), (- 1, 8), (0, 6), (1, 4), (2, 2)}
27. {(- 2, 3), (- 1, 2), (0, 1), (1, 2), (2, 3)}
28. {(- 1, 4), (0, 1), (1, - 2)}
29. {(- 3, 10), (- 2, 7), (- 1, 4), (0, 1), (1, -2), (2, -5), (3, - 8)}
30.



31. Solution set is {(-2, -7), (-1, -4), (0, -1), (1, 2),(2,5)}
32. Solution set is {(-2, -6), (-2, -5), (-2, -4),..., (-1, -3), (-1, -2), ...,(0,0), (0,1),...,(1,3), (1, 4),...,(2, 6),, (2, 7),...}
33. Solution set is {(-2, -8), (-2, -9), (-2, -10),..., (-1, -5), (-1, -6),..., (0, -2), (0, -3),..., (1, 1), (1, 0),..., (2, 4), (2, 3),...}

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34. **Solution set is** {(-2, 6), (- 1, 4), (0, 2), (1,0),(2,2)}
35. Solution set is {(-2, 2), (-2, 3), (-2, 4),..., (-1,1), (- 1, 2), (-1, 3),...,(0,0), (0,1),..., (1, 1), (1, 2),..., (2, 2), (2, 3),...}
36. Solution set is {(-2, 4), (-2, 5), (-2, 6),..., (-1, 3), (-1, 4),..., (0, 2), (0, 3),..., (1, 1), (1, 2),..., (2, 2), (2, 3),...}

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37. Solution set is { ..., (-1, -1), (0, -2), (1, -1), (2, 0),...}
38. Solution set is { ..., (-1,2), (0,1), (1,0), (2,1),...}
39. Solution set is { ..., (-1, 3), (0, 2), (1, 1),(2,0),...}
40. Solution set is { ..., (-1,0), (0,0), (1,0), (2,0), ...}
41. Solution set is { ..., (2, -1), (2,0), (2,1), (2,2),...}
42. Solution set is { ..., (3,-3), (2, -2), (1, -1), (0, 0), (1, 1),...}
43. The graph is all points above and including the x-axis.
44. The graph is a vertical strip between and including the lines $x = -1$ and $x = 1$.
45. The graph is two lines, one bisecting quadrants I and III, the other bisecting quadrants II and IV.
46. The graph is the two axes.
47. The graph is a line bisecting quadrants I and III with (0, 0) removed.
48. The graph is a line bisecting quadrants II and IV with (0, 0) removed.
49. The graph is a parabola with vertex (0, 0), axis the y-axis, and directrix $y = \frac{1}{4}$.
50. The graph is the reflection of Exercise 49 about the x-axis.

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Class Exercises

1. the x-axis.
2. the y-axis.
3. perpendicular; The x- and y- axes are perpendicular.
4. right triangle.
5. $|x_2 - x_1|$; $(x_2 - x_1)^2$
6. $|y_2 - y_1|$; $(y_2 - y_1)^2$
7. $(PR)^2 + (QR)^2$
8. $\sqrt{(PR)^2 + (QR)^2}$ or $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

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1. $\sqrt{(2 - 1)^2 + (4 - 2)^2} = \sqrt{1 + 4} = \sqrt{5}$
2. $\sqrt{34}$ 3. $\sqrt{178}$ 4. $\sqrt{82}$
5. $\sqrt{130}$ 6. $\frac{\sqrt{37}}{4}$ 7. $\sqrt{4x^2 + 4y^2}$
8. $\sqrt{x^2 + y^2}$ 9. $\sqrt{9x^2 + y^2}$

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Exercises continued

10. $\frac{1}{2}(7/2 + 4/3) = 29/12$
$\frac{1}{2}(-3 + 3) = 0$
Midpoint is (29/12, 0)

11. (-1, - 7/2) 12. (½, -2) 13. (5, 7/2)

14. $(x, 0)$ 15. $(3x, y)$

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16. No, because Theorem 5-4.1 is violated.

17. A(2, 5), B(-1, 8), C(-3,10)

$AB = \sqrt{(-1-2)^2 + (8-5)^2} = \sqrt{9+9} = 3\sqrt{2}$

$AC = \sqrt{(-3-3)^2 + (10-5)^2} = \sqrt{25+25} = 5\sqrt{2}$

$BC = \sqrt{(-3+1)^2 + (10-8)^2} = \sqrt{4+4} = 2\sqrt{2}$

AC = AB + BC (Recall Theorem 5-4.1)

Exercises 18-20 are done in a way similar to Exercise 17

18. No. 19. Yes. 20. Yes.

21. (x_1, y_1) (x_m, y_m)

$P_1(-3, 10)$, $M(-2, 9)$, $P_2(x_2, y_2)$

$x_m = \frac{x_1 + x_2}{2}$ $y_m = \frac{y_1 + y_2}{2}$

$-2 = \frac{-3 + x_2}{2}$ $9 = \frac{10 + y_2}{2}$

$x_2 = -1$ $y_2 = 8$

$P_2(-1, 8)$

22. $AB = \sqrt{(-3-1)^2 + (5+1)^2} = \sqrt{52} = 2\sqrt{13}$

$AC = \sqrt{(5-1)^2 + (5+1)^2} = \sqrt{52} = 2\sqrt{13}$

$BC = \sqrt{(5+3)^2 + (5-5)^2} = \sqrt{64} = 8$

AB = AC

Therefore △ABC is isosceles

23. $AB = \sqrt{(0+4)^2 + (4-0)^2} = 4\sqrt{2}$

$AC = \sqrt{(4+4)^2 + (0-0)^2} = 8$

$BC = \sqrt{(4-0)^2 + (0-4)^2} = 4\sqrt{2}$

$(AC)^2 \stackrel{?}{=} (AB)^2 + (BC)^2$

$(8) \stackrel{?}{=} (4\sqrt{2})^2 + (4\sqrt{2})^2$

$64 \stackrel{?}{=} 32 + 32$

$64 = 64$

Therefore △ABC is a right triangle.

24. PQ = 8, SR = 8
$PS = 3\sqrt{2}$, $QR = 3\sqrt{2}$
Quadrilateral PQRS is a parallelogram (Theorem 7-2.1)

25. $WX = 2\sqrt{10}$
$XY = \sqrt{29}$
$YZ = 2\sqrt{10}$
$WZ = \sqrt{29}$
Quadrilateral WXYZ is a parallelogram (Theorem 7-2.1) but it is *not* a rhombus (Theorem 7-4.1)

26. $P_1(3, 2)$; $M(x_m, y_m)$; $P_2(5, 2)$
$x_m = \frac{3+5}{2} = 4$

$y_m = \frac{2+2}{2} = 2$

(continued next page)

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26. *continued*

C = (4, 2)
$r = P_1M = MP_2 = 1$

27. $P_1(3, 4)$; $M(x_m, y_m)$; $P_2(-3, 4)$

$x_m = \frac{3-3}{2} = 0$

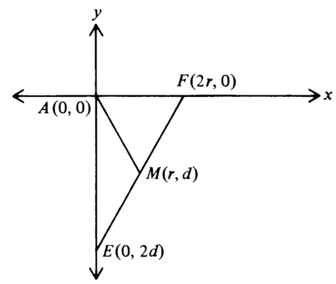
$y_m = \frac{-4+4}{2} = 0$

C(0, 0)

$r = P_1M + MP_2 = 5$

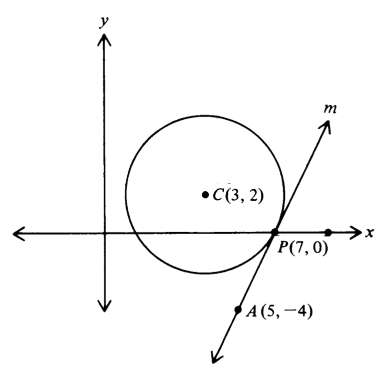
28. AB = DC = a
$AD = BC = \sqrt{b^2 + c^2}$

29.



$AM = \sqrt{r^2 + d^2} = EM$

30.



$CP = \sqrt{20} = 2\sqrt{5}$
$AP = \sqrt{20} = 2\sqrt{5}$
$CA = \sqrt{40} = 2\sqrt{10}$
$(CA)^2 \stackrel{?}{=} (CP)^2 + (AP)^2$

40 = 40
Therefore △CPA is a right triangle and $\overline{CP} \perp m$
Thus, $\overline{CP}$ tangent to m (Theorem 9-3.1)

31. A(2, 3); B(14, 3); C(7, 8)
AB = 12
$AC = 5\sqrt{2}$
$BC = \sqrt{74}$
P = AB + AC + BC
$P = 12 + 5\sqrt{2} + \sqrt{74}$

Exercises 32-33 are done in a way similar to Exercise 31.

32. $11 + 3\sqrt{2} + \sqrt{13}$ 33. $8 + \sqrt{29} + \sqrt{37} + 3\sqrt{2}$

34. $PS = 2\sqrt{10}$, and $PQ = 4\sqrt{2}$
Quadrilateral PQRS is *not* a square (Definition 7-7).

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1. $m = \frac{\Delta y}{\Delta x} = \frac{9-5}{3-0} = \frac{4}{3}$

Exercises 2-6 are done in a way similar to Exercise 1

2. - 3 3. ½ 4. 0

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5. 1

6. Undefined

7. Since m is the slope of $y = mx + b$, the slope of $y = 2x + 2$ is 2.

Exercises 8-12 are done in the same way as Exercise 7

8. ½ 9. - 2 10. 1

11. 2 12. ⅓

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Exercises 1-12 are done in the same way as the Class Exercises on page 446

1. 9/2 2. 8/5 3. -1 4. y/x

5. $-\frac{2y}{3x}$ 6. $\frac{4y}{x}$ 7. 2 8. -4

9. 7 10. -2 11. -3 12. -5

13. Each of the points is found by finding the values which will satisfy the equation:

$\frac{y-1}{x-1} = \frac{3}{2}$. Two possible points are: (3,4), (5, 7).

Exercises 14-16 are done in the same way as Exercise 13. Answers may vary. Possible answers are given below:

14. (5, -1), (10, -3) 15. (0, -4), (2, -5)
16. (9, 5), (12, 8)
17. Reverse the steps in the proof of the first part.
18. Reverse the steps in the proof of the first part.

19. A(-3.4) B(3, 8) C(0, 6)
m_1 of $\overleftrightarrow{AB} = \frac{8-4}{3+3} = \frac{2}{3}$
m_2 of $\overleftrightarrow{BC} = \frac{6-8}{0-3} = \frac{2}{3}$

$m_1 = m_2$
Therefore A, B, C are collinear (Postulate 6-1)

Exercises 20-24 are done in the same way as Exercise 19.

20. No. 21. Yes. 22. No. 23. Yes. 24. No. 25. ½

25. (2, k) (3, 2k)

$\frac{2k-k}{3-2} = \frac{1}{2}$

$k = ½$

26. Just find the slope of the line determined by the given points:

$\frac{\Delta y}{\Delta x} = \frac{7-3}{2-6} = \frac{4}{-4} = -1$

27. Find the negative reciprocal of the slope of the line determined by the given points.

$\frac{\Delta y}{\Delta x} = \frac{6-1}{4-(-2)} = \frac{5}{6}$. The negative reciprocal is $-\frac{6}{5}$.

28. A(1, 1) B(0, 4) C(3,0)
m_1 of $\overline{AB} = \frac{-4-1}{0-1} = 5$
m_2 of $\overline{AC} = \frac{0-1}{3-1} = -½$
m_3 of $\overline{BC} = \frac{0+4}{3-0} = \frac{4}{3}$

A, B, C, are not vertices of a right triangle since no two sides are perpendicular (Theorem 10-3.2).

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29. A(-2, 3) B(2, 7) C(8, 5) D(4, 1)
m_1 of $\overline{AD} = \frac{-2}{6} = -\frac{1}{3}$
m_2 of $\overline{BC} = \frac{-2}{6} = -\frac{1}{3}$

m_3 of $\overline{AB} = \frac{4}{4} = 1$
m_4 of $\overline{DC} = \frac{4}{4} = 1$

Therefore $\overline{AD} \parallel \overline{BC}$, $\overline{AB} \parallel \overline{DC}$.
Thus ABCD is a parallelogram (Definition 7-1).

30. For P(3, 7) and C(-3, 2).
The slope of $\overline{CP} = \frac{5}{6}$

Therefore the slope of the tangent is $-\frac{6}{5}$.

(Theorem 9-3.2 and Theorem 10-3.2)

31. N(4, 4) is midpoint of $\overline{AC}$
M(6, 3) is midpoint of $\overline{AB}$
Slope of $\overline{MN} = \frac{4-3}{4-6} = -½$

Slope of $\overline{BC} + \frac{7-5}{5-9} = -½$
Therefore $\overline{MN} \parallel \overline{BC}$ (Theorem 10-3.1)

32. Draw △ABC with coordinates A(x_1, y_1), B(x_2 , y_2), C(x_3 • y_3).
Let D and E be the respective midpoints of $\overline{AB}$ and $\overline{BC}$.
D$(x_2 + x_1)/2$, $(y_2 + y_1)/2$ and E$(x_3 + x_2)/2$, $(y_3 + y_2)/2)$ (Theorem 10-2.2)

$DE = \sqrt{[(x_2 + x_1 - x_3 - x_2)/2]^2 + [(y_2 + y_1 - y_3 - y_2)/2]^2}$

$= (½)\sqrt{x_1 - x_3)^2 + (y_1 - y_3)^2}$ (Theorem 10.2.1);

$AC = \sqrt{x_1 - x_3)^2 + (y_1 - y_3)^2}$ (Theorem 10-2.1);

Hence, DE = (½) AC (Postulate 2-1).

33. Draw quadrilateral ABCD such that $\overline{AC}$ intersects $\overline{BD}$ at E.
Let A(x_1,y_1), B(x_2, y_2), C(x_3,y_3), and D(x_4,y_4).
E$[(x_1 + x_3)/2, (y_1 + y_3)/2]$, and E$[(x_2 + x_4)/2, (y_2 + y_4)/2]$ (Theorem 10-2.2);

$x_1 + x_3 = x_2 + x_4$, $y_1 + y_3 = y_2 + y_4$ (Postulate 2-1);

$AD = \sqrt{(x_1 - x_4)^2 + (y_1 - y_4)^2}$;

$BC = \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2}$ (Theorem 10-2.1);

$AD = \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2}$ (Postulate 2-1)
AD = BC (Transitive property)
Similarly, AB = DC.

34. Draw △ABC such that A(0,a), B(a, 0), C(b, b).
AC = BC (Theorem 10-2.1)
△ABC is isosceles (Definition 3-12)
Let D be the midpoint of $\overline{AB}$, then D(a/2, a/2) (Theorem 10-2.2)
Slope $\overline{CD} = (b-a/2)/(b-a/2) = 1$
Slope $\overline{AB} = a/-a = -1$ (Definition 10-5)
Slope CD • slope $\overline{AB}$ = 1 • - 1 = - 1
Thus, CD ⊥ AB (Theorem 10-3.2).

35. Draw equilateral quadrilateral PQRS with P(0,0), Q(a,0), R($c + a$, b) and S(c, b) where $c < a$. $b^2 = a^2 - c^2$ (Given, Theorem 8-8.1)
Slope $\overline{SQ} = b/(c - a)$,
Slope $\overline{RP} = b/(c + a)$ (Definition 10-5)
$\overline{SQ} \perp \overline{RP}$, when Slope SQ • (-1)(Slope $\overline{RP}$) = -1 (Theorem 10-3.2)
Slope $\overline{SQ}$ • (-1)(Slope $\overline{RP}$) = $[b/(c-a)] \cdot [(c + a)/-b] = -1$
Therefore $\overline{SQ} \perp \overline{RP}$.

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Exercises continued

36. Draw trapezoid TRAP such that P(0, 0), A(a, 0), T($2b$, $2c$) R($2a - 2d$, $2c$).
Then if O and E are the respective midpoints of $\overline{TP}$ and $\overline{RA}$, O(b,c), E($2a - d$,c)
Slope $\overline{TR}$ = slope $\overline{AP}$ = 0 (Definition 10-5)
Slope $\overline{DE}$ = $(c-c)/(2a - d - b)$ (Definition 10-5)
Slope $\overline{TR}$ = slope $\overline{AP}$ = slope $\overline{OE}$ (Transitive property)
$\overline{OE} \parallel \overline{TR}$ and $\overline{OE} \parallel \overline{AP}$ (Theorem 10-3.1)

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Exercises 1-6 are done in the same way as Example 4 on page 452

1. $y - 9 = 3(x - 1)$
2. $y = - 4x + 5$
3. $y - 8 = 5(x + 8)$
4. $y - 7 = (2/7)(x - 4)$
5. $y - 9 = 2(x - 3)$
6. $y = - 5$

Exercises 7-12 are done in the same way as Example 6 on page 453

7. $y = 3$
8. $y = - 2x$
9. $y = x + 2$
10. $y - 1 = (2/3)(x - 7)$
11. $y = 2x - 9$
12. $y + 1 = (4/3)(x - 5)$

For Exercises 13-18 substitute the appropriate values for m and b in y = mx + b.

13. $y = (3/4)x + 2$
14. $y = 3x + 17$
15. $y = - x - 9$
16. $y = -(\frac{1}{2})x + 10$
17. $y = -(3/4)x - 8$
18. $y = -(1/5)x + 6$

For Exercises 19-24 substitute the appropriate values for a and b in x/a + y/b = 1 (Theorem 10 4.2)

19. $x/7 + y/21 = 1$
20. $x/2 + y/5 = 1$
21. $x/4 - y/3 = 1$
22. $x/6 + y/6 = 1$
23. $x/3 - y/1 = 1$
24. $x/-2 - y/4 = 1$

25. Divide all terms by 4.
$x/4 + y/2 = 1$

26. Divide all terms by 6.
$x/2 + y/3 = 1$

27. Divide all terms by 4.
$y/4 - x/2 = 1$

28. Use Theorem 10-4.2 to get the equation in the Two-Intercept form, by dividing each term by 15.
$y/15 - 2x/15 = 1$

Which can be written as:
$$\frac{x}{\frac{-15}{2}} + \frac{y}{15} = 1$$
The slope is $-\frac{b}{a}$, which is $-\frac{15}{\frac{-15}{2}} = 2$
Where $a = -15/2$ and $b = 15$, the x and y intercepts, respectively.

Exercises 29-36 are done the same way as Exercise 28.

29. m = -(3/2), b = 3, a = 2
30. m = 1/7, b = -2, a = 14.
31. m = 3, b = -6, a = 2
32. m = 4/5, b = -4, a = 5
33. m = -2, b = 5, a = $2\frac{1}{2}$
34. m = $-\frac{1}{2}$, b = -5, a = -10
35. m = 2, b = -6, a = 3
36. m = -(3/7), b = 9/7, a = 3
37. Since the slope of the given line is -2, the slope of the perpendicular is $\frac{1}{2}$.

$y - 7 = \frac{1}{2}(x - 1)$ (Theorem 10-4.1)
or
$2x + y = 6$

38. The slopes of parallel lines are equal. The slope of $y + 2 = \frac{1}{2}(x - 7)$ (Theorem 10-4.1)
or, $x - 2y = 11$
39. *This is done in the same way as Exercise 37.*
$y = 5$.

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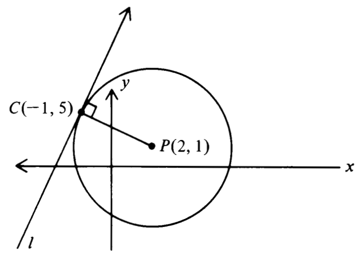
40. Slope of $\overline{AB} = \frac{-3-1}{2-1} = -4$
Slope of the perpendicular to $\overline{AB} = \frac{1}{4}$
The midpoint of $\overline{AB}$ is (3/2, - 1) (Theorem 10-2.2)
The equation of $\overline{AB}$: $y + 1 = \frac{1}{4}\left(x - \frac{3}{2}\right)$

41. *This is done in the same way as Exercise 40*
$y + 2 = \frac{2}{5}(x - 4)$

42.



Slope of $\overline{PC} = \frac{5-1}{-1-2} = -\frac{4}{3}$

Slope of $l = \frac{3}{4}$
Equation of l: $y - 5 = \frac{3}{4}(x + 1)$ (Theorem 10-4.1)

43. A(2, -5) B(-2,5)
Slope of $\overline{AB} = \frac{5 + 5}{-2-2} = -\frac{5}{2}$
Equation of $\overline{AB}$: $y + 5 = -\frac{5}{2}(x-2)$ (Theorem 10-4.1)
$y = \frac{-5x}{2}$
Since both x and y intercepts are 0, a = 0, and b = 0. Therefore we cannot express the equation in the form $x/a + y/b = 1$ (division by 0!)

44. M is the midpoint of $\overline{BC}$
A(3, 2) M(7/2, 4) (Theorem 10-2.2)
Slope of $\overline{AM} = \frac{4 - 2}{\frac{7}{2} - 3} = 4$
$y - 2 = 4(x - 3)$; and $y = 4x - 10$ (Theorem 10-4.1)

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Exercises 1-5 are done in the same way as Example 1 on page 456.

1. $x^2 + y^2 = 9$
2. $x^2 + y^2 = 3$
3. $x^2 + y^2 = 1/16$
4. $x^2 + y^2 = 1$
5. $x^2 + y^2 = 36$

Exercises 6-10 are done in the same way as Example 3 on page 456.

6. $(x-2)^2 + (y-5)^2 = 9$
7. $x^2 + (y-3)^2 = 9$
8. $(x-\frac{1}{2})^2 + (y+\frac{1}{2})^2 = 9$
9. $(x+2)^2 + (y-5)^2 = 9$
10. $(x-1)^2 + (y-1)^2 = 9$

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Exercises 1-10 are done in the same way as Example 3 on page 456

1. $x^2 + y^2 = 4$
2. $x^2 + y^2 = 25/4$
3. $x^2 + y^2 = 3$
4. $x^2 + y^2 = 144$
5. $x^2 + y^2 = 20$
6. No.
7. Yes.
8. Yes
9. No.
10. Yes.

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Exercises 11-16 are done in the same way as Example 3 on page 456

11. $(x+1)^2 + (y-5)^2 = \frac{1}{4}$
12. $(x-8)^2 + (y-5)^2 = 81$
13. $x^2 + (y-3)^2 = 16$
14. $(x+5)^2 + (y-4)^2 = 11$
15. $x^2 + y^2 = 4/9$
16. $(x-6)^2 + (y-5)^2 = 36$

17. C(8, 5) P(2, 1)
$r = CP = \sqrt{52} = 2\sqrt{13}$ (Theorem 10-2.1)
$(x-h)^2 = (y-k)^2 = r^2$
$(x-8)^2 + (y-5)^2 = 52$

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18. C(3, -4) P(-3, 0)
$r = CP = \sqrt{52} = 2\sqrt{13}$ (Theorem 10-2.1)
$(x-3)^2 + (y+4)^2 = 52$

19. A(-5, 3), $C(h,k)$ B(3, 6)
$C(-1, \frac{9}{2})$ (Theorem 10-2.2)
$r = AC = CB = \sqrt{\frac{73}{4}} = \frac{1}{2}\sqrt{73}$ (Theorem 10-2.1)

20. $r = 5$
$(x-5)^2 + (y-13)^2 = 25$

21. $r = 6$
$(x-5)^2 + (y-6)^2 = 36$

22. Draw quadrilateral ABCD with A(0, 0), B($2b$, $2c$), C($2d$, $2e$), D($2a$, 0).
If E is the midpoint of $\overline{BD}$ and $\overline{CA}$, then E($b-a, c$) and E (d,e);
Thus $c = e$, and $b - a = d$ (Postulate 2-1)
Slope $\overline{BC} = (2c-2e)/(2b-2d) = 0/(b-d)$ (Definition 10-5, Postulate 2-1)
Slope $\overline{AD}$ = 0 (Definition 10-5;
$\overline{AD} \parallel \overline{BC}$ (Theorem 10-3.1)
Similarly, $\overline{CD} \parallel \overline{BA}$.

23. Let $m_1 \perp l$ and $m_2 \perp l$ such that slope $l = a$.
Slope $m_1 = -1/a$,
Slope $m_2 = -1/a$, (Theorem 10-3.2)
$m_1 \parallel m_2$ (Theorem 10-3.1).

24. Let $l \parallel k$ such that slope l = slope $k = a$.
If $m \perp l$, then slope $m = -1/a$ (Theorem 10-3.2)
Slope m • slope $k = -1/a \cdot a = -1$ (Postulate 2-1)
$m \perp k$ (Theorem 10-3.2).

25. Let $l \parallel k$ and $m \parallel k$.
If slope $k = a$, then slope $l = a$, and slope $m = a$ (Theorem 10-3.1)
$l \parallel m$ (Theorem 10-3.1)

26. Draw quadrilateral PQRS such that P(0,0), Q(b, 0), R($c + b$, $\sqrt{a^2 - c^2}$), S(c,$\sqrt{a^2 - c^2}$).
Then PS = QR and PQ = SR (Theorem 10-2.1)
Slope $\overline{SP} = \sqrt{a^2 - c^2}/c$,
Slope $\overline{RQ} = \sqrt{a^2 - c^2}/(c + b - b)$ (Definition 10-5)
$\overline{SP} \parallel \overline{RQ}$ (Theorem 10-3.1)
Similarly, $\overline{PQ} \parallel \overline{SR}$.

27. Let ABCD be a quadrilateral with $\overline{AB} \parallel \overline{DC}$ and $\overline{AB} \cong \overline{DC}$.
If A(0,0), B(a, 0) and D(b,c), then C($a + d$,c) (Theorem 10-3.1)
Slope $\overline{DA} = c/b$
Slope $\overline{CB} = c/d$ (Definition 10-5)
$a^2 = (a + d - b)^2$ (Theorem 10-2.1)
$d - b = 0$ (Property of exponents)
$\overline{DA} \parallel \overline{CB}$ (Theorem 10-3.1)
ABCD is a parallelogram (Definition 7-1)

28. Draw rectangle XYZW such that X(0,0), Y(a, 0), W(0,b), Z(a,b).
$ZX = \sqrt{a^2 + b^2}$ (Theorem 10-2.1)
$WY = \sqrt{a^2 + b^2}$ (Theorem 10-2.1)
$\overline{ZX} \cong \overline{WY}$ (Transitive property).

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29. Draw $\triangle$QRT with right $\angle$ Q such that Q(0,0), R($2a$, 0), T(0, $2b$).
If S is the midpoint of $\overline{TR}$, S(a,b) (Theorem 10-2.2)
$QS = \sqrt{a^2 + b^2}$ (Theorem 10-2.1)
$TR = \sqrt{4a^2 + 4b^2}$ (Theorem 10-2.1)
TR = 2QS (Transitive property).

30. Draw $\triangle$MOP such that M(0, 0), O($2a$, 0), P($2b$, $2c$).
If T is the midpoint of $\overline{PO}$ then T ($b + a$,c) (Theorem 10-2.2)
$\overline{TA} \parallel \overline{OM}$ where A is in $\overline{PM}$ (Given)
A (x,c) (Theorem 10-3.1)
Midpoint $\overline{PM}$ (b,c) (Theorem 10-2.2)
A (b,c) (Postulate 2-4, Transitive property).

31. *See solution for Exercise 36 on page 450.*

32. Draw trapezoid DRAT such that D (0, 0), R($2a$,0), A($2d$, $2c$), T($2b$, $2c$).
If E and F are the respective midpoints of $\overline{TD}$ and $\overline{AR}$, E (b,c), and F($d + a$,c) (Theorem 10-2.2)
$EF = d+a - b$, (Theorem 10-2.1)
$DR = 2a$, $TA = 2d - 2b$ (Theorem 10-2.1)
$DR + TA = 2a + 2d - 2b = 2(d + a - b)$ (Addition property)
EF = ($\frac{1}{2}$)(DR + TA) (Transitive property)

33. Draw trapezoid DING with D(0,0), I(a, 0), N(a-d,c), G(b,c)
$GD = \sqrt{b^2 + c^2}$
$NI = \sqrt{d^2 + c^2}$ (Theorem 10-2.1)
GD = NI (Given)
$b = d$ (Postulate 2-1)
$GI = \sqrt{(b-a)^2 + c^2}$ (Theorem 10-2.1)
$ND = \sqrt{(a-d)^2 + c^2}$ (Theorem 10-2.1)
$GI = \sqrt{(d-a)^2 + c^2}$ (Postulate 2-1)
$\overline{GI} \cong \overline{ND}$ (Transitive property)

34. Draw $\triangle$ABC with $\angle$ A a right angle, and A(0, 0), B(a, 0), C(0,a).
$BC = \sqrt{a^2 + a^2} = \sqrt{2a^2}$ (Theorem 10-2.1)
$AB = \sqrt{a^2}$ (Theorem 10-2.1)
$\sqrt{2}$AB = BC (Multiplication property).

35. Draw $\odot$P with P(0,0), and $r = a$.
Inscribe $\angle$ ABC in $\odot$P with A(0,a), B(x,y), and C(0, $-a$)
Slope $\overline{AB} = (y - a)/x$, (Definition 10-5)
Slope $\overline{BC} = (y + a)/x$ (Definition 10-5)
$x^2 + y^2 = a^2$ (Definition 9-21)
$(y - a)/x \cdot (y + a)/x = (y^2 - a^2)/x^2 = -x^2/x^2$ (Multiplication property, Postulate 2-1)
$\overline{BC} \perp \overline{AB}$ (Theorem 10-3.2)
$\angle$ ABC is a right angle (Theorem 2-6.5).

36. Draw trapezoid CAMP with C(0,0), A($2a$, 0), M($2a - 2d$, $2c$) and P($2b$, $2c$).
If S, T, E, and W are the midpoints of the sides (reading counterclockwise) then S (a, 0), T($2a - d$, c), E($a - d + b$, $2c$), and W(b,c) (Theorem 10-2.2)
CAMP is an isoscles trapezoid (Given)
$b = d$ (Postulate 2-1)
$ST = \sqrt{(a-d)^2 + c^2}$,
$TE = \sqrt{(a-b)^2 + c^2}$ (Theorem 10-2.1)
ST = TE (Transitive property)
Slope $\overline{ST} = c/(a - d)$
Slope $\overline{WE} = c/(a - d)$ (Definition 10-5)
$\overline{ST} \parallel \overline{WE}$ (Theorem 10-3.1)
Similarly, $\overline{TE} \parallel \overline{SW}$.
STEW is a rhombus (Definition 7-6).

37. Draw parallelogram MEAT such that M(0, 0), E(a, 0), A($b + a$, c), T(b , c).
TE = MA (Given)
$(b - a)^2 + c^2 = c^2 = (b + a)^2 + c^2$ (Postulate 2-1)
$2b(-2a) = 0$, then, either $b = 0$ or $a = 0$ (Multiplication property)
$a \neq 0$, otherwise we have no parallelogram
$b = 0$ (Property of disjunction)
A(a,c), T(0,c) (Postulate 2-1)
MEAT is a rectangle (Definition 7-5)
$c/-a \cdot c/a = -1$ (Given, Theorem 10-3.2), or
$c^2 = a^2$ (Multiplication property)
MT = ME, and MEAT is a rhombus (Definition 7-6)
MEAT is a square (Definition 7-7).

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Exercises continued

38. Draw $\triangle POT$ with P(0, 0), O($2a$, 0), T($2b$, $2c$).
If R, A, and G are the respective midpoints of $\overline{PO}$, $\overline{OT}$, and $\overline{TP}$, then R(a, 0), A($b+a, c$), and G(b, c) (Theorem 10-2.2)
GP = $\sqrt{b^2 + c^2}$, (Theorem 10-2.1)
PR = a, RG = $\sqrt{(b-a)^2 + c^2}$, (Theorem 10-2.1)
OA = $\sqrt{(b-a)^2 + c^2}$, (Theorem 10-2.1)
OR = a, AR = $\sqrt{b^2 + c^2}$ (Theorem 10-2.1)
$\triangle GPR \cong \triangle ARO$ (SSS)
Use a similar procedure to prove the remaining triangles congruent.

39. Draw rhombus APEX such that A(0, 0), P(2a,0), E($2a + 2b$, $2c$), X($2b$, $2c$).
If L, I, N, and T are midpoints of the respective sides, L(a, 0), I($2a + b, c$), N($a + 2b$, $2c$), and T(b, c) (Theorem 10-2.2)
Slope $\overline{LI}$ = $c/(a + b)$ (Definition 10-5)
Slope $\overline{NI}$ = $c/(b - a)$ (Definition 10-5)
Slope $\overline{NT}$ - $c/(a + b)$ (Definition 10-5)
$\overline{LI} \parallel \overline{NT}$ (Theorem 10-3.1)
$a^2 = b^2 + c^2$ (Theorem 7-4.1)
$c/(b - a) \cdot c/(a + b) = c^2/(b^2 - a^2) = c^2 / -c^2 = -1$ (Postulate 2-1)
$\overline{LI} \perp \overline{NI}$ (Theorem 10-3.2)
$m\angle$ LIN = 90 (Theorem 2-6.5)
LINT is a rectangle (Definition 7-5).

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Review Exercises

1. (0, 0) 2. (-4.5, 0) 3. (5.5, -5.5) 4. (0, 4.5)
5. B 6. G 7. E 8. H
9. $\sqrt{36x^2 + 16y^2}$ 10. $\sqrt{\frac{13}{3}}$
11. $\sqrt{31.04}$ 12. $5 + \sqrt{85} + \sqrt{170}$

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13. A(5,2) B(8, -1) C(10,-3)
Slope of $\overline{AB} = \frac{-1-2}{8-5} = -1$
Slope of $\overline{AC} = \frac{-3-2}{10-5} = -1$
Therefore A, B, C, are collinear (Postulate 6-1)

14. A(0, -2) (B)6, -2) C(3, 6)
AB = 6
AC = $\sqrt{(3-0)^3 + (6+2)^2} = \sqrt{73}$
BC = $\sqrt{(3-6)^2 + (6+2)^2} = \sqrt{73}$
AC = BC
Therefore $\triangle ABC$ is isosceles

15. M(4, $\frac{5}{4}$) is the midpoint of $\overline{BC}$
AM = $\sqrt{(4-0)^2 + (\frac{5}{2} - 4)^2}$
AM = $\sqrt{16 + 9/4}$
AM = $\sqrt{73/4} = \frac{1}{2}\sqrt{73}$

16. A(-5, 13), C(h,k) B(3,7)
Midpoint of $\overline{AB}$ is C(-1,10)
r = AC = CB = 5 (Theorem 10-2.1)

17. -3 18. $-\frac{5}{8}$ 19. $\frac{8}{7}$
20. 2 21. $\frac{1}{2}$ 22. $\frac{1}{3}$

23. A(-2, 3), B(-3, -2), C(9, 6)
Slope of $\overline{AB} = \frac{-5}{-1} = 5$
Slope of $\overline{AC} = \frac{3}{11}$

continued next page

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23. *continued*

Therefore A, B, C, are *not* collinear since slopes are not equal.

24. This is not a right triangle since *no* two sides have slopes which are negative reciprocals of each other.

25. $6x + 5y - 7 = 0$
$y = -\frac{6}{5}x + \frac{7}{5}$

Slope of given line is -6/5
Therefore slope of the perpendicular is 5/6.

26. $y + 8 = (2/3)(x - 2)$ 27. $y - 9 = (3/10)(x - 3)$
28. $y = -(2/5)x - \frac{1}{2}$ 29. $(x/7) + (y/-2) = 1$
30. $y = 2x - 6$ 31. $y = x - \frac{1}{2}$
32. $m = 2/3$, $a = 3$, $b = -2$ 33. $m = 5/3$, $a = 3$, $b = -5$
34. $m = 7/12$, $a = 16/7$, $b = -(4/3)$ 35. $m = -10$, $a = 2/5$, $b = 4$
36. $m = 7/5$, $a = 1/7$, $b = -1/5$ 37. $m = -2/3$, $a = 7/2$, $b = 7/3$
38. $\overleftrightarrow{AC}$: $2y = -7x + 21$;
$\overleftrightarrow{BD}$: $x + 3y = 14$.

39. A(2, 3), B(-2,5)
Slope of $\overline{AB} = \frac{5-3}{-2-2} = -\frac{1}{2}$
The midpoint of $\overline{AB}$ is (0, 4)
Therefore the equation of the perpendicular bisector is $y - 4 = \frac{1}{2}(x-0)$, or
$y = \frac{1}{2}x + 4$

40. $x^2 + y^2 = r^2$
$x^2 + y^2 = \left(\frac{7}{2}\right)^2$
$x^2 + y^2 = \frac{49}{4} = 12.25$

Exercises 41-43 are done in a way similar to Example 40.
41. $x^2 + y^2 = 12$. 42. $x^2 + y^2 = \frac{1}{4}$ 43. $x^2 + y^2 = 1$
44. (7, 4); 2
$(x - 7)^2 + (y - 4)^2 = 4$

Exercises 45-47 are done in a way similar to Example 44.
45. $x^2 + (y + 2)^2 = 9$ 46. $(x + 2)^2 + (y + 3)^2 = 25$
$x^2 + y^2 = 1/16$

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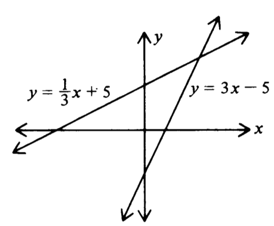
Chapter Test

1.



2. Find the midpoint of the given segment and then find the midpoints of the two remaining segments. These points are (-6,0), (0,0), (6,0).

3. *See solution of Exercise 35 on page 460.*

4. $y = 2x - 1$

5. $(x-3)^2 + (y + 2)^2 = 58$

6. $y = x - 1$

7. All of the slopes are $\frac{-2}{3}$; (1,1) is between (3,$-\frac{1}{3}$) and (-2, 3), since $\frac{2}{3}\sqrt{13} + \sqrt{13} = \frac{5}{3}\sqrt{13}$.

8. $\frac{\sqrt{85}}{2} + \sqrt{17} + \frac{\sqrt{65}}{2}$

9. *See solution of Exercise 48 on page 462.*

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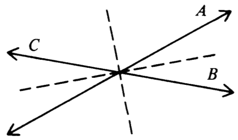
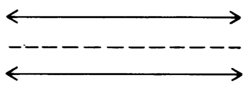
Class Exercises

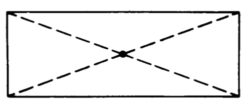
1. The locus is two intersecting lines that bisect the angles formed by $\overleftrightarrow{BC}$ and $\overleftrightarrow{AD}$.



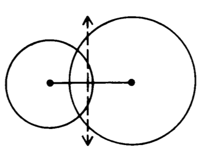
2. The locus is a line parallel to both lines and halfway between them.



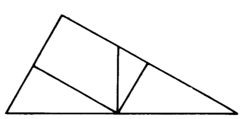
3. The locus is the point of intersection of the diagonals.



4. The locus is the perpendicular bisector of the line of centers.



5. The locus is the point of intersection of the perpendicular bisectors of the sides.



6. The locus is a circle with center (-1, -3) and radius 7. $(x+1)^2 + (y+3)^2 = 49$.

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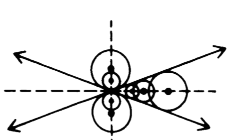
Exercises

1. *See Class Exercise 2.*
2. *See Class Exercise 1.* If $\overleftrightarrow{AD} \perp \overleftrightarrow{CB}$, the lines forming the locus will be perpendicular also.
3. Locus = $\{(x,y): x^2 + y^2 < 49\}$.
4. Locus = $\{(x,y): x^2 + y^2 > 4\}$.
5. Locus = $\{(x,y): 4 < x^2 + y^2 < 49\}$.
6. The locus is any point in a line parallel to and 3 units away from $\overline{AB}$.
7. The locus is the line perpendicular to $\overline{AB}$ at C.
8. The locus is a circle whose equation is $x^2 + y^2 = 9$.
9. The locus is the interior of a circle whose equation is $x^2 + y^2 = 9$.
10.



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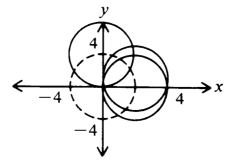
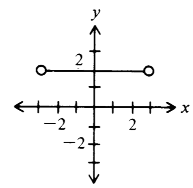
11. No locus exists.
12. The midpoint of $\overline{AB}$ is the locus.
13. The dashed circle is the required locus.



14. The locus is a pair of lines whose intersection is the given point.

15.



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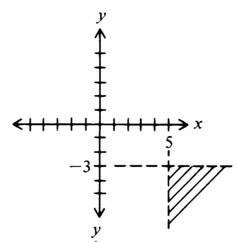

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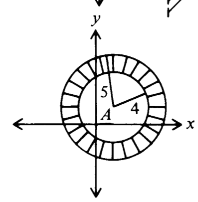

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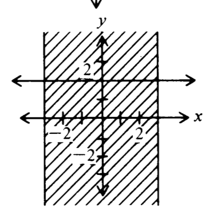

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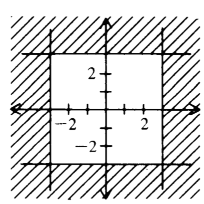

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20. We must find the perpendicular bisector of the given segment. First find the midpoint, (11/2, 5) then the negative reciprocal of the slope of the given segment, -7/2. The equation of this perpendicular bisector is $14x + 4y = 97$.

21. $(x-7)^2 + (y+2)^2 = 5$. *See solution of Exercise 17 on page 459.*

22. $(x-5)^2 + (y+2)^2 = 13$. *See solution of Exercise 19 on page 460.*

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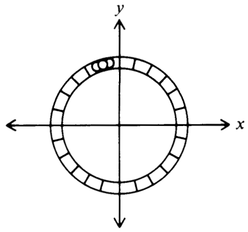
23.



This curve is called a hypercycloid of four cusps, other hypercycloids are possible. Although further investigation of this curve is beyond the scope of this course, careful drawing should produce this interesting locus.

24. Find the intersection of the perpendicular bisector of $\overline{PR}$ and $\overleftrightarrow{PQ}$.

25. Find the intersection of the angle bisector of $\angle$ R and $\overleftrightarrow{PQ}$.

26. The locus of points at a distance of more than 13 units from the origin is $x^2 + y^2 > 169$. The point (-5,12) *is* 13 units from the origin. Therefore the point (-5,12) is not in the locus.

27. *See Section 11-3, Construction 1.*

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23. The medians always intersect in a common point inside the triangle.

24. The medians always intersect in a common point inside the triangle.

25. The medians always intersect in a common point inside the triangle.

26. The altitudes always intersect in a common point not necessarily in the interior of the triangle.

27. Construct a right triangle with sides 1, 1, $\sqrt{2}$.

28. Construct a right triangle with sides 1, $\sqrt{3}$, 2.

29. The third angle of the triangle in Exercise 28 may be used.

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3. They are corresponding angles.
4. Corollary 6-2.1a

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15. Add a 45° angle to a 90° angle.

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22. The angle bisectors intersect in a common point.

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22. The circle is inscribed in the triangle in both cases.

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Exercises

2. *See Construction 18 on page 491.*

3. *See Construction 19 on page 491.*

4. To prove this corollary simply have students prove that since the perpendicular bisectors of the sides of the triangle determined by the three given noncollinear points are unique, and by Theorem 11-7.1 are concurrent at a unique point, the circumcircle of a triangle (with center at this point of concurrence) is also unique.

5. Assume that two nonconcentric distinct circles intersect in more than two points, say three points. Then these three points would determine two distinct circles. This is *impossible* by Corollary 11-7.1a. Therefore reject the hypothesis and reach the desired conclusion.

6. 4   7. 8   8. 15/2   9. 7/2

11. (5, 4)   12. (0,0)   13. $(9-\sqrt{41}, 9-\sqrt{41})$ or approximately (2.6, 2.6)

14. $\left(\frac{10}{3}, \frac{8}{3}\right)$

16. 3

17. It is the length of the perpendicular from X to $\overline{BC}$.

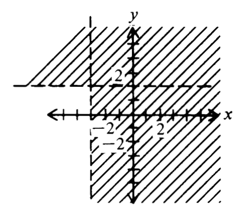
Review Exercises

1. $y = 3$ and $y = -3$.

2. A line parallel to the two given lines and midway between them.

3. The locus is all points in the open disc with radius three; that is, $x^2 + y^2 < 9$.

4.



5. Locus {(6, 6), (6, -6), (-6, -6), (-6, 6)}.

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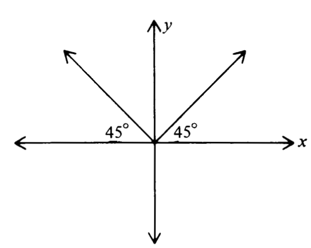
45. $\frac{20}{3}$ 46. 9   47. 3

Chapter Test

1.



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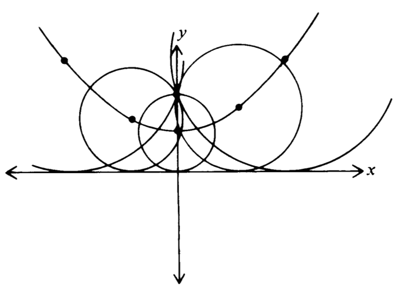
Chapter Test continued

2.



A Parabola.

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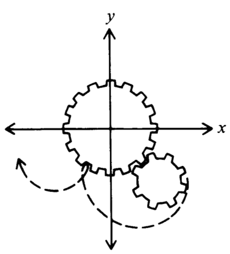
3.



4. $y = |x|$
5. $y = x$
6. $(x-a)^2 + (y-b)^2 = 9$

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Exercises

7. $(a + b\sqrt{c})^3 - 3(a + b\sqrt{c}) + 1 = 0$ (Given)
   $a^3 + 3ab^2c - 3a + 1 = 0$, and
   $3a^2b + b^3c - 3b = 0$ (Given)
   $(a - b\sqrt{c})^3 - 3(a - b\sqrt{c}) + 1 = a^3 + 3ab^2c - 3a + 1 -$
   $(3a^2b\sqrt{c} + b^3c\sqrt{c} - 3b\sqrt{c}) = 0 - c(3a^2b + b^3c - 3b) = 0$
   $-\sqrt{c} \cdot 0 = 0$ (Postulate 2-1).

8. $(x - r_1)(x - r_2)(x - r_3) = x^3 - r_1x^2 - r_2x^2 - r_3x^2 + r_1r_2x + r_1r_3x + r_2r_3x - r_1r_2r_3 = x^3 + ax^2 \; bx + c$
   (Transitive property)
   $a = -(r_1 + r_2 + r_3)$, or $-a = r_1 + r_2 + r_3$
   (Postulate 2-1).

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Exercises

1. True. 2. True. 3. False.
4. False. PQ • QR or PQ • PS or QR • RS or PS • RS
5. True
6. False; it also includes the triangle.
7. False; a pentagon and its interior is a polygonal region.
8. The area of a square is $b \cdot h$; but $b = h$, so $\mathcal{A} = b^2$ or s^2, where $s = b$ = length of a side.
9. *See Example 1 on page 505.*
10. Student drawing (answer may vary).
11. Student drawing (answer may vary).
12. Student drawing (answer may vary).

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13. (R_2); (R_3); (R_6).
14. (R_4); (R_7).
15. (R_3); (R_6).
16. (R_3); (R_4); (R_5).
17. No, add $\mathcal{A}(R_4)$.
18. No, add $\mathcal{A}(R_6)$.
19. Yes.
20. Because △ABD ≅ △ACD.
21. 9
22. 56 23. $25\frac{1}{60}$ 24. x^2
25. 9 26. $12\frac{1}{4}$ 27. $\frac{1}{4}x^2y^2$
28. $4x^4$ 29. 72 30. $26\frac{9}{32}$
31. $\frac{1}{32}x^2y^2$ 32 $8x^4$ 33. six times greater.
34. unchanged 35. nine times greater.
36. Sixteen times greater.
37. $\overline{AD} \cong \overline{BC}$ (Theorem 7-1.2)
    $\overline{AM} \cong \overline{BM}$
    m∠ A = 90 = m∠ B (Theorem 7-3.1)
    △AMD ≅ △BMC (SAS)
    $\mathcal{A}$△AMD = $\mathcal{A}$△BMC (Postulate 12-2)
38. Draw $\overline{ME} \perp \overline{DC}$ such that $\overline{DEC}$.
    △MAD ≅ △DEM (HL)
    △MBC ≅ △CEM (HL)
    $\mathcal{A}$△MAD = $\mathcal{A}$△DEM (Postulate 12-2)
    $\mathcal{A}$△MBC = $\mathcal{A}$△CEM (Postulate 12-2)
    $\mathcal{A}$△MAD + $\mathcal{A}$△MBC = $\mathcal{A}$△DEM + $\mathcal{A}$△CEM (Postulate 12-3)
    $\mathcal{A}$△DMC = ($\frac{1}{2}$) $\mathcal{A}$ rectangle ABCD (Postulate 2-1)
39. $\overline{BC} \cong \overline{ED}$ (Addition property)
    $\overline{GD} \cong \overline{GC}$ (Theorem 3-4.2)
    ∠ GDC ≅ ∠ GCD (Theorem 3-4.2)
    △ABC ≅ △FED (ASA)
    $\mathcal{A}$△ABC = $\mathcal{A}$△FED (Postulate 12-2)
    $\mathcal{A}$ quadrilateral AGDB = $\mathcal{A}$ quadrilateral GFEC
    (Postulate 12-3).

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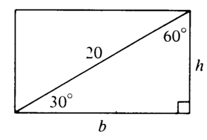
40. Let $h = 4$
    $\mathcal{A} = bh$, $P = 2h + 2b$
    $30 = 8 + 2b$
    $b = 11$
    Therefore $\mathcal{A} = 44$

Exercises 41 and 42 are done in a way similar to Example 40.

41. 50 42. 36
43. $\mathcal{A} = bh$
    $48 = 16h$, and $h = 3$.
44. $\mathcal{A} = bh$
    $120 = 15h$
    $h = 8$
    The length of the diagonal (17) is found using Theorem 8-8.1.
45.



$h = 10$, $b = h\sqrt{3} = 10\sqrt{3}$ (Corollary 8-9.3b)
$\mathcal{A} = bh = 100\sqrt{3}$

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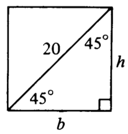
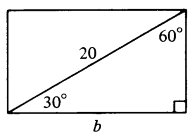
Exercises continued

46.



$b = h$ (the rectangle is a square).
$A = \frac{1}{2}(20)^2 = 200$
or
$20 = h\sqrt{2}$ (Theorem 8-9.1)
$h = 10\sqrt{2} = b$
$A = bh = 200$

47.



$h = 10$, $b = 10\sqrt{3}$ (Corollary 8-9.3b)
$A = bh$
$A = 100\sqrt{3}$

48. $y^2 + 8 = (y + 4)(y - 2)$
$y^2 + 8 = y^2 + 2y - 8$
$y = 8$
The altitude is $y + 4 = 12$
The base is $y - 2 = 6$

49. $\angle F \cong \angle BEC$ (Corollary 6-3.1a)
$\angle ADF \cong \angle C$ (Corollary 6-3.1a)
$\overline{BE} \cong \overline{AF}$ (Theorem 7-1.2)
$\triangle BCE \cong \triangle ADF$ (AAS)
$A\triangle BCE = A\triangle ADF$ (Postulate 12-2)
$A\triangle BCE + A$ quadrilateral ABED = $A\triangle ADF + A$ quadrilateral ABED (Postulate 12-3)
A rectangle ABCD = A parallelogram ABEF (Postulate 2-1).

50. By drawing $\overline{MK}$ and $\overline{NL}$, students may prove the eight triangles congruent.
Apply Theorem 12-1.1 and Corollary 12-1.1a,
A quadrilateral MNKL = $\frac{1}{2}(MK)^2 = \frac{1}{2}(AD)^2 = \frac{1}{2} A$ square ABCD.

Class Exercises

1. bh
2. They are congruent and equal in area.
3. $A\triangle ABC = \frac{1}{2} A$ ABCD.
4. $(b)\ (h)$
5. Right triangles.
6. $(\frac{1}{2})$; (p)
7. $(\frac{1}{2})$; (q)
8. ADC; BDC (Postulate 12-3)
9. $(\frac{1}{2})$; (p); $(\frac{1}{2})$; (q)
10. $(p + q)$; (b).

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11. BDC; ADC
12. $(\frac{1}{2})$; (q); $(\frac{1}{2})\ (p)$
13. $(q - p)$; (b).

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Exercises

1. $A = \frac{1}{2}bh$
$A = \frac{1}{2}(3)(8) = 12$

2. $\frac{153}{24}$

3. $\frac{3x(x + 2)}{2}$

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4. $k = \frac{1}{2}bh$
$10 = \frac{1}{2}(2)h$
$h = 10$

5. $\frac{2}{3}$

6. $\frac{x - 5}{x}$

7. $A = \frac{1}{2}\, l_1 \cdot l_2$
$A = \frac{1}{2}(3)(4) = 6$ (The legs are the two shorter lengths)

8. 330

9. $\frac{xy}{2}$

10. $A\triangle ABC = \frac{1}{2}(5\sqrt{3})(6)(\sin 60)$ (Theorem 12-2.5)
$= 15\sqrt{3}\left(\frac{\sqrt{3}}{2}\right) = 45/2$

11. $A = \frac{1}{2}(8)(16) = 64$ (Theorem 12-2.1)

12. $A = \frac{s^2\sqrt{3}}{4}$ and $s = 3$
$A = \frac{9\sqrt{3}}{4}$

Exercises 13-15 are done in a way similar to Example 12

13. $27\sqrt{3}$

14. $96\sqrt{3}$

15. $36\,x^2\sqrt{3}$

16. $A = \frac{h^2\sqrt{3}}{3}$, and $h = 2$
$A = \frac{4\sqrt{3}}{3}$

Exercises 17-19 are done in a way similar to Example 16.

17. $81\sqrt{3}$

18. $72\sqrt{3}$

19. $48x^2\sqrt{3}$

20. Since DE = BE, and both $\triangle AED$ and $\triangle AEB$ share the same altitude from A to $\overline{BD}$ (Corollary 12-2.2a)
$A\triangle AED = A\triangle AEB$ (Corollary 12-2.2a)
Similarly, $A\triangle AEB = A\triangle CEB$, and $A\triangle CEB = A\triangle DEC$.

21. $A\triangle ACB = (\frac{1}{2})(AC \cdot BC)$ (Theorem 12-2.1)
$A\triangle ACB = (\frac{1}{2})(CD \cdot AB)$ (Theorem 12-2.2)
$(\frac{1}{2})(AC \cdot BC) = (\frac{1}{2})(CD \cdot AB)$ (Transitive property)
$AC \cdot BC = CD \cdot AB$ (Multiplication property).

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22. An altitude from P to $\overline{QR}$ is an altitude of $\triangle PQR$, $\triangle MPN$ and $\triangle MPQ$.

$\frac{A\triangle MPN}{A\triangle PQR} = \frac{MN}{QR}$ (Theorem 12-2.4)	$\frac{A\triangle MPQ}{A\triangle PQR} = \frac{QM}{QR}$
$\frac{A\triangle MPN}{24} = \frac{2}{6}$	$\frac{A\triangle MPQ}{24} = \frac{3}{6}$
$A\triangle MPN = 8$	$A\triangle MPQ = 12$

23. *See Class Exercises 1-4 on Page 507.*
24. *See Class Exercises 5-13 on Pages 507-8.*
25. Use either Postulate 2-1 and Theorem 12-2.2, or Corollary 12-2.2a.
26. Refer to the proof of Theorem 12-2.3 on page 510.

27. Refer to the figure for Example 4.
If DE = f, DF = e, and EF = d, then $\sin \angle F = DN/e$ (Definition 8-9), or
DN = $e(\sin \angle F)$ (Multiplication property);
$A\triangle DEF = (\frac{1}{2})(EF \cdot DN) = (\frac{1}{2})d\,[e(\sin \angle F)] = (\frac{1}{2})de\,(\sin \angle F)$ (Theorem 12-2.2, Postulate 2-1, Closure).

28. Let $\overline{AD}$ be an altitude of equilateral $\triangle ABC$ where AC = AB = BC = s.
$AD = (s\sqrt{3})/2$ (Theorem 8-9.3)
$A\triangle ABC = (\frac{1}{2})(BC \cdot AD) = \frac{1}{2} \cdot s \cdot (s\sqrt{3})/2 = (s^2\sqrt{3})/4$ (Theorem 12-2.2, Postulate 2-1, Closure).

29. Let $\overline{AD}$ be an altitude of equilateral $\triangle ABC$ such that AD = h.
$AB = (2h\sqrt{3})/3$ (Corollary 8-9.3a)
$BC = (2h\sqrt{3})/3$ (Transitive property)
$A\triangle ABC = (\frac{1}{2})(BC \cdot AD) = (\frac{1}{2})([2h\sqrt{3})/3] \cdot h = h^2\sqrt{3})/3$ (Theorem 12-2.2, Postulate 2-1, Closure).

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Exercises continued

30. $\overline{BC}$ is base of $\triangle DBC$ and $\triangle ABC$
$\frac{\mathcal{A}\triangle DBC}{\mathcal{A}\triangle ABC} = \frac{DF}{AE} = \frac{3}{8}$ (Theorem 12-2.3)

31. $\frac{\mathcal{A}\triangle DBC}{\mathcal{A}\triangle ABC} = \frac{3}{8}$ from Exercise 30.
Therefore $\mathcal{A}\triangle DBC$ is $\frac{3}{8}\mathcal{A}\triangle ABC$.
Thus the area of the shaded region is $\frac{5}{8}\mathcal{A}\triangle ABC$.

32. $m\angle B = m\angle BAM$ (Theorem 3-4.2)
$\overline{AM} \cong \overline{CM}$ (Transitive property)
$m\angle C = m\angle CAM$ (Theorem 3-4.2)
$m\angle B + m\angle BAM + m\angle C + m\angle CAM = 180$ (Theorem 6-4.2)
$m\angle BAM + m\angle CAM = 90$ (Subtraction property)
$\mathcal{A}\triangle ABC = (\frac{1}{2})(AB \cdot AC)$ (Theorem 12-2.1).

33. $\mathcal{A}\triangle ADE = \frac{1}{2}(2)(8) \sin \angle A = 8 \sin \angle A$
$\mathcal{A}\triangle ABC = \frac{1}{2}(10)(12) \sin \angle A = 60 \sin \angle A$

$\frac{\mathcal{A}\triangle ADE}{\mathcal{A}\triangle ABC} = \frac{8 \sin \angle A}{60 \sin \angle A} = \frac{8}{60} = \frac{2}{15}$

34. $\mathcal{A}\triangle FIJ = \frac{1}{2}(5)(4) \sin \angle F = 10 \sin \angle F$
$\mathcal{A}\triangle FGH = \frac{1}{2}(7)(10) \sin \angle F = 35 \sin \angle F$
$\frac{\mathcal{A}\triangle FIJ}{\mathcal{A}\triangle FGH} = \frac{10}{35} = \frac{2}{7}$

35. $\mathcal{A}\triangle KMN = \frac{1}{2}(9)(5) \sin \angle K = \frac{45}{2} \sin \angle K$

$\mathcal{A}\triangle KLP = \frac{1}{2}(11)(9) \sin \angle K = \frac{99}{2} \sin \angle K$

$\frac{\mathcal{A}\triangle KMN}{\mathcal{A}\triangle KLP} = \frac{\frac{45}{2}}{\frac{99}{2}} = \frac{45}{99} = \frac{5}{11}$

36. Draw altitude $\overline{AE}$, $BE = EC = 5$
$(AB)^2 = (AE)^2 + (BE)^2$ (Theorem 8-8.1)
$(13)^2 = (AE)^2 + (5)^2$
$AE = 12$
$\mathcal{A}\triangle ABC = \frac{1}{2}(BC)(AE) = \frac{1}{2}(10)(12) = 60$
$\mathcal{A}\triangle ABC = \frac{1}{2}(AC)(BD)$
$60 = \frac{1}{2}(13)(BD)$
$BD = \frac{120}{13}$

37. Draw $\overline{DE} \perp \overleftrightarrow{AC}$ at E, and $\overline{BF} \perp \overleftrightarrow{AC}$ at F.
$\triangle AED \cong \triangle CFB$ (AAS)
$\overline{DE} \cong \overline{BF}$ (Definition 3-3)
$\mathcal{A}\triangle ADP = \mathcal{A}\triangle ABP$ (Corollary 12-2.2a).

38. Draw quadrilateral ABCD, where $\overline{AC}$ and $\overline{BD}$ meet at E.
$\mathcal{A}\triangle AED + \mathcal{A}\triangle DEC = \mathcal{A}\triangle BEC + \mathcal{A}\triangle DEC$ (Postulate 12-3)
$\mathcal{A}\triangle ADC = \mathcal{A}\triangle BDC$ =(Subtraction property)
Draw $\overline{AF} \perp \overleftrightarrow{DC}$ and $\overline{BH} \perp \overline{DC}$.
$\overline{AF} \cong \overline{BH}$ (Theorem 12-2.2)
$\overline{AF} \parallel \overline{BH}$ (Theorem 6-1.1)
Quadrilateral AFHB is a parallelogram (Theorem 7-2.2)
$\overline{AB} \parallel \overline{DC}$ (Definition 7-1)
Similarly, $\overline{AD} \parallel \overline{BH}$
Quadrilateral ABCD is a parallelogram (Definition 7-1)

39. An altitude from A to $\overline{BE}$ is an altitude of $\triangle AFB$ and $\triangle ABE$.
$\frac{\mathcal{A}\triangle ABF}{\mathcal{A}\triangle ABE} = \frac{BF}{BE} = \frac{3}{9} = \frac{1}{3}$ (Theorem 12-2.4)

Therefore $\mathcal{A}\triangle ABF = \frac{1}{3}\mathcal{A}\triangle ABF$ *(I)*

An altitude from B to $\overline{AC}$ is an altitude of $\triangle ABE$ and $\triangle ABC$.

$\frac{\mathcal{A}\triangle ABE}{\mathcal{A}\triangle ABC} = \frac{AE}{AC} = \frac{4}{10} = \frac{2}{5}$

Therefore $\mathcal{A}\triangle ABE = 2/5\ \mathcal{A}\triangle ABC$ *(II)*
Substitute *(II)* into *(I)*:

$\mathcal{A}\triangle ABF = \frac{1}{3}\ (2/5\ \mathcal{A}\triangle ABC)$
$\mathcal{A}\triangle ABF = \frac{2}{15}\ \mathcal{A}\triangle ABC.$

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40. An altitude from A to $\overline{BC}$ is an altitude of $\triangle ABD$ and $\triangle ABC$.
$\frac{\mathcal{A}\triangle ABD}{\mathcal{A}\triangle ABC} = \frac{BD}{BC} = \frac{1}{6}$ (Theorem 12-2.4)

Therefore $\mathcal{A}\triangle ABD = \frac{1}{6}\mathcal{A}\triangle ABC$ *(I)*

Similarly, $\frac{\mathcal{A}\triangle ABE}{\mathcal{A}\triangle ABC} = \frac{AE}{AC} = \frac{4}{10} = \frac{2}{5}$

Therefore $\mathcal{A}\triangle ABE = \frac{2}{5}\mathcal{A}\triangle ABC$. *(II)*

Similarly, $\frac{\mathcal{A}\triangle AFE}{\mathcal{A}\triangle ABE} = \frac{6}{9} = \frac{2}{3}$.

Therefore $\mathcal{A}\triangle AFE = \frac{2}{3}\mathcal{A}\triangle ABE$ ***(III)***
Substitute *(II)* into *(III)*:
$\mathcal{A}\triangle AFE = \frac{2}{3}\left(\frac{2}{5}\mathcal{A}\triangle ABC\right) = \frac{4}{15}\mathcal{A}\triangle ABC$ *(IV)*
$\mathcal{A}ECDF = \mathcal{A}\triangle ABC - (\mathcal{A}\triangle ABD + \mathcal{A}\triangle AFE)$
$\mathcal{A}ECDF = \mathcal{A}\triangle ABC - (\frac{1}{6}\mathcal{A}\triangle ABC + \frac{4}{15}\mathcal{A}\triangle AFE) = \frac{17}{30}\mathcal{A}\triangle ABC$

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41. An altitude from Q to $\overline{PR}$ is an altitude for $\triangle QNR$ and $\triangle PQR$.
$\frac{\mathcal{A}\triangle QNR}{\mathcal{A}\triangle PQR} = \frac{3}{8}$ (Theorem 12-2.4)

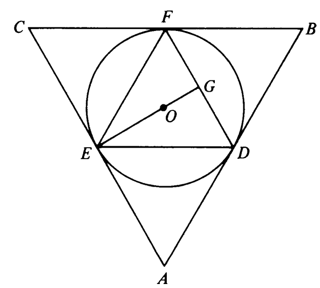
Therefore $\mathcal{A}\triangle QNR = \frac{3}{8}\mathcal{A}\triangle PQR$ *(I)*
Similarly, $\frac{\mathcal{A}\triangle QKR}{\mathcal{A}\triangle QNR} = \frac{8}{12} = \frac{2}{3}$

Therefore $\mathcal{A}\triangle QKR = \frac{2}{3}\mathcal{A}\triangle QNR$ *(II)*
Substitute *(I)* into *(II)*:
$\mathcal{A}\triangle QKR = \frac{2}{3}\left(\frac{3}{8}\mathcal{A}\triangle PQR\right) = \frac{1}{4}\mathcal{A}\triangle PQR.$

42. *This exercise is done in the same way as Exercise 40.*

25/56

43.



Equilateral $\triangle ABC$ is circumscribed about circle O, which is circumscribed about equilateral $\triangle DEF$. The difference of the areas of $\triangle ABC$ and $\triangle DEF$ is $\mathcal{A}\triangle ADE + \mathcal{A}\triangle BDF + \mathcal{A}\triangle CEF$. Since these are congruent triangles and each is congruent to $\triangle DEF$, we need simply find OE, where $\mathcal{A}\triangle DEF = 25/3$. Since $\mathcal{A}\triangle DEF = \frac{EG^2\sqrt{3}}{3} = 25/3$,

$EG = \sqrt{\frac{25\sqrt{3}}{3}}$. But, $OE = 2EG/3$.

Therefore $OE = \frac{2}{3}\sqrt{\frac{25\sqrt{3}}{3}} = \frac{10}{3}\sqrt{\frac{\sqrt{3}.3}{3\ 3}} = \frac{10}{9}\sqrt{3\sqrt{3}}$

44. Draw $\triangle ABC$ with medians $\overline{AE}$, $\overline{CD}$, and $\overline{BF}$ meeting at G.
$GE = (\frac{1}{3})(AE)$ (Theorem 11-7.3)
$\mathcal{A}\triangle BGE = (\frac{1}{3})\ \mathcal{A}\triangle ABE$ (Theorem 11-7.3)
$\mathcal{A}\triangle ABE = (\frac{1}{2})\ \mathcal{A}\triangle ABC$ (*See Example 2*)
$\mathcal{A}\triangle BGE = (\frac{1}{3})(\frac{1}{2})\ \mathcal{A}\triangle ABC$ (Postulate 2-1)
$\mathcal{A}\triangle BGE = (1/6)\ \mathcal{A}\triangle ABC$ (Closure)
$\mathcal{A}\triangle BGE = \mathcal{A}\triangle EGC$ (*See Example 2*).
Repeat this procedure to show that the remaining triangles are equal in area.

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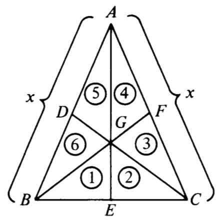
Exercises continued

45.



Draw median $\overline{AE}$
From Exercise 44: $\mathcal{A}\triangle 1 = \mathcal{A}\triangle 2 = \mathcal{A}\triangle 3 = \mathcal{A}\triangle 4 = \mathcal{A}\triangle 5 = \mathcal{A}\triangle 6$
$\mathcal{A}\triangle BGC = 48$
$\mathcal{A}\triangle 1 + \mathcal{A}\triangle 2 = 48$
$\mathcal{A}\triangle 1 = \mathcal{A}\triangle 2 = 24$
$\mathcal{A}\triangle ABC = 6(24) = 144$

$\mathcal{A}\triangle ABC = \frac{1}{2}(AB)(AC)\sin \angle A$ (Theorem 12-2.5)
$144 = \frac{1}{2}(x)\ (x)\ \sin 30$
$144 = \frac{1}{2}x^2(\frac{1}{2})$
$x = 24 = BA = CA$

46. Construct $\odot P \cong \odot Q$ with diameter d.
Inscribe $\triangle ABC$ in $\odot P$ and $\triangle DEF$ in $\odot Q$.
Let $\overline{AG}$ and $\overline{DH}$ be the respective altitudes of $\triangle ABC$ and $\triangle DEF$.
$AB \cdot AC = AG \cdot d$ and $DE \cdot DF = DH \cdot d$ (Ex.43, Section 9-6)
$\mathcal{A}\triangle ABC = (\frac{1}{2})(AG \cdot BC)$ (Theorem 12-2.2)
$AG = (2\mathcal{A}\triangle ABC)/BC$ (Multiplication property)
$AB \cdot AC = [(2\mathcal{A}\triangle ABC)/BC]d$ (Postulate 2-1)
Similarly, $DE \cdot DF = [(2\mathcal{A}\triangle DEF)/EF]d$
$\mathcal{A}\triangle ABC/\mathcal{A}\triangle DEF = (AB \cdot AC \cdot BC)/(DE \cdot DF \cdot EF)$
(Transitive property, Division property).

47. $AC/CE = \mathcal{A}\triangle AHC/\mathcal{A}\triangle EHC$ (Theorem 12-2.4)
$AH/HF = \mathcal{A}\triangle AHC/\mathcal{A}\triangle FCH$ (Theorem 12-2.4)
$\mathcal{A}\triangle EHC = \mathcal{A}\triangle FCH$ (Theorem 12-2.4)
$\mathcal{A}\triangle EHC = \mathcal{A}\triangle FCH$ (Corollary 12-2.2b)
$AC/CE = AH/HF$ (Transitive property).

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Class Exercises

1. $\mathcal{A} = \frac{1}{2}(7)(7) = 49/2.$
2. $\mathcal{A} = \frac{1}{2}(3)(8 + 14) = 33$
3. $\mathcal{A} = \frac{1}{2}\ (4)\ (10) = 20.$

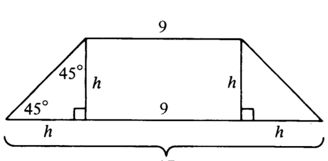
Exercises

1. $\mathcal{A} = \frac{1}{2}(3)(8) = 12$ 2. 75   3. $10x^2$
4. $20 = 4h; h = 5$ 5. $\frac{1}{4}$ 6. $3x$
7. $\mathcal{A} = \frac{1}{2}(12)(16) = 96$ 8. 147/16   9. $49x^2$
10. $\mathcal{A} = \frac{1}{2}(3)(5 + 7) = 18$
11. 76
12. Since the diagonal of a parallelogram partitions it into two congruent triangles, find the area of one of these triangles and then double this result.

$\mathcal{A}\triangle = \frac{1}{2}ab\ \text{Sin}\ \angle C$ (Theorem 12-2.5)
$\mathcal{A}\triangle = \frac{1}{2}(8)(12)\ \text{Sin}\ 30 = 24$
$2\mathcal{A}\triangle = 48$ = Area of the parallelogram.

13. $48\sqrt{2}$ 14. $48\sqrt{3}$ 15. 96

16.



$h + 9 + h = 17$
$h = 4$
$\mathcal{A} = \frac{1}{2}(4)(9 + 17) = 52$

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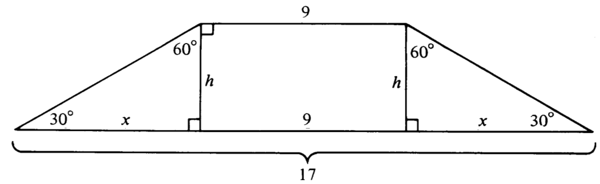
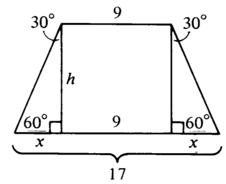
17.



$x + 9 + x = 17$
$x = 4$
$x = h\sqrt{3},\quad h = \frac{4\sqrt{3}}{3}$
$\mathcal{A} = \frac{1}{2}(\frac{4\sqrt{3}}{3})\ (9 + 17) = \frac{52\sqrt{3}}{3}$

18.

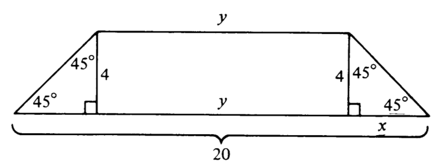


$x + 9 + x = 17$
$x = 4$
$h = x\sqrt{3} = 4\sqrt{3}$
$\mathcal{A} = \frac{1}{2}(4\sqrt{3})(9 + 17) = 52\sqrt{3}$

Exercise 19 is done in the same way as Exercise 16

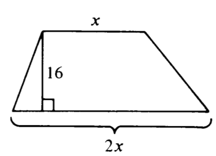
19. 52

20.



$x = 4$
$x + y + x = 20$
$y = 12$
$\mathcal{A} = \frac{1}{4}(4)\ (y + 20)$
$\mathcal{A} = 2(12 + 20) = 64$

21.



$\mathcal{A} = \frac{1}{2}(16)(x + 2x)$
$144 = 8(3x)$
$144 = 24x$
$x = 6$
$2x = 12$

Page 518

22. Let x = length of shorter base.
$x + 1$ = length of longer base.
$\mathcal{A} = \frac{1}{2}h(b_1 + b_2)$
$27 = \frac{1}{2}(3)(x + x + 1)$
$x = 8\frac{1}{2}$, and $x + 1 = 9\frac{1}{2}$

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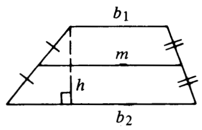
Exercises continued

23.



$\mathcal{A} = \frac{1}{2}h(b_1 + b_2)$ (Theorem 12-3.3)
$\mathcal{A} = h \cdot \frac{1}{2}(b_1 + b_2)$
$m = \frac{1}{2}(b_1 + b_2)$ (Theorem 7-7.2)
Therefore $\mathcal{A} = hm$ (Substitution property)

24. $\mathcal{A} = hm$ from Exercise 23
$20 = 5m$, and $m = 4$

Exercises 25-27 are done in a way similar to Exercise 24.

25. 3.2 26. 5.1 27. $3\frac{5}{31}$

28. Draw parallelogram ABCD with diagonal $\overline{AC}$.
$\mathcal{A}\triangle ADC = (\frac{1}{2})(AD \cdot DC \cdot \sin \angle D)$ (Theorem 12-2.5)
$\triangle ADC \cong \triangle CBA$ (Theorem 7-1.1)
$\mathcal{A}\triangle ADC = \mathcal{A}\triangle CBA$ (Postulate 12-2)
$\mathcal{A}\triangle ADC = (\frac{1}{2})\ \mathcal{A}$ parallelogram ABCD (Postulate 12-3)
$\mathcal{A}$ parallelogram ABCD = $AD \cdot DC \cdot \sin \angle D$ (Postulate 2-1)

29. See the solution for Exercise 12 on page 517.

$\mathcal{A} = 10(15) \sin 30$ (Theorem 12-2.5)
$\mathcal{A} = 150(\frac{1}{2}) = 75$

30. $\mathcal{A} = 100(15) \sin 45$
$\mathcal{A} = 150(\frac{1}{2}\sqrt{2}) = 75\sqrt{2}$

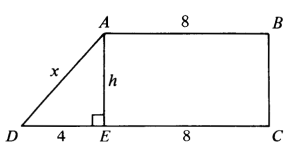
31. $\mathcal{A} = (10)(15) \sin 60$
$\mathcal{A} = 150(\frac{1}{2}\sqrt{3}) = 75\sqrt{3}$

32. $\mathcal{A} = (10)(15) = 150$

33. The diagonals of a square are congruent;
so $\mathcal{A} = \frac{1}{2}\ d \cdot d = \frac{1}{2}d^2$.

34. $\mathcal{A} = \frac{1}{2}(SQ)(RP) = \frac{1}{2}(16)(12) = 96$ (Theorem 12-3.2)
SR = 10 (Theorem 8-8.1)
$\mathcal{A} = (PT)(SR)$
$96 = x(10)$
$x = 9.6$

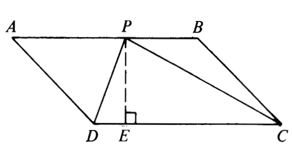
35.



$30 = \frac{1}{2}h(8 + 12)$, and $h = 3$
$x^2 = h^2 + (4)^2$; and $x = 5$

36. KG = 15 (Theorem 8-8.1)
$\mathcal{A}$EFGJ = $\frac{1}{2}(16)(30) = 240$ (Theorem 12-3.2)
$\mathcal{A}$EFGJ = $17x$ (Theorem 12-3.1)
$240 = 17x$
$x = 240/17$

37.



Construct parallelogram ABCD with $\overline{PE} \perp \overline{DC}$ at E.
$\mathcal{A}\triangle DPC = (\frac{1}{2})(PE \cdot DC)$ (Theorem 12-2.2)
$\mathcal{A}$ parallelogram ABCD = $PE \cdot DC$ (Theorem 12-3.1)
$\mathcal{A}$ parallelogram ABCD = $2\mathcal{A}\triangle DPC$ (Transitive property)
The remaining half of $\mathcal{A}$ parallelogram ABCD is $\mathcal{A}\triangle APD$ + $\mathcal{A}\triangle BPC$ (Postulate 12-3)
$\mathcal{A}\triangle APD + \mathcal{A}\triangle BPC = \mathcal{A}\triangle DPC$ (Postulate 2-1).

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38. Construct parallelogram ABCD to meet the given conditions.
$\overline{AE}$ is a median of $\triangle ABD$ (Theorem 7-1.5)
$\mathcal{A}\triangle DEA = (\frac{1}{2})\mathcal{A}\triangle ABD$ (Theorem 12-2.4)
$\triangle ABD \cong \triangle CDB$ (Theorem 7-1.1)
$\mathcal{A}\triangle ABD = (\frac{1}{2})\ \mathcal{A}$ parallelogram ABCD (Postulate 12-3)
$\mathcal{A}\triangle DEA = (\frac{1}{4})\mathcal{A}$ parallelogram ABCD (Postulate 2-1).

39. Construct altitude $\overline{AH}$ from A to $\overline{DC}$.
EP = FP (Theorem 7-6.1)
$\overline{MP}$ is a median of trapezoid AEFD and $\overline{NP}$ is a median of trapezoid CFEB (Definition 7-11)
$\mathcal{A}$ trapezoid AEFD = MP • AH, where $\overline{AH} \perp \overline{DC}$ at H,
and $\mathcal{A}$ trapezoid CFEB = NP • AH (See Exercise 23).
$\mathcal{A}$ trapezoid AEFD = $\mathcal{A}$ trapezoid CFEB (Transitive property).

40. The altitude of this trapezoid has length $4\sqrt{3}$ (Theorem 8-9.3)
$\mathcal{A} = \frac{1}{2}(4\sqrt{3})\ (15 + 23) = 76\sqrt{3}$

41. $\mathcal{A}\triangle ADB = \mathcal{A}\triangle CBD$ (Theorem 7-1.1, Postulate 12-2)
$\mathcal{A}\triangle PGB = \mathcal{A}\triangle BFP$ (Theorem 7-1.1, Postulate 12-2)
$\mathcal{A}\triangle PED = \mathcal{A}\triangle DHP$ (Theorem 7-1.1, Postulate 12-2).
$\mathcal{A}$ parallelogram AGPE = $\mathcal{A}\triangle ADB - \mathcal{A}\triangle PGB - \mathcal{A}\triangle PED = \mathcal{A}\triangle CBD - \mathcal{A}\triangle BFP - \mathcal{A}\triangle DHP$ (Postulate 12-3) = $\mathcal{A}$ parallelogram CFPH.

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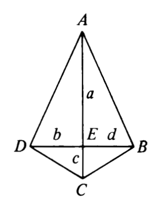
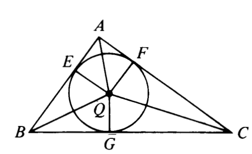
42. Construct $\overleftrightarrow{EPF} \perp \overleftrightarrow{AD}$ at F.
Let $\overleftrightarrow{BC}$ intersect $\overleftrightarrow{EPF}$ at E.
$\mathcal{A}\triangle APD = (\frac{1}{2})(PF \cdot AD)$ (Theorem 12-2.2)
$\mathcal{A}\triangle BPC = (\frac{1}{2})(PE \cdot BC)$ (Theorem 12-2.2)
BC = AD (Definition 7-1)
$\mathcal{A}\triangle APD + \mathcal{A}\triangle BPC = (\frac{1}{2})(PF \cdot AD) + (\frac{1}{2})(PE \cdot BC) = (\frac{1}{2})BC\ (PF + PE) = (\frac{1}{2})BC\ (EF) = (\frac{1}{2})\ \mathcal{A}$ parallelogram ABCD (Theorem 12-3.1).

43.



Draw quadrilateral ABCD such that $\overline{AC} \perp \overline{BD}$ at E.
Let AE = a, DE = b, BE = d, and EC = c.
$\mathcal{A}\triangle AED = (\frac{1}{2})ab$, $\mathcal{A}\triangle AEB = (\frac{1}{2})ad$, $\mathcal{A}\triangle DEC = (\frac{1}{2})bc$,
$\mathcal{A}\triangle BEC = (\frac{1}{2})cd$ (Theorem 12-2.1).
$\mathcal{A}$ quadrilateral ABCD = $\mathcal{A}\triangle AED + \mathcal{A}\triangle AEB + \mathcal{A}\triangle DEC + \mathcal{A}\triangle BEC$ (Postulate 12-3)
$\mathcal{A}$ quadrilateral ABCD = $(\frac{1}{2})ab + (\frac{1}{2})ad + (\frac{1}{2})bc + (\frac{1}{2})cd$ (Postulate 2-1)
$\mathcal{A}$ quadrilateral ABCD = $(\frac{1}{2})[a(b + d) + c(b + d)]$ (Distributive property)
$\mathcal{A}$ quadrilateral ABCD = $(\frac{1}{2})(b + d)\ (a + c)$ (Distributive property)
$\mathcal{A}$ quadrilateral ABCD = $(\frac{1}{2})BD \cdot AC$ (Postulate 2-1)

44.



Draw $\triangle ABC$ with inscribed $\odot Q$ such that E, F, and G are the points of tangency with $\overline{AEB}$, $\overline{AFC}$, and $\overline{BGC}$.
$\overline{QE} \perp \overline{AB}$ (Theorem 9-3.2)
$\overline{QG} \perp \overline{BC}$ (Theorem 9-3.2)
$\overline{QF} \perp \overline{AC}$ (Theorem 9-3.2)
$\overline{QE} \cong \overline{QF} \cong \overline{QG}$ (Radii)
$\mathcal{A}\triangle ABC = \mathcal{A}\triangle AQB + \mathcal{A}\triangle BQC + \mathcal{A}\triangle AQC$ (Postulate 12-3)
$\mathcal{A}\triangle ABC = (\frac{1}{2})(AB)(QE) + (\frac{1}{2})(BC)(QG) + (\frac{1}{2})(AC)(QF)$ (Theorem 12-2.2)
$\mathcal{A}\triangle ABC = (\frac{1}{2})(QE)(AB + BC + AC)$ (Distributive property).

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45. Construct parallelogram ABCD with DM = MC.
Draw $\overline{AM}$.
Construct $\overline{AE} \perp \overline{DC}$ at E.
$\mathcal{A}\triangle ADM = (\frac{1}{2})(AE \cdot DM)$ (Theorem 12-2.2)
$\mathcal{A}$ parallelogram ABCD = AE • DC (Theorem 12-3.1)
DM = $(\frac{1}{2})$DC (Definition 1-15)
$\mathcal{A}\triangle ADM = (\frac{1}{4})(AE \cdot DC)$ (Postulate 2-1)
$\mathcal{A}\triangle ADM = (\frac{1}{4})\ \mathcal{A}$ parallelogram ABCD (Postulate 2-1)
$3\mathcal{A}\triangle ADM = \mathcal{A}$ trapezoid MCBA (Subtraction property).

46. Construct trapezoid ABCD such that AB < CD.
Draw $\overline{PM}$ and $\overline{PN}$.
Construct $\overline{PF} \perp \overline{DC}$ at F such that $\overline{PF}$ intersects $\overline{MN}$ at E.
PE = EF (Theorem 7-6.1)
PE = $(\frac{1}{2})$PF (Definition 7-11)
$\mathcal{A}\triangle MPN = (\frac{1}{2})(PE \cdot MN)$ (Theorem 12-2.1)
$\mathcal{A}\triangle MPN = (\frac{1}{2})[(\frac{1}{2})PF] \cdot MN = (\frac{1}{4})(PF \cdot MN)$ (Postulate 2-1, Closure)
$\mathcal{A}$ trapezoid ABCD = PF • MN (Exercise 23)
$\mathcal{A}\triangle MPN = (\frac{1}{4})\ \mathcal{A}$ trapezoid ABCD (Postulate 2-1).

47. Construct △ABC subject to the given conditions.
Draw $\overline{MN}$.
Construct $\overline{AF} \perp \overline{BC}$ at F.
Let $\overline{MN}$ intersect $\overline{AF}$ at E.
$\overline{MN} \parallel \overline{DE}$ (Theorem 7-6.2)
Quadrilateral MNED is a parallelogram (Definition 7-1)
AE = EF (Theorem 7-6.4)
EF = $(\frac{1}{2})$AF (Theorem 7-6.4)
$\mathcal{A}$ parallelogram MNED = EF • MN (Theorem 12-3.1)
$\mathcal{A}\triangle ABC = (\frac{1}{2})(BC \cdot AF)$ (Theorem 12-2.2)
MN = $(\frac{1}{2})$BC (Theorem 7-6.3)
$\mathcal{A}$ parallelogram MNED = $[(\frac{1}{2})(AF)][(\frac{1}{2})(BC)] = \frac{1}{4}(AF \cdot BC)$,
$\mathcal{A}$ parallelogram MNED = $(\frac{1}{2})\ \mathcal{A}\triangle ABC$ (Postulate 2-1).

48. Construct trapezoid ABCD with AB < DC and M as the midpoint of $\overline{BC}$.
Draw $\overline{AM}$ and $\overline{DC}$.
Construct $\overline{AH} \perp \overline{DC}$ at H
$\overline{MF} \perp \overline{AH}$ at F
$\overline{DE} \perp \overline{MF}$ at E.
$\overline{ME}$ intersects $\overline{AD}$ at N.
△AFN ≅ △DEN (AAS)
AF = DE (Definition 3-3)
$\mathcal{A}\triangle DNM = (\frac{1}{2})(MN \cdot DE)$ (Theorem 12-2.2)
$\mathcal{A}\triangle ANM = (\frac{1}{2})(MN \cdot AF)$ (Theorem 12-2.2)
$\mathcal{A}\triangle DAM = \mathcal{A}\triangle DNB + \mathcal{A}\triangle ANM$ (Postulate 12-3)
$\mathcal{A}\triangle DAM = (\frac{1}{2})MN\ (AF + DE)$ (Postulate 2-1)
AF + DE = AH (Definition 7-1, Postulate 2-1)
$\mathcal{A}\triangle DAM = (\frac{1}{2})MN \cdot AH$ (Postulate 2-1)
$\mathcal{A}$ trapezoid ABCD = MN • AH (Exercise 23)
$\mathcal{A}\triangle AMD = (\frac{1}{2})\ \mathcal{A}$ trapezoid ABCD (Transitive property).

49. $\mathcal{A}$ parallelogram TJBQ = $\mathcal{A}$ parallelogram LTQH (both share base $\overline{TQ}$ and a common altitude) (Theorem 7-1.6)
Similarly, $\mathcal{A}$ parallelogram LTQH = $\mathcal{A}$ parallelogram QERN
$\mathcal{A}$ parallelogram TJBQ = $\mathcal{A}$ parallelogram QERN (Transitive property)
Similarly, $\mathcal{A}$ parallelogram ECPR = $\mathcal{A}$ parallelogram TIAC
$\mathcal{A}$ parallelogram TIAC + $\mathcal{A}$ parallelogram TJBQ =
$\mathcal{A}$ parallelogram QERN + $\mathcal{A}$ parallelogram ECPR =
$\mathcal{A}$ parallelogram QCPN (Postulate 2-1).

50. Since △AGD and parallelogram ABCD have the same base ($\overline{AD}$) and the same altitude, $\mathcal{A}\triangle AGD = (\frac{1}{2})\ \mathcal{A}$ parallelogram ABCD (Theorem 12-2.2, Theorem 12-3.1)
Similarly, $\mathcal{A}\triangle AGD = (\frac{1}{2})\ \mathcal{A}$ parallelogram EFGD
$(\frac{1}{2})\ \mathcal{A}$ parallelogram ABCD = $(\frac{1}{2})\ \mathcal{A}$ parallelogram EFGD (Transitive property)
$\mathcal{A}$ parallelogram ABCD = $\mathcal{A}$ parallelogram EFGD (Multiplication property)

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Exercises

1. $5as/2$ 2. $4as$ 3. $5as$ 4. $6as$

5. $\mathcal{A} = \frac{1}{2}(5)(20) = 50$ 6. 168 7. 333/2

8. $34x^2$ 9. $16 = \frac{1}{2}(a)(6)$; $a = 16/3$

10. 8/9 11. $4\frac{6}{7}$ 12. $3x$

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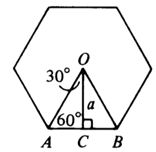
13. The area of the square is $4^2 = 16$, $a = 2$ (isosceles right triangle)

14. 3;36 15. $3\frac{1}{2}$, 49 16. $6x$, $144x^2$

17.



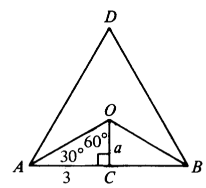
$m\angle AOB = 360/6 = 60$
$m\angle AOC = m\angle BOC = 30$
AB = s; $p = 6s$
$36 = 6s$; $s = 6$
AC = CB = 3
$a = 3\sqrt{3}$
$\mathcal{A} = \frac{1}{2}\ ap$
$\mathcal{A} = \frac{1}{2}(3\sqrt{3})\ (36) = 54\sqrt{3}$

Exercises 18-20 are done in a way similar to Exercise 17.

18. $2\sqrt{3}$; $24\sqrt{3}$ 19. $5\sqrt{3}/2$; $75\sqrt{3}/2$

20. $25\sqrt{3}/6$; $625\sqrt{3}/2$

21.



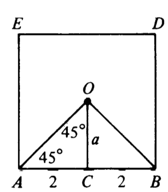
$m\angle AOB = 360/3 = 120$
$m\angle AOC = m\angle BOC = 60$
$p = 3s$, $18 = 3s$, $s = 6$
AC = CB = 3
$3 = a\sqrt{3}$; $a = \sqrt{3}$
$\mathcal{A} = \frac{1}{2}\ ap$
$\mathcal{A} = \frac{1}{2}(\sqrt{3})\ (18) = 9\sqrt{3}$
One could also use
$\mathcal{A} = \frac{s^2\sqrt{3}}{4}$ to find area.

Exercises 22-24 are done in a way similar to Exercise 21

22. $2\sqrt{3}/3$; $4\sqrt{3}$ 23. $5\sqrt{3}/6$; $25\sqrt{3}/4$

24. $25\sqrt{3}/18$; $625\sqrt{3}/36$

25.

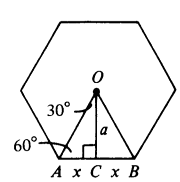


$m\angle AOB = 360/4 = 90$
$m\angle AOC = m\angle BOC = 45$
AC = CB = 2
$a = 2$
$p = 4s = 16$

Exercises 26-28 are done in a way similar to Exercise 25

26. $2\frac{1}{2}$; 20 27. 3; 24 28. $2\sqrt{2}$; $16\sqrt{2}$

29.



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29. *continued*

$m\angle AOB = 360/6 = 60$
$m\angle AOC = m\angle BOC = 30$
$a = x\sqrt{3}$; $x = a\sqrt{3}/3$
$AB = 2x$, $p = 6\,(AB) = 12x = 4a\sqrt{3}$
$\mathcal{A} = \frac{1}{2}ap$
$18\sqrt{3} = \frac{1}{2} \cdot a \cdot 4a\sqrt{3}$
$18\sqrt{3} = 2a^2\sqrt{3}$
$a^2 = 9$; and $a = 3$
$p = 4a\sqrt{3} = 12\sqrt{3}$

Exercises 30-32 are done in a way similar to Exercise 29

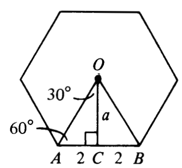
30. $3\sqrt{3}$; 36 31. $5\sqrt{3}$; 60
32. $\sqrt{3}/2$; $48\sqrt{3}$ (Use Theorem 12-2.6 and Theorem 12-2.7, or Theorem 12-4.1.

33. $\mathcal{A}_1 = s_1^2$; $p_1 = 4s$

$36 = s_1^2$; $p_1 = 24$

$s_1 = 6$

$p_2 = 6s_2$; $24 = 6s_2$; $s_2 = 4$



$m\angle AOB = 360/6 = 60$
$m\angle AOC = m\angle BOC = 30$
$AC = CB = 2$; $a = 2\sqrt{3}$
$\mathcal{A}_2 = \frac{1}{2}ap_2$
$\mathcal{A}_2 = \frac{1}{2}(2\sqrt{3})(24) = 24\sqrt{3}$

Exercises 34-36 are done in a way similar to Exercise 33

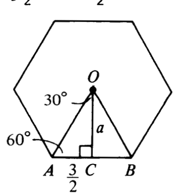
34. $32\sqrt{3}/3$ 35. $128\sqrt{3}/3$ 36. $64\sqrt{3}/3$

37. $\mathcal{A}_1 = \dfrac{s_1^2\sqrt{3}}{4}$

$9\sqrt{3} = \dfrac{s_1^2\sqrt{3}}{4}$

$s_1^2 = 9(4)$; $s_1 = 6$; $p_1 = 3s = 18$

$p_2 = 6s_2$; $18 = 6s_2$; $s_2 = 3$



$m\angle AOB = 360/6 = 60$
$m\angle AOC = m\angle BOC = 30$
$a = \frac{3}{2}\sqrt{3}$
$\mathcal{A}_2 = \frac{1}{2}ap_2$
$\mathcal{A}_2 = \frac{1}{2}(\frac{3}{2}\sqrt{3})(18)$
$\mathcal{A}_2 = \dfrac{27\sqrt{3}}{2}$

Exercises 38-40 are done in a way similar to Exercise 37.

38. $36\sqrt{3}$ 39. $9\sqrt{3}/2$ 40. $54\sqrt{3}$

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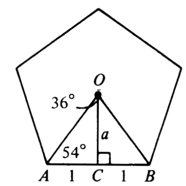
41.

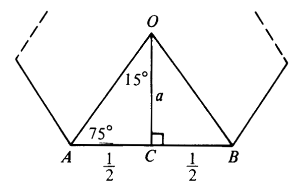


$m\angle AOB = 360/5 = 72$
$m\angle AOC = m\angle BPC = 36$
$AC = CB = 1$
$\tan 54 = a/1$
$a = \tan 54 = 1.3764$
$p = 5(AB) = 10$
$\mathcal{A} = \frac{1}{2}ap$
$\mathcal{A} = \frac{1}{2}(1.3764)(10)$
$\mathcal{A} = 6.8820 \approx 7$

Exercises 42-44 are done in a way similar to Exercise 41.

42. $61.938 \approx 62$ sq. in. 43. $688.2 \approx 688$ sq. in.
44. $290.7645 \approx 291$ sq. in.

45. $n = 12$; $p = 12s$; $12 = 12s$; $s = 1$

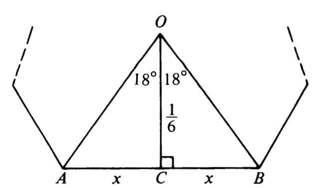


$m\angle AOB = 360/12 = 30$
$m\angle AOC = m\angle BOC = 15$
$\tan 75 = \dfrac{a}{\frac{1}{2}}$
$2a = \tan 75$
$2a = 3.7321$
$a = 1.86605$
$\mathcal{A} = \frac{1}{2}ap$
$\mathcal{A} = \frac{1}{2}(1.86605)(12)$
$\mathcal{A} = 11.19630 \approx 11.$

Exercises 46-48 are done in a way similar to Exercise 45

46. $100.764 \approx 101$ sq. in.
47. $10{,}076.4 \approx 10{,}076$ sq. in.
48. $69.975 \approx 70$ sq. in.

49. $n = 10$; $a = 2$ in. $= 1/6$ ft. $= OC$



$AB = 2x$, $p = 10(AB) = 20x$
$m\angle AOB = 360/10 = 36$
$\tan 18 = \dfrac{x}{\frac{1}{6}}$
$x = \frac{1}{6}\tan 18$
$p = 20x = \frac{10}{3}\tan 18$
$\mathcal{A} = \frac{1}{2}ap$
$\mathcal{A} = \frac{1}{2}(\frac{1}{6})(\frac{10}{3}\tan 18)$
$\mathcal{A} = \frac{5}{18}\tan 18$

$\mathcal{A} = \frac{5}{18}(.3249) = .09 \approx .1$ sq. ft.
If rounded off to nearest square foot, the answer is 0 or 1, but cannot be 0, therefore $\mathcal{A} = 1$.

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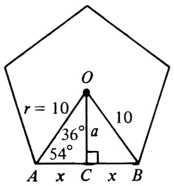
Exercise 50-52 are done in the same way as Exercise 49.

50. 13 sq. ft. 51. 27 sq. ft. 52. 61 sq. ft.

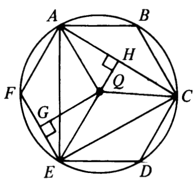
53.



m∠ AOB = 360/5 = 72
m∠ AOC = m∠ BOC = 36
AC = CB; p= 5(AB) = 10x
sin 36 = x/10; sin 54 = a/10

x = 10(.5878) | a = 10(.8090)
x = 5.878 | a = 8.090
$\mathcal{A} = \frac{1}{2}ap$
$\mathcal{A} = \frac{1}{2}(8.09)(10x)$; $\mathcal{A} = (8.09)(5x)$; $\mathcal{A} = (8.09)(29.39)$
$\mathcal{A} = 237.8 \approx 238$

54.



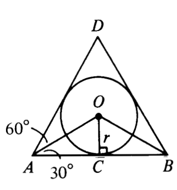
Inscribe regular hexagon ABCDEF in ⊙Q.
Inscribe equilateral △ACE in ⊙Q.
Draw $\overline{AQ}$, $\overline{CQ}$, and $\overline{EQ}$.
Construct $\overline{QH} \perp \overline{AC}$ at H and $\overline{QG} \perp \overline{FE}$ at G.
△AQE ≅ △AFE (ASA)
△EQC ≅ △EDC (ASA)
△AQC ≅ △ABC (ASA)
$\mathcal{A}$△ACE = $\mathcal{A}$△AQE + $\mathcal{A}$△EQC + $\mathcal{A}$△AQC = $\mathcal{A}$△AFE + $\mathcal{A}$△EDC + $\mathcal{A}$△ABC (Postulate 12-3)
$\mathcal{A}$△ACE = (½) $\mathcal{A}$ hexagon ABCDEF (Transitive property).

Alternate solution: QG = QE $\sqrt{3}/2$ (Theorem 8-9.3)
QE = FE (Definition 3-12)
$\mathcal{A}$ hexagon ABCDEF = (½)QG(6FE) = (½)(QE $\sqrt{3}/2$)(6QE) = (3/2)QE² $\sqrt{3}$ (Theorem 12-4.1)
$\mathcal{A}$△ABC = EH² $\sqrt{3}/3$ (Theorem 12-2.7)
$\mathcal{A}$△ABC = [(3/2)² QE² $\sqrt{3}$]/3 = (3/4) QE² $\sqrt{3}$ (Postulate 2-1)
$\mathcal{A}$△ABC = (½) $\mathcal{A}$ hexagon ABCDEF (Transitive property).

55. Let r = The length of the radius of the circle. The diagonal of the square has length $2r$. Therefore its area = $\frac{1}{2}d^2 = \frac{1}{2}(2r)^2 = 2r^2$.
The hexagon is composed of six equilateral triangles, each having sides of length r.
Each has area = $\frac{r^2\sqrt{3}}{4}$. Thus the area of the six triangles = $6\left(\frac{r^2\sqrt{3}}{4}\right) = \frac{3r^2\sqrt{3}}{2}$.

The ratio of the required areas is $\dfrac{2r^2}{\frac{3r^2\sqrt{3}}{2}} = \dfrac{4\sqrt{3}}{9}$.

56.



m∠AOB = 360/3 = 120
m∠AOC = 60
AC = CB; AC =$r\sqrt{3}$; AB = $2r\sqrt{3}$

$\mathcal{A}_1 = \frac{(AB)^2\sqrt{3}}{4} = \frac{12r^2\sqrt{3}}{4} = 3r^2\sqrt{3}$

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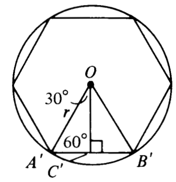
56. *continued*



m∠ A'OB' = 360/6 = 60; m∠ A'OC' = 30
A'C' = C'B' = $\frac{r}{2}$; A'B' =r, OC =$\frac{r\sqrt{3}}{2}$

p = 6(A'B') = 6r
$\mathcal{A}_2 = \frac{1}{2}(OC)(p)$

$\mathcal{A}_2 = \frac{1}{2}\left(\frac{r\sqrt{3}}{2}\right)(6r)$

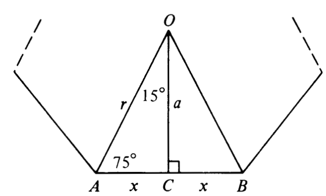
$\mathcal{A}_2 = \frac{3r^2\sqrt{3}}{2}$

$$\frac{\mathcal{A}_1}{\mathcal{A}_2} = \frac{\frac{3r^2\sqrt{3}}{1}}{\frac{3r^2\sqrt{3}}{2}} = \frac{3r^2\sqrt{3}}{1} \cdot \frac{2}{3r^2\sqrt{3}} = \frac{2}{1}$$

57. 8/6 = $r_1/r_2 = a_1/a_2 = p_1/p_2$

$\mathcal{A}_1/\mathcal{A}_2 = \frac{1}{2}a_1p_1/\frac{1}{2}a_2p_2$ = 8•8/6•6 = 16/9.

58.

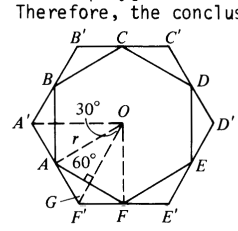


n = 12, m∠ AOB = 360/12 = 30
m∠ AOC = 15
AC = CB = x, AB = 2x
p = 12(AB) = 24x
sin 75 = a/r
$a = r$ sin 75
sin 15 = x/r
$x = r$ sin 15
Therefore $p = 24\,r \cdot$ sin 15

$\mathcal{A} = \frac{1}{2}ap$
3 = ½(r • sin 75) (24r • sin 15)
3 = 12r^2 sin 75 • sin 15
3/12 = r^2 (sin 75)(sin 15)
.25 = r^2 (.9659)(.2588); (.9659)(.2588) ≈ .25
.25 = .25r^2
r^2 = 1; and r = 1

59. Refer to the proof of Theorem 12-4.1. Replace statement 5 with n triangles of equal area. Replace statement 7 with $\mathcal{A}$ polygon ABCDE = n• [(½)a • DC]. Replace statement 8 with $\mathcal{A}$ polygon ABCDE = (½)a • n (DC). $p = n$ • DC. Therefore, the conclusion is $\mathcal{A}$ polygon ABCDE = (½)ap.

60.



$\mathcal{A}_1 = \mathcal{A}$ hexagon ABCDEF = $\frac{225\sqrt{3}}{2}$
m∠ AOF = 360/6 = 60
m∠ AOG = 30
AG = r/2; OG = $\frac{r}{2}\sqrt{3}$
AF = 2(AG) = r
p_1 = 6(AF) = 6r
$\mathcal{A}_1 = \frac{1}{2}(OG)p_1$

continued next page

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60. *continued*

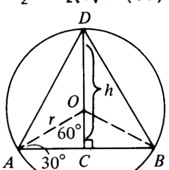
$\frac{225\sqrt{3}}{2} = \frac{1}{2}(\frac{r\sqrt{3}}{6})(6r)$

$\frac{225\sqrt{3}}{2} = \frac{3r^2\sqrt{3}}{2}$

$r^2 = 75$
$r = 5\sqrt{3} = OA$ [$\overline{OA}$ is an apothem of Regular Hexagon A'B'C'D'E'F']

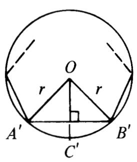
In right $\triangle$ ADA', AA' = 5 (Corollary 8-9.3b)
Therefore $p_2 = 12 \cdot 5 = 60$
$\mathcal{A}_2 = \frac{1}{2}(5\sqrt{3})(60) = 150\sqrt{3}$

61.



$\mathcal{A}\triangle ABD = K$ $\qquad K = \frac{h^2\sqrt{3}}{3}$ (Theorem 12-2.7)

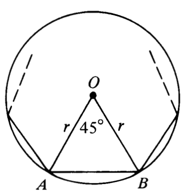
Therefore $h = \sqrt{K\sqrt{3}}$ $\qquad r = \frac{2}{3}h\sqrt{K\sqrt{3}}$



m$\angle$ A'OB' = 360/12 = 30
We have 12 sides, therefore 12 congruent triangles (if all sides were drawn).
$\mathcal{A}\triangle A'OB' = \frac{1}{2}(A'O)(B'O) \sin\angle A'OB'$
$\mathcal{A}\triangle A'OB' = \frac{1}{2}(r)(r) \sin 30 = \frac{r^2}{4}$

$\mathcal{A}_2$ of Regular Dodecagon $= 12(\frac{r^2}{4}) = 3r^2 = 3(\frac{2}{3}\sqrt{K\sqrt{3}})^2 = 3\left(\frac{4}{9}K\sqrt{3}\right) = \frac{4K\sqrt{3}}{3}$

62. m$\angle$ AOB = 360/8 = 45
$\mathcal{A}\triangle AOB = \frac{1}{2}(r)(r) \sin 45 = \frac{r^2\sqrt{2}}{4}$



We have 8 congruent triangles (if all sides were drawn)
Area of Regular Octagon $= 8(\frac{r^2\sqrt{2}}{4}) = 2r^2\sqrt{2}$

63. Refer to the proof of Exercise 44 of Section 12-3, and extend it to a polygon with any number of sides.

64. The proof given here is for a regular pentagon. However, a similar proof could be given for any other regular polygon. Construct regular pentagon ABCDE and choose any point P in the interior of the pentagon.
Draw $\overline{PK} \perp \overline{AB}$ at K
$\overline{PM} \perp \overline{BC}$ at M
$\overline{PN} \perp \overline{DC}$ at N
$\overline{PH} \perp \overline{ED}$ at H, and
$\overline{PR} \perp \overline{AE}$ at R.
$\mathcal{A}\triangle APB = (\frac{1}{2})(AB \cdot PK)$ (Theorem 12-2.2)
$\mathcal{A}\triangle BPC = (\frac{1}{2})(BC \cdot PM)$ (Theorem 12-2.2)
$\mathcal{A}\triangle CPD = (\frac{1}{2})(DC \cdot PN)$ (Theorem 12-2.2)
$\mathcal{A}\triangle DPE = (\frac{1}{2})(DE \cdot PH)$ (Theorem 12-2.2)
$\mathcal{A}\triangle EPA = (\frac{1}{2})(EA \cdot PR)$ (Theorem 12-2.2)
$\mathcal{A}$ pentagon ABCDE $= (\frac{1}{2})AB(PK + PM + PN + PH + PR) = (\frac{1}{2})AB(S)$ (Postulate 12-3)
$\mathcal{A}$ pentagon ABCDE $= (\frac{1}{2})a(5AB) = (5a)(\frac{1}{2})AB$ (Theorem 12-4.1)
$(\frac{1}{2})AB(S) = (5a)(\frac{1}{2})AB$ (Transitive property)
$S = 5a$, or $PK + PM + PN + PH + PR = 5a$ (Postulate 2-1)

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Exercises

1. $d = 2r$, $r = 2$
$\mathcal{A} = \pi r^2 = 4\pi$

Exercises 2-4 are done in the same way as Exercise 1

2. 100π 3. $225\pi/4$ 4. $12\frac{1}{2}\pi$

5. $C = 2\pi r$
$10\pi = 2\pi r$
$r = 5$
$\mathcal{A} = \pi r^2 = 25\pi$

Exercises 6-8 are done in the same way as Exercise 5

6. 324π 7. $625\pi/4$ 8. $81/2\pi$

9. $\mathcal{A} = \pi r^2$
$25\pi = \pi r^2$
$r^2 = 25$
$r = 5$

Exercises 10-12 are done in the same way as Exercise 9

10. 9 11. $2\sqrt{6}$ 12. $\sqrt{8\sqrt{2}}$

13. $\mathcal{A} = \pi r^2$
$49\pi = \pi r^2$
$r = 7$
$C = 2\pi r = 14\pi$

Exercises 14-16 are done in the same way as Exercise 13.

14. 24π 15. $6\pi\sqrt{2}$ 16. $8\sqrt{\pi}$

17. $\mathcal{A} = n/360 \cdot \pi r^2$
$= 30/360 \cdot 36\pi$
$= 3\pi$

Exercises 18-20 are done in the same way as Exercise 17.

18. $9\pi/2$ 19. $36\pi/5$ 20. 8.4π

21. $\mathcal{A} = n/360 \cdot \pi r^2$ (Theorem 12-5.2)
$24\pi = 90/360 \cdot \pi r^2$
$24\pi = \pi r^2/4$
$r^2 = 96$
$\mathcal{A} = \pi r^2 = 96\pi$

Exercises 22-24 are done in the same way as Exercise 21.

22. 72 23. 60 24. 108

25. $\mathcal{A} = n/360 \cdot \pi r^2$ (Theorem 12-5.2)
$16\pi = n/360 \cdot 144\pi$

$n = \frac{360(16\pi)}{144\pi}$
$n = 40$

Exercises 26-28 are done in the same way as Exercise 25.

26. 90 27. $22\frac{1}{2}$ 28. 30

29. $\mathcal{A} = \frac{n}{360} \cdot \pi r^2$

$24\pi = \frac{30}{360} \cdot \pi r^2$

$24\pi = \frac{\pi r^2}{12}$
$r^2 = 12(24)$
$r = 12\sqrt{2}$
$d = 2r = 24\sqrt{2}$

Exercises 30-32 are done in the same way as Exercise 29.

30. 24 31. $12\sqrt{2}$ 32. $24\sqrt{\frac{6}{5}}$

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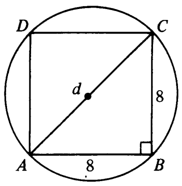
Exercises continued

33. 

$d = 8\sqrt{2}$
$d = 2r,\ r = 4\sqrt{2}$
$\mathcal{A} = \pi r^2 = 32\pi$

Exercises 34-36 are done in the same way as Exercise 33.

34. 16π 35. 50π 36. 96π

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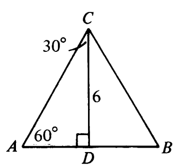
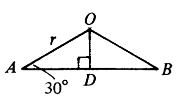
37. 

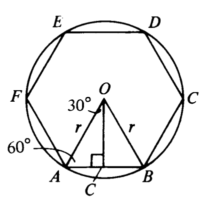
$CD = (AD)(\sqrt{3})$
$6 = (AD)(\sqrt{3})$
$AD = 2\sqrt{3}$
$AB = 4\sqrt{3}$



$m\angle AOB = \frac{360}{3} = 120$
$m\angle AOD = 60$
$AD = (OD)\sqrt{3}$
$2\sqrt{3} = (OD)\sqrt{3}$; and $OD = 2$
$r = 2(OD) = 4$
$\mathcal{A} = \pi r^2 = 16\pi$

Exercises 38-40 are done in the same way as Exercise 37.

38. 36π 39. 48π 40. $\frac{400\pi}{9}$

41. 

$AB = 2$
$AC = CB = 1$
$m\angle AOB = 360/6 = 60$
$m\angle AOC = 30$
$r = 2(AC) = 2$
$\mathcal{A} = \pi r^2 = 4\pi$

Exercises 42-44 are done in the same way as Exercise 41.

42. 36π 43. 400π 44. 162π

45. $\mathcal{A}_1 = \pi r_1^2$ | $\mathcal{A}_2 = \pi r_2^2$
$d_1 = AB = 20$ | $d_2 = CD = 10$
$r_1 = 10$ | $r_2 = 5$
$\mathcal{A}_1 = 100\pi$ | $\mathcal{A}_2 = 25\pi$

$\mathcal{A}_1 - \mathcal{A}_2 = 75\pi$

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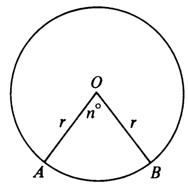
46. Area of sector $= \frac{150}{360} \cdot \pi r^2 = \frac{5}{12} \cdot 144\pi = 60\pi$

47. Area of sector $= \frac{90}{360} \cdot \pi(14)(14) = 49\pi$
$\mathcal{A}\triangle DQE = \frac{1}{2}(14)(14) = 98$
Area of sector $- \mathcal{A}\triangle DQE =$ Area of segment
$49\pi - 98 =$ Area of segment
$49(\pi - 2) =$ Area of segment

48. 

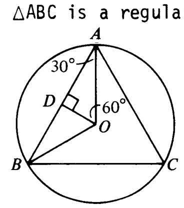
Length of $\widehat{AB} = \frac{n}{360} \cdot 2\pi r$
$L = \frac{\pi rn}{180}$
Area of sector $= \frac{n}{360} \cdot \pi r^2$

$K = \frac{\pi rn}{180} \cdot \frac{r}{2} = L \cdot \frac{r}{2} = \frac{rL}{2}$

49. $9\pi = \frac{n}{360} \cdot 2\pi r = \frac{n}{360} \cdot \pi r^2,\ r = 2$

50. $K = \frac{1}{2}rL$ (from Exercise 48)
$K = (\frac{1}{2})\left(\frac{8}{\pi}\right)(4) = \frac{16}{\pi}$
$A = \pi r^2 = (\pi)\left(\frac{64}{\pi^2}\right) = \frac{64}{\pi}$
$\frac{K}{A} = \frac{\frac{16}{\pi}}{\frac{64}{\pi}} = \frac{1}{4}$

51. $\triangle ABC$ is a regular Polygon



$m\angle AQB = \frac{360}{3} = 120$
$m\angle AQD = 60$
$AB = 12;\ AD = DB = 6$
$AD = (QD)\sqrt{3};\ 6 = (QD)\sqrt{3};\ QD = 2\sqrt{3}$
$AQ = 2(QD) = 4\sqrt{3}$
$\mathcal{A}\triangle AQB = \frac{1}{2}(QD)(AB) = \frac{1}{2}(2\sqrt{3})(12) = 12\sqrt{3}$

Area of sector $= \frac{m\angle AQB}{360} \cdot \pi(AQ)^2 = \frac{120}{360} \cdot \pi(48) = 16\pi$

Area of segment = Area of sector $- \mathcal{A}\triangle AQB = 16\pi - 12\sqrt{3}$

52. Area of Semicircle P $= \frac{1}{2}\pi(AP)^2 = \frac{1}{2}\pi(3)^2 = \frac{9\pi}{2}$

Area of Semicircle Q $= \frac{1}{2}\pi(BQ)^2 = \frac{1}{2}\pi(4)^2 = 8\pi$

Area of Semicircle R $= \frac{1}{2}\pi(AR)^2 = \frac{1}{2}\pi(7)^2 = \frac{49\pi}{2}$

Area of shaded region $= \frac{49\pi}{2} - (\frac{9\pi}{2} + 8\pi) = 12\pi$

53. Area of Semicircle P $= \frac{1}{2}\pi(AP)^2 = \frac{1}{2}\pi(4)^2 = 8\pi$
Area of Semicircle Q $= \frac{1}{2}\pi(AQ)^2 = \frac{1}{2}\pi(6)^2 = 18\pi$
Area of Semicircle R $= \frac{1}{2}\pi(BR)^2 = \frac{1}{2}\pi(2)^2 = 2\pi$
Shaded area = (area of Semicircle Q - area of Semicircle P) + area of Semicircle R $= (18\pi - 8\pi) + 2\pi = 12\pi$

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Exercises continued

54. PQ = 3 (radius of each circle)
PA = PB = QQ = QA = PQ = 3 radii
APBQ is a Rhombus
△APQ and △BQP are equilateral
m∠ AQP = 60

Area of sector AQP = $\frac{60}{360} \cdot \pi(PQ)^2 = \frac{1}{6}\pi(3)^2 = \frac{3\pi}{2}$

$\mathcal{A}\triangle APQ = \frac{(PQ)^2\sqrt{3}}{4} = \frac{(3)^2\sqrt{3}}{4} = \frac{9\sqrt{3}}{4}$

Area of segment formed by $\overline{AP} = \frac{3\pi}{2} - \frac{9\sqrt{3}}{4}$

There are four equal segments

$4\left(\frac{3\pi}{2} - \frac{9\sqrt{3}}{4}\right) = 6\pi - 9\sqrt{3}$ segments formed by $\overline{AP}$, $\overline{PB}$, $\overline{BQ}$, $\overline{AQ}$.

$MQ = \frac{3}{2}$, $AM = (MQ)\sqrt{3} = 3\sqrt{3}/2$
$AB = 2(AM) = 3\sqrt{3}$
Area of rhombus APBQ = $\frac{1}{2}(AB)(PQ) = \frac{1}{2}(3\sqrt{3})(3) = \frac{9\sqrt{3}}{2}$

Area of shaded = area of rhombus APBQ + area of the four segments = $\frac{9\sqrt{3}}{2} + 6\pi - 9\sqrt{3} = 6\pi - \frac{9\sqrt{3}}{2}$

55. Area sector SQR = $\frac{1}{4}\pi r^2 = \frac{1}{4}\pi \times 16 = 4\pi$
Area sector PQS also = 4π
Area shaded region = area sector SQR + Area sector PQS - Area PQRS
Area shaded region = $4\pi + 4\pi - 16 = 8\pi - 16$

56. $AB = 8\sqrt{2}$, $AP = r = 4\sqrt{2}$
$\mathcal{A}$ Semicircle P = $\frac{1}{2}\pi(AP)^2 = \frac{1}{2}\pi(32) = 16\pi$
$\mathcal{A}\triangle ACB = \frac{1}{2}(8)(8) = 32$
$\mathcal{A}$ Sector CAB of circle C = $90/360 \cdot \pi(AC)^2 = \frac{1}{4}\pi(8)^2 = 16\pi$
($\mathcal{A}\triangle ACB$ + $\mathcal{A}$ semicircle P) = $32 + 16\pi$ = ($\mathcal{A}$ shaded + $\mathcal{A}$ unshaded)
($\mathcal{A}\triangle ACB$ + $\mathcal{A}$ semicircle P) - $\mathcal{A}$ sector of circle C = $(32 + 16\pi) - 16\pi = 32$ (area of shaded region).

57. Let AC = $2b$, BC = $2a$, and AB = $2c$.
$\mathcal{A}$ semicircle APC = $(\frac{1}{2})b^2\pi$ (Theorem 12-5.2)
$\mathcal{A}$ semicircle CQB = $(\frac{1}{2})a^2\pi$ (Theorem 12-5.2)
$\mathcal{A}$ semicricle ARB = $(\frac{1}{2})c^2\pi$ (Theorem 12-5.2)
$(2a)^2 + (2b)^2 = (2c)^2$ (Thereom 8-8.1)
$4a^2 + 4b^2 = 4c^2$ (Theorem 8-8.1)
$(\frac{1}{2})a^2\pi + (\frac{1}{2})b^2\pi = (\frac{1}{2})c^2\pi$ (Division property)
$\mathcal{A}$ semicircle APC + $\mathcal{A}$ semicircle CQB = $\mathcal{A}$ semicircle ARB (Postulate 2-1)

58. Using the same lengths as in Exercise 57, $\mathcal{A}$ semicircle APC = $(\frac{1}{2})b^2\pi$ (Theorem 12-5.2)
$\mathcal{A}$ semicircle CQB = $(\frac{1}{2})a^2\pi$ (Theorem 12-5.2)
$\mathcal{A}$ semicircle ASB = $(\frac{1}{2})c^2\pi$ (Theorem 12-5.2)
$AR_1 + AR_2$ = $\mathcal{A}$ semicircle APC + $\mathcal{A}$ semicircle CQB + $\mathcal{A}\triangle ABC$ - $\mathcal{A}$ semicircle ASB (Postulate 12-3)
$AR_1 + AR_2 = (\frac{1}{2})b^2\pi + (\frac{1}{2})a^2\pi + \mathcal{A}\triangle ABC - (\frac{1}{2})c^2\pi$ (Postulate 2-1)
$(\frac{1}{2})b^2\pi + (\frac{1}{2})a^2\pi = (\frac{1}{2})c^2\pi$ (Theorem 8-8.1)
$AR_1 + AR_2 = \mathcal{A}\triangle ABC$ (Postulate 2-1)

59. Pick point E on the boundary of the shaded region with $\widehat{AEC}$, and point F on the boundary of the unshaded region with $\widehat{AFC}$.
Let BQ = x
AQ = r
AB = $r + x$, and
BC = $r - x$.
$\mathcal{A}$ shaded region = $\mathcal{A}$ semicircle AEC - $\mathcal{A}$ semicircle ANB + $\mathcal{A}$ semicircle BMC (Postulate 12-3)
$\mathcal{A}$ shaded region = $(\pi/8)(2r)^2 - (\pi/8)(r + x)^2 + (\pi/8)(r-x)^2$ (Postulate 2-1) = $\mathcal{A}$ shaded region = $(\pi/8)(4r^2 - r^2 - 2rx - x^2 + r^2 - 2rx + x^2) = (\pi/8)(4r^2 - 4rx) = (\pi r/2)(r - x)$ (Multiplication and Division properties)
$\mathcal{A}$ non-shaded region = $\mathcal{A}$ semicircle AFC - $\mathcal{A}$ semicircle BMC + $\mathcal{A}$ semicircle ANB (Postulate 12-3)
$\mathcal{A}$ non-shaded region = $(\pi/8)(2r)^2 - (\pi/8)(r - x)^2 + (\pi/8)(r + x)^2$ (Postulate 2-1)

(continued)

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59. *continued*

$\mathcal{A}$ non-shaded region = $(\pi/8)(4r^2 - r^2 + 2rx - x^2 + r^2 + 2rx + x^2) = (\pi/8)(4r^2 + 4rx) = (\pi r/2)(r + x)$
(Multiplication and Division properties)
$\mathcal{A}$ shaded region/$\mathcal{A}$ non-shaded region = $(r - x)/(r + x)$ = BC/AB (Postulate 2-1)

60. Draw $\overline{TQ}$ and $\overline{BQ}$.
$\overline{TQ} \perp \overline{ATB}$ (Theorem 9-3.2)
$\mathcal{A}$ shaded region = $\mathcal{A}$ larger circle - $\mathcal{A}$ smaller circle (Given)
$\mathcal{A}$ shaded region = $\pi \cdot BQ^2 - \pi \cdot TQ^2 = \pi(BQ^2 - TQ^2)$ (Postulate 2-1)
$TQ^2 + TB^2 = BQ^2$, (Theorem 8-8.1)
$TB^2 = BQ^2 - TQ^2$ (Theorem 8-8.1)
$\mathcal{A}$ shaded region = $\pi \cdot TB^2 = \pi \cdot (\frac{1}{2})AB^2 = (\frac{1}{4})\pi \cdot AB^2$ (Postulate 2-1)

61. Draw $\overline{AP}$ and $\overline{BP}$.
Pick point R on the boundary of the shaded region such that $\widehat{ARB}$.
$\overline{AP} \perp \overline{BP}$ (Corollary 9-6.1c)
$\overline{AP} \cong \overline{BP}$ (Theorem 9-5.4)
$\mathcal{A}$ shaded region = $\mathcal{A}$ semicircle ARB + $\mathcal{A}\triangle APB$ - $\mathcal{A}$ sector ANBP (Postulate 12-3)
$\mathcal{A}$ shaded region = $(\frac{1}{2})\pi \cdot AQ^2 + (\frac{1}{2})AP^2 - (\frac{1}{4})\pi \cdot AP^2$ (Postulate 2-1)
In right △AQP, $AQ^2 = (\frac{1}{2})AP^2$ (Theorem 8-8.1)
$\mathcal{A}$ shaded region = $(\frac{1}{4})\pi \cdot AP^2 + (\frac{1}{2})AP^2 - (\frac{1}{4})\pi \cdot AP^2 = (\frac{1}{2})AP^2$ (Postulate 2-1)
$AP^2 = 2(PQ^2)$ (Theorem 8-8.1)
$\mathcal{A}$ shaded region = PQ^2 (Postulate 2-1)

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1. $\frac{\mathcal{A}_1}{\mathcal{A}_2} = \left(\frac{s_1}{s_2}\right)^2$

$\frac{1}{4} = \left(\frac{s_1}{s_2}\right)^2$

$\frac{s_1}{s_2} = \frac{1}{2}$

Exercises 2-4 are done in the same way as Exercise 1

2. 4/5
3. 5/12
4. $\frac{\sqrt{2}}{3}$

5. $\frac{\mathcal{A}_1}{\mathcal{A}_2} = \left(\frac{a_1}{a_2}\right)^2$

$\frac{\mathcal{A}_1}{\mathcal{A}_2} = (\frac{1}{2})^2 = \frac{1}{4}$

Exercises 6-8 are done in the same way as Exercise 5

6. 256/625
7. 25/49
8. 4/169

9. $\frac{\mathcal{A}_1}{\mathcal{A}_2} = \left(\frac{3}{10}\right)^2 = \frac{9}{100}$

Exercises 10-11 are done in the same way as Exercise 9.

10. 25/64
11. 9/16

12. $\frac{m_1}{m_2} = \frac{h_1}{h_2} = \frac{15}{20} = \frac{3}{4}$

13. $\frac{h_1}{h_2} = \frac{p_1}{p_2} = \frac{15}{20} = \frac{3}{4}$

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Exercises continued

14. $\frac{\mathcal{A}_1}{\mathcal{A}_2} = \left(\frac{h_1}{h_2}\right)^2$

$\frac{\mathcal{A}_1}{80} = \left(\frac{15}{20}\right)^2$

$\frac{\mathcal{A}_1}{80} = \frac{9}{16}$

$\mathcal{A}_1 = 45$

15. $\frac{\mathcal{A}_1}{\mathcal{A}_2} = \left(\frac{s_1}{s_2}\right)^2$

$\frac{1x}{4x} = \left(\frac{s_1}{s_2}\right)^2$

$s_2 = 2s_1$

16. $\frac{\mathcal{A}_1}{\mathcal{A}_2} = \left(\frac{s_1}{s_2}\right)^2$

$\frac{\mathcal{A}_1}{\mathcal{A}_2} = \left(\frac{1x}{4x}\right)^2$

$\frac{\mathcal{A}_1}{\mathcal{A}_2} = \frac{1}{16}$

$\mathcal{A}_2 = 16\mathcal{A}_1$

17. Let $x = \mathcal{A}_1$

$636 - x = \mathcal{A}_2$

$\frac{\mathcal{A}_1}{\mathcal{A}_2} = \left(\frac{M_1}{M_2}\right)^2$

$\frac{x}{636 - x} = \left(\frac{5}{9}\right)^2$

$x = 150 = \mathcal{A}_1$

$636 - x = 486 = \mathcal{A}_2$

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18. In the proof of Theorem 12-6.1 include the following step: ratio of similitude

$= b/b' = \sqrt{\mathcal{A}\triangle ABC/\mathcal{A}\triangle A'B'C'}$ (taking the square root of each side of the equality).

19. Refer to Exercise 18.

20. The ratio of similitude of any two circles equals the ratio of their radii, say r/r' (Definition 8-5)
$\mathcal{A}\odot P = \pi r^2$ (Theorem 12-5.1)
$\mathcal{A}\odot Q = \pi(r')^2$ (Theorem 12-5.1)
$\mathcal{A}\odot P/\mathcal{A}\odot Q = \pi r^2/\pi(r')^2 = r^2/(r')^2$ (Postulate 2-1)

21. Refer to Exercise 18.

22. Let $x = \mathcal{A}_1$
$x + 320 = \mathcal{A}_2$

$\frac{\mathcal{A}_1}{\mathcal{A}_2} = \left(\frac{s_1}{s_2}\right)^2$

$\frac{x}{x + 320} = \left(\frac{7}{21}\right)^2$

$x = 40 = \mathcal{A}_1$
$x + 320 = 360 = \mathcal{A}_2$

23. $x = \mathcal{A}_1$; $25x = \mathcal{A}_2$
$y = d_1$; $y + 12 = d_2$

$\frac{\mathcal{A}_1}{\mathcal{A}_2} = \left(\frac{d_1}{d_2}\right)^2$

$\frac{1x}{25x} = \left(\frac{y}{y + 12}\right)^2$

$\frac{1}{5} = \frac{y}{y + 12}$

$y = 3 = d_1$
$y + 12 = 15 = d_2$

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24. $x = s_1$; $2x = s_2$

$\frac{\mathcal{A}_1}{\mathcal{A}_2} = \left(\frac{s_1}{s_2}\right)^2$

$\frac{\mathcal{A}_1}{\mathcal{A}_2} = \frac{1}{4}$; Since one area is 4 times the other this represents a 300% increase.

25. Draw $\triangle ABC$ with midline $\overline{DE} \parallel \overline{BC}$.
$\triangle ADE \sim \triangle ABC$ (Corollary 8-5.1c)
$DE = (\frac{1}{2})BC$ (Theorem 7-6.3)
$\mathcal{A}\triangle ADE/\mathcal{A}\triangle ABC = \frac{1}{4}$ (Theorem 12-6.1)
$\mathcal{A}\triangle ADE/\mathcal{A}$ quadrilateral DECB $= \frac{1}{3}$ (Theorem 8-1.3).

26. $s = s_1$; $s + x = s_2$
$y = \mathcal{A}_1$; $2y = \mathcal{A}_2$

$\frac{\mathcal{A}_1}{\mathcal{A}_2} = \left(\frac{s_1}{s_2}\right)^2$

$\frac{1y}{2y} = \left(\frac{s}{s + x}\right)^2$

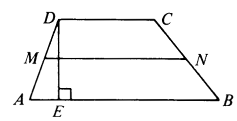
$\frac{1}{2} = \left(\frac{s}{s + x}\right)^2$, or $\frac{1}{\sqrt{2}} = \frac{s}{s + x}$

$s + x = s\sqrt{2}$

$s\sqrt{2} - s = x$

$x = s(\sqrt{2} - 1)$

27.

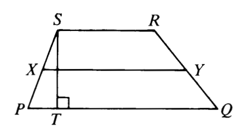


$h_1 = DE$

$\mathcal{A}_1 = \frac{1}{2}(DE)(AB + DC)$ (Theorem 12-3.3)

$\mathcal{A}_1 = (DE)\frac{(AB + DC)}{2}$

$\mathcal{A}_1 = h_1$ (MN) (Theorem 7-7.2)



$h_2 = ST$

$\mathcal{A}_2 = \frac{1}{2}(ST)(PQ + SR)$

$\mathcal{A}_2 = (ST)\frac{(PQ + SR)}{2}$; $\mathcal{A}_2 = h_2$ (XY)

$\frac{\mathcal{A}_1}{\mathcal{A}_2} = \frac{h_1(MN)}{h_2(XY)}$

$\frac{\mathcal{A}_1^2}{\mathcal{A}_2^2} = \left(\frac{h_1}{h_2}\right)^2 \cdot \left(\frac{MN}{xy}\right)^2$

$\frac{\mathcal{A}_1}{\mathcal{A}_2} = \left(\frac{h_1}{h_2}\right)^2$

$\frac{\mathcal{A}_1^2}{\mathcal{A}_2^2} = \frac{\mathcal{A}_1}{\mathcal{A}_2} \cdot \left(\frac{MN}{XY}\right)^2$

$\frac{\mathcal{A}_2}{\mathcal{A}_1} \cdot \frac{\mathcal{A}_1^2}{\mathcal{A}_2^2} = \left(\frac{MN}{XY}\right)^2$

$\frac{\mathcal{A}_1}{\mathcal{A}_2} = \left(\frac{MN}{XY}\right)^2$

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Exercises continued

28. AD/AB = 5/7 (Theorem 8-1.2)
$\mathcal{A}\triangle ADE/\mathcal{A}\triangle ABC = 25/49$ (Theorem 12-6.1)
$\mathcal{A}\triangle ADE/\mathcal{A}$ quadrilateral DECB = 25/24 (Theorem 8-1.3).

29. $\frac{AB}{BC} = \frac{2}{3}$

Let $2x = AB$, $3x = BC$, $AC = AB + BC = 5x$

Semicircle Q, $d_1 = AB = 2x$
$r_1 = AB/2 = x$

$\mathcal{A}_1 = \frac{1}{2}\pi r_1^2 = \frac{1}{2}\pi x^2$

Semicircle Q, $d_2 = BC = 3x$
$r_2 = BC/2 = 3x/2$

$\mathcal{A}_2 = \frac{1}{2}\pi r_2^2 = \frac{9\pi x^2}{8}$

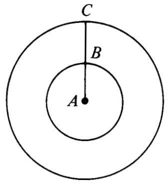
Semicircle R, $d_3 = AC = 5x$ (the largest semicircle R)
$r_3 = AC/2 = 5x/2$

$\mathcal{A}_3 = \frac{1}{2}\pi r_3^2 = \frac{25\pi x^2}{8}$

$\mathcal{A}_4$ = area of shaded $= \mathcal{A}_3 - (\mathcal{A}_1 + \mathcal{A}_2)$

$$= \frac{25\pi x^2}{8} - \left(\tfrac{1}{2}\pi x^2 + \frac{9\pi x^2}{8}\right) = \frac{3\pi x^2}{2}$$

$$\frac{\mathcal{A}_4}{\mathcal{A}_3} = \frac{\frac{3\pi x^2}{2}}{\frac{25\pi x^2}{8}} = \frac{3\pi x^2}{2} \cdot \frac{8}{25\pi x^2} = \frac{12}{25}$$

30. 

$r = AB = BC$
$AC = AB + BC = 2r$
$\mathcal{A}_1 = \pi(AB)^2 = \pi r^2$
$\mathcal{A}_2 = \pi(AC)^2 = 4\pi r^2$
$\mathcal{A}_2 - \mathcal{A}_1 = 4\pi r^2 - \pi r^2 = 3\pi r^2 = \mathcal{A}_3$(area formed by both circles)

$$\frac{\mathcal{A}_3}{\mathcal{A}_2} = \frac{3\pi r^2}{4\pi r^2} = \frac{3}{4}$$

31. $\triangle KMP \sim \triangle QRP$

$$\frac{\mathcal{A}\triangle KMP}{\mathcal{A}\triangle QRP} = \left(\frac{5}{8}\right)^2 = \frac{25}{64}$$

$\triangle MNP \sim \triangle RSP$

$$\frac{\mathcal{A}\triangle MNP}{\mathcal{A}\triangle RSP} = \left(\frac{MP}{PR}\right)^2 = \left(\frac{KP}{QP}\right)^2 = \left(\frac{5}{8}\right)^2 = \frac{25}{64}$$

$$\frac{\mathcal{A}KMNP}{\mathcal{A}QRSP} = \frac{\mathcal{A}\triangle MNP + \mathcal{A}\triangle KMP}{\mathcal{A}\triangle QRP + \mathcal{A}\triangle RSP} = \frac{25}{64} \quad \text{(Theorem 8-1.4)}$$

32. Since $\triangle ADE \sim \triangle AFG \sim \triangle ABC$ (Theorem 8-6.1)
$\mathcal{A}DEFG = 3/4\ \mathcal{A}\triangle AFG$
$\mathcal{A}\triangle AFG = 4/9\ \mathcal{A}\triangle ABC$
$\mathcal{A}DEFG = \frac{3}{4} \cdot \frac{4}{9}\mathcal{A}\triangle ABC = \frac{1}{3}\mathcal{A}\triangle ABC$

33. $\triangle ADE \sim \triangle ABC \sim \triangle EFC$ (Corollary 8-5.1c)
AD/AB = ¼, Thus EC/AC = 3/4 (Corollary 8-2.1a)
$\mathcal{A}\triangle ADE/\mathcal{A}\triangle ABC = 1/16$ (Postulate 2-1)
$\mathcal{A}\triangle ADE = (1/16)\mathcal{A}\triangle ABC$, and $\mathcal{A}\triangle EFC = (9/16)\ \mathcal{A}\triangle ABC$ (Multiplication property)
$\mathcal{A}$ parallelogram ABCD = $(6/16)\mathcal{A}\triangle ABC = (3/8)\ \mathcal{A}\triangle ABC$ (Postulate 2-1)

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34. $m\angle A = (\frac{1}{2})m\ \overparen{TB}$ (Theorem 9-6.1)
$m\angle BTP = (\frac{1}{2})\ m\ \overparen{TB}$ (Theorem 9-8.1)
$\triangle ATP \sim \triangle TBP$ (Corollary 8-5.1a)
$\mathcal{A}\triangle ATP/\mathcal{A}\triangle TBP = TP^2/BP^2$ (Theorem 12-6.1)
$\mathcal{A}\triangle ATP/\mathcal{A}\triangle TBP = (AP \cdot BP)/BP^2 = AP/BP$ (Postulate 2-1)

Review Exercises

1. $\mathcal{A} = bh$
$\mathcal{A} = (3\sqrt{2})(5\sqrt{3}) = 15\sqrt{6}$

2. $\mathcal{A} = \frac{1}{2}d^2$
$\mathcal{A} = \frac{1}{2}(6)^2 = 18$

3. $\mathcal{A} = bh$
$b' = 2b$
$h' = h/2$

$\mathcal{A}' = b'\ h' = (2b)\ (h/2) = bh$
Therefore $\mathcal{A} = \mathcal{A}'$

4. $\mathcal{A} = bh$
$b' = 3b$
$h' = 3h$
$\mathcal{A}' = b'\ h' = (3b)(3h) = 9bh$
Therefore $\mathcal{A}' = 9\mathcal{A}$

5. $\mathcal{A} = bh$
$b' = 4b$
$h' = h$
$\mathcal{A}' = b'h' = (4b)(h) = 4bh$
$\mathcal{A} = 4\mathcal{A}'$

6. $\angle GEB \cong \angle GBE$ (Theorem 3-4.2)
$\overline{CB} \cong \overline{DE}$ (Theorem 3-4.2)
$\triangle ABC \cong \triangle FED$ (ASA)
$\mathcal{A}\triangle ABC = \mathcal{A}\triangle FED$ (ASA)
$\mathcal{A}$ quadrilateral AGEC + $\mathcal{A}\triangle EGB$ = $\mathcal{A}$ quadrilateral FGBD + $\mathcal{A}\triangle EGB$ (Addition property)
$\mathcal{A}$ quadrilateral AGEC = $\mathcal{A}$ quadrilateral FGBD (Subtraction property).

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7. $\mathcal{A} = \frac{1}{2}(8)(6) = 24$

8. $\mathcal{A} = \frac{1}{2}\ bh$
$38\sqrt{2} = \frac{1}{2}b\ (6)$

$b = \frac{38\sqrt{2}}{3}$

9. $\mathcal{A} = \frac{1}{2}(4)(6)\sin 45 = 6\sqrt{2}$

10. $\mathcal{A} = \frac{s^2\sqrt{3}}{4}$

$\mathcal{A} = \frac{(8x)^2\sqrt{3}}{4}$
$\mathcal{A} = 16x^2\sqrt{3}$

11. $\mathcal{A} = \frac{h^2\sqrt{3}}{3}$ (Theorem 12-2.7)

$\mathcal{A} = \frac{(6)^2\sqrt{3}}{3} = 12\sqrt{3}$

12. AD = 4, BE = 3, AC = 8
$\mathcal{A}\triangle ABC = \frac{1}{2}(AC)(BE) = \frac{1}{2}(BC)(AD)$
$\frac{1}{2}(8)(3) = \frac{1}{2}(BC)(4)$
BC = 6

13. Use the result of Exercise 44 on page 513, after drawing $\overline{CG}$.

14. $\mathcal{A} = bh$
$48 = (8\sqrt{2})h$
$h = \frac{6}{\sqrt{2}} = 3\sqrt{2}$

15. $\mathcal{A} = \frac{1}{2}(13)(7) = 91/2$

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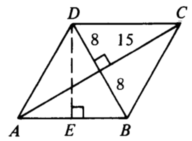
Review Exercises continued

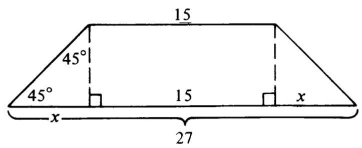
16.



Area = ½(AC)(BC) = 240
$(AB)^2 = (15)^2 + (8)^2$
AB = 17
Area = (AB)(DE)
240 = 17(DE)
DE = 240/17

17. $\mathcal{A} = \frac{1}{2}(6)(12 + 17) = 87$

18.

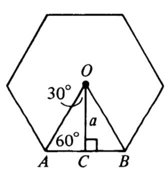


$x + 15 + x = 27$
$x = 6 = h$
$\mathcal{A} = \frac{1}{2}(6)(15 + 27) = 126$

19. Draw $\overline{QF} \perp \overline{AFB}$ and $\overline{PE} \perp \overleftrightarrow{BC}$ at F.
$\mathcal{A}\triangle AQB = (\frac{1}{2})(QF \cdot AB)$ (Theorem 12-2.2)
$\mathcal{A}$ parallelogram ABCD = QF • AB (Theorem 12-3.1)
$\mathcal{A}\triangle AQB = (\frac{1}{2})\ \mathcal{A}$ parallelogram ABCD (Postulate 2-1)
Similarly, $\mathcal{A}\triangle BCP = (\frac{1}{2})(PE \cdot BC)$, and $\mathcal{A}$ parallelogram ABCD = PE • BC
$\mathcal{A}\triangle BCP = (\frac{1}{2})\ \mathcal{A}$ parallelogram ABCD (Postulate 2-1)
$\mathcal{A}\triangle AQB = \mathcal{A}\triangle BCP$ (Transitive property).

20. $\mathcal{A} = \frac{1}{2}ap$
$\mathcal{A} = \frac{1}{2}(8)(30) = 120$

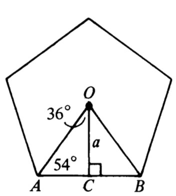
21.



m∠ AOB = 360/6 = 60
m∠ AOC = 30
AC = CB = $5\sqrt{2}$
$a = (AC)\sqrt{3} = (5\sqrt{2})(\sqrt{3}) = 5\sqrt{6}$
$p = 6(AB) = 60\sqrt{2}$
$\mathcal{A} = \frac{1}{2}ap$
$\mathcal{A} = \frac{1}{2}(5\sqrt{6})(60\sqrt{2}) = 150\sqrt{12} = 300\sqrt{3}$

22. $\mathcal{A} = \frac{1}{2}ap$
$60 = \frac{1}{2}a(5\sqrt{2})$
$a = \dfrac{120}{5\sqrt{2}} = 12\sqrt{2}$

23.



m∠ AOB = 360/5 = 72
m∠ AOC = 36
$p = 5(AB) = 250$
tan 54 = $a/25$
a = 25 tan 54

(*continued*)

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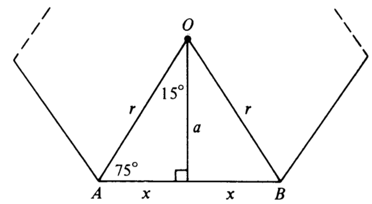
23. *continued*

$\mathcal{A} = \frac{1}{2}ap$
$\mathcal{A} = \frac{1}{2}(25 \tan 54)(250) = (25)(125)(1.3764) = 4301$

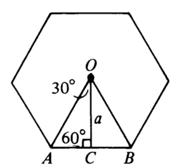
24.



m∠ AOB = 360/12 = 30
m∠ AOC = 15
AC = CB = x
$p = 12(AB) = 24x$
$\sin 15 = x/r$; $x = r \sin 15$
$\sin 75 = a/r$; $a = r \sin 75$
$\mathcal{A} = \frac{1}{2}ap$
$72 = \frac{1}{2}(r \sin 75)(24x)$
$72 = 12rx \sin 75$
$72 = 12r(r \sin 15)(\sin 75)$
$6 = r^2 \sin 15 \sin 75$

$$r = \sqrt{\frac{6}{\sin 15 \sin 75}} = \sqrt{\frac{6}{(.2588)(.9659)}} = \sqrt{6/.25} = \sqrt{24} \approx 5.$$

25.



m∠ AOB = 360/6 = 60
m∠ AOC = 30
AC = CB = $s/2$
$p = 6(AB) = 6s$
$a = \frac{s}{2}\sqrt{3}$
$\mathcal{A} = \frac{1}{2}ap$
$\mathcal{A} = \frac{1}{2}\ \frac{s\sqrt{3}}{2} \cdot 6s$

$\mathcal{A} = \dfrac{3s^2\sqrt{3}}{2}$

26. $\mathcal{A} = \pi r^2$
$60 = \pi r^2$
$r = \sqrt{60} = 2\sqrt{15}$
$d = 2r = 4\sqrt{15}$

27. Area of sector $= \dfrac{n}{360} \cdot \pi r^2$

$= \dfrac{40}{360} \cdot 100\pi = \dfrac{100\pi}{9}$

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28. Area of sector $= \dfrac{n}{360} \cdot \pi r^2$

$72\pi = \dfrac{270}{360} \cdot \pi r^2$
$r^2 = 4(24)$

$r = \sqrt{4(24)} = 4\sqrt{6}$

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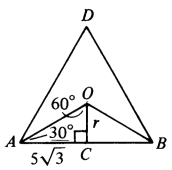
Review Exercises continued

29. $h = \frac{s\sqrt{3}}{2}$

$15 = \frac{s\sqrt{3}}{2}$

$s = 10\sqrt{3}$
$\overline{OC}$ is radius of inscribed circle.



$m\angle AOB = 360/3 = 120$
$m\angle AOC = 60$
$5\sqrt{3} = r\sqrt{3}$
$r = 5$
$\mathcal{A} = \pi r^2$; $\mathcal{A} = 25\pi$

30. Area of sector $= \frac{45}{360} \cdot \pi(16)^2 = 32\pi$
Area $\triangle AOB = \frac{1}{2}(16)(16) \sin 45 = 64\sqrt{2}$
Area of segment = Area of sector - Area of triangle
Area of segment $= 32\pi - 64\sqrt{2}$

31. $\mathcal{A}$ Sector (with radius CQ = 8) $= \frac{80}{360} \cdot \pi(8)^2 = \frac{128\pi}{9}$

$\mathcal{A}'$ Sector (with radius AQ = 4) $= \frac{80}{360} \cdot \pi(4)^2 = \frac{32\pi}{9}$

$\mathcal{A} - \mathcal{A}' = \frac{128\pi}{9} - \frac{32\pi}{9} = \frac{96\pi}{9} = \frac{32\pi}{3}$.

32. $m\angle C = 90$
$(AB)^2 = (12)^2 + (5)^2$; $AB = 13$; $BQ = 13/2$
$\mathcal{A}$ of semicircle $Q = \frac{1}{2}\pi(BQ)^2 = \frac{1}{2}\pi\left(\frac{13}{2}\right)^2 = \frac{169\pi}{8}$

$\mathcal{A}'$ of $\triangle ABC = \frac{1}{2}(12)(5) = 30$
$\mathcal{A}''$ of shaded region $= \mathcal{A} - \mathcal{A}' = \frac{169\pi}{8} - 30$

33. $\mathcal{A}_1/\mathcal{A}_2 = (s_1/s_2)^2$

$1/9 = (s_1/s_2)^2$

$1/3 = s_1/s_2$

34. $\mathcal{A}_1/\mathcal{A}_2 = (h_1/h_2)^2$

$\mathcal{A}_1/\mathcal{A}_2 = (9/16)^2 = 81/256$

35. $\mathcal{A}_1/\mathcal{A}_2 = (a_1/a_2)^2$

$16/\mathcal{A}_2 = (2/7)^2$

$16/\mathcal{A}_2 = 4/49$

$\mathcal{A}_2 = 196$

36. $\mathcal{A}_1/\mathcal{A}_2 = (M_1/M_2)^2$

$20/45 = (14/M_2)^2$

$2/3 = 14/M_2$
$M_2 = 21$

37. $\frac{\mathcal{A}'}{\mathcal{A}} = \left(\frac{c'}{c}\right)^2 = \left(\frac{9}{25}\right)^2 = \frac{81}{625}$

38. Let $x = d_1$
$3x = d_2$

$\frac{\mathcal{A}_1}{\mathcal{A}_2} = \left(\frac{d_1}{d_2}\right)^2$

$\frac{\mathcal{A}_1}{\mathcal{A}_2} = \left(\frac{1x}{3x}\right)^2 = \frac{1}{9}$

$\mathcal{A}_2 = 9\mathcal{A}_1$

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39. $\triangle KMN \sim \triangle ABC$ (Theorem 8-6.2)
$KM = \frac{1}{2}(AB)$ (Theorem 7-6.3)

$\frac{\mathcal{A}\triangle KMN}{\mathcal{A}\triangle ABC} = \left(\frac{KM}{AB}\right)^2 = \left[\frac{1(KM)}{2(KM)}\right]^2 = (\frac{1}{2})^2 = \frac{1}{4}$

Chapter Test

1. True

2. False; $\mathcal{A} = \frac{1}{2} d^2$

3. False; it is quadrupled if the resulting pentagon is regular.

4. True.

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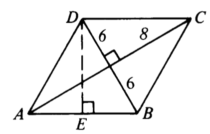
5. $\mathcal{A} = \frac{1}{2} d^2$

$\mathcal{A} = \frac{1}{2}\left(\frac{5}{2}\right)^2 = \frac{25}{8}$

6.



$\mathcal{A} = \frac{1}{2}(AC)(BD)$
$\mathcal{A} = \frac{1}{2}(16)(12) = 96$
$(AB)^2 = (8)^2 + (6)^2$; $(AB)^2 = 100$; $AB = 10$
$\mathcal{A} = (AB)(DE)$

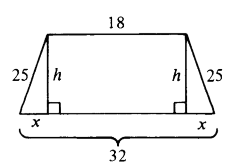
$96 = (10)(DE)$
$DE = 9.6$

7. $\mathcal{A} = \frac{h^2\sqrt{3}}{3}$ (Theorem 12-2.7)

$9\sqrt{3} = \frac{h^2\sqrt{3}}{3}$
$h^2 = 27$
$h = 3\sqrt{3}$

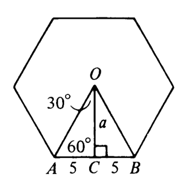
8. $\mathcal{A} = \frac{1}{2}(4)(5)(\sin 30)$
$\mathcal{A} = 10(\frac{1}{2}) = 5$

9.



$x + 18 + x = 32$
$x = 7$
$(25)^2 = x^2 + h^2$
$h = 24$
$\mathcal{A} = \frac{1}{2}(24)(18 + 32)$
$\mathcal{A} = 600$

10.



$m\angle AOB = 360/6 = 60$
$m\angle AOC = 30$
$a = 5\sqrt{3}$
$p = 6(AB) = 60$
$\mathcal{A} = \frac{1}{2} ap$
$\mathcal{A} = 150\sqrt{3}$

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Chapter Test continued

11. Area of sector $= \frac{n}{360} \cdot \pi r^2$

$12\pi = \frac{n}{360} \cdot 16\pi$

$n = 270$

12. $\left(\frac{\mathcal{A}_1}{\mathcal{A}_2}\right) = \left(\frac{s_1}{s_2}\right)^2$

$\frac{\mathcal{A}_1}{48} = \left(\frac{5}{20}\right)^2 = \frac{1}{16}$

$\mathcal{A}_1 = 3$

13. $16\pi - 4\pi = 12\pi$

14. $\mathcal{A}$ Semicircle P $= \frac{1}{2}\pi\,(3)^2 = \frac{9\pi}{2}$

$\mathcal{A}$ Semicircle R $= \frac{1}{2}\pi\,(3)^2 = \frac{9\pi}{2}$

$\mathcal{A}$ Semicircle (with $\overline{AB}$ as diameter) $= \frac{1}{2}\pi(9)^2 = \frac{81\pi}{2}$

$\mathcal{A}$ Semicircle Q (with diameter 6) $= \frac{1}{2}\pi(3)^2 = \frac{9\pi}{2}$

Area of shaded $= \frac{9\pi}{2} + \frac{9\pi}{2} + \left[\frac{81\pi}{2} - \frac{9\pi}{2}\right] = 45\pi$

15. $\triangle ABQ$ is equilateral, $m\angle Q = 60$, $r = 24 = s$

Area of sector $= \frac{60}{360} \cdot \pi(24)^2 = 96\pi$

$\mathcal{A}\triangle ABQ = \frac{(24)^2\sqrt{3}}{2} = 144\sqrt{3}$

Area of segment $= 96\pi - 144\sqrt{3}$

16. See solution to Exercise 37 on page 512

17. $\triangle BDM \sim \triangle BAC \sim \triangle MEC$ (Corollary 8-5.1c)
$\mathcal{A}\triangle BDM / \mathcal{A}\triangle BAC = \frac{1}{4}$ (Theorem 12-6.1)
$\mathcal{A}\triangle BDM = (\frac{1}{4})\ \mathcal{A}\triangle BAC$ (Multiplication property)
Similarly, $\mathcal{A}\triangle MEC = (\frac{1}{4})\ \mathcal{A}\triangle BAC$
$\mathcal{A}\triangle BDM + \mathcal{A}\triangle MEC = (\frac{1}{2})\ \mathcal{A}\triangle BAC$ (Addition property)
$\mathcal{A}$ parallelogram ADME $= (\frac{1}{2})\ \mathcal{A}\triangle ABC$ (Postulate 12-3)

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Class Exercises

1. Yes; No
2. 60, 90, 108, 120.
3. Yes; no; no.
4. Yes.
5. No.
6. Exercises 1, 3, and 4 yield triangular faces; Exercise 5 shows there are no more.
7. Exercise 1 yields square faces; Exercise 3 shows there are no more.
8. Exercise 1 yields pentagonal faces; Exercise 3 shows there are no more.

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Exercises

1. No; Theorem 13-1.2
2. None
3. $30 + 70 > x$; therefore $x < 100$
   $30 + x > 70$; therefore $x > 40$
4. Yes, Theorem 13-1.1, Theorem 13-1.2
5. No, Theorem 13-1.2 violated
6. No, Theorem 13-1.1 violated
7. A line formed by the intersection of planes which bisect the dihedral angles.

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8. We have 4 congruent triangles for lateral faces.
   Area of square $= 6^2 = 36$
   Area of one lateral face $= \frac{1}{2}(4)(6) = 12$
   $4(12) = 48$
   Total area $= 36 + 48 = 84$

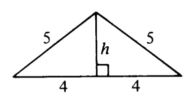
9. We have 4 congruent equilateral triangles, one is a base and three are lateral faces.

   Area of an equilateral triangle $= \frac{s^2\sqrt{3}}{4} = \frac{(4)^2\sqrt{3}}{4} = 4\sqrt{3}$;

   Area of base or one lateral face.
   Total area $= 4(4\sqrt{3}) = 16\sqrt{3}$.

10. Draw $\overline{AC}$ (Postulate 2-3)
   In face AVC, construct $\angle AVD \cong \angle AVB$ (Postulate 2-9)
   Locate B on $\overrightarrow{VB}$ such that $\overline{VB} \cong \overline{VD}$ (Postulate 1-2)
   $\triangle VAD \cong \triangle VAB$ (SAS)
   $AB + BC > AC$ (Theorem 5-4.1)
   $BC > DC$ (Subtraction property)
   $m\angle BVC > m\angle DVC$ (Theorem 5-5.2)
   $m\angle AVB + m\angle BVC > m\angle AVD + m\angle DVC$ (Addition property)
   $m\angle AVB + m\angle BVC > m\angle AVC$ (Postulate 2-1).

11. We have a square base and 4 congruent triangles for lateral faces.
   Area of square $= s^2 = 64$



   $(5)^2 = h^2 + (4)^2$; and $h = 3$
   Area of one lateral face $= \frac{1}{2}(8)(h) = 12$
   Area of the 4 lateral faces $= 4(12) = 48$
   Total area $= 64 + 48 = 112$

12. Three planes. Two planes intersect in a line. The line intersects the third plane in a point.

13. Yes; when the plane of the perpendicular lines is parallel to the other plane.
   No; Theorem 4-5.6.

14. No, three are required for a polyhedral angle.

15. Let plane AVX bisect B-AV-C, and plane CVY bisect B-CV-A.
   Plane AVX intersects plane CVY in $\overleftrightarrow{VZ}$ (Postulate 2-6)
   $\overline{VZ}$ is equidistant from planes AVC and AVB, and is on the bisector of C-BV-A (Definition 4-14).

16. Measure the same distance from the vertex on each edge. Planes perpendicular to the edges will determine congruent triangles on the faces. Then, triangles on the perpendicular planes are congruent.

17. Not if two are in the same plane containing the flagpole.

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Class Exercises

1. Volume of a single card multiplied by the number of cards.
2. Unchanged.
3. They are equal.
4. If and only if the cards have the same area as in the first deck.

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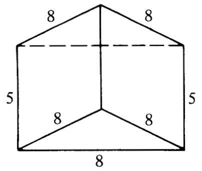
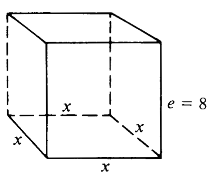
Exercises

1. Base is an equilateral triangle with side 8; lateral edge = 5 = e



lateral area = eP, P = 3(8) = 24
lateral area = 5(24)
lateral area = 120

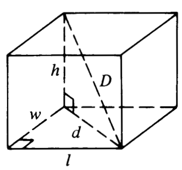
2.



lateral area = eP
96 = 8(4x)
x = 3
$\mathcal{A} = x^2 = 9$

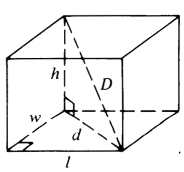
3. lateral area = eP
360 = e(30)
e = 12

4.



$e = l = w = h$
$d^2 = l^2 + w^2$ (Theorem 8-8.1)
$D^2 = d^2 + h^2$ (Theorem 8-8.1)
$D^2 = l^2 + w^2 + h^2$ (Substitution postulate)
$D^2 = e^2 + e^2 + e^2$ (Substitution postulate)
$D = e\sqrt{3}$; $D = 4\sqrt{3}$

5.



There are 6 congruent squares.
e^2 = area of each square
Total area = $6e^2$
$D = e\sqrt{3}$
$8\sqrt{3} = e\sqrt{3}$
$e = 8$
Total area = $6(8)^2 = 384$

6. Total area = $2(lw + lh + wh)$
= 2(150 + 120 + 80) = 2(350) = 700

7. Let B = area of base
$\mathcal{V} = Bh$
$\mathcal{V} = (lw)\,h$
$\mathcal{V} = (9 \cdot 10)(15) = 1350$

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8. Let $x = e_1$
$3x = e_2$

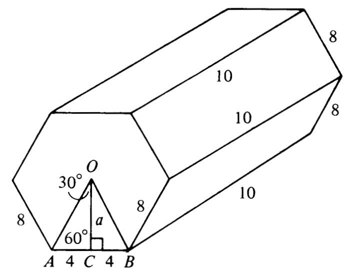
$\mathcal{A}/\mathcal{A}' = (e/e')^2$
$\mathcal{A}/\mathcal{A}' = (1x/3x)^2$
$\mathcal{A}/\mathcal{A}' = 1/9$
$\mathcal{A}_2 = 9\mathcal{A}_1$

$\mathcal{V}/\mathcal{V}' = (e/e')^3$
$\mathcal{V}/\mathcal{V}' = (1x/3x)^3$
$\mathcal{V}/\mathcal{V}' = 1/27$
$\mathcal{V}_2 = 27\mathcal{V}_1$

9. $\mathcal{V} = Bh$
$\mathcal{V} = (lw)h$
$\mathcal{V} = (8 \cdot 12)(20) = 1920$

10. lateral area = eP, P = 3 + 4 + 4 + 5 + 6 + 7 = 29
e = 7
lateral area = 7(29) = 203

11. We have two congruent regular hexagonal bases.
We have six congruent rectangles for lateral faces.



m∠ AOB = 360/6 = 60
m∠ AOC = 30
$a = 4\sqrt{3}$
$\mathcal{A}_1 = \frac{1}{2}a$P, P = 6(AB) = 48
$\mathcal{A}_1 = \frac{1}{2}(4\sqrt{3})(48)$
$\mathcal{A}_1 = 96\sqrt{3}$

$\mathcal{A}_2 = 10(8) = 80$
$6\mathcal{A}_2 = 6(80) = 480$ area of the six lateral faces

$2\mathcal{A}_1 = 192\sqrt{3}$ (which is the area of top and bottom faces)
Total area = $2\mathcal{A}_1 + 6\mathcal{A}_2 = 192\sqrt{3} + 480$

12. $\mathcal{V} = Bh$
$\mathcal{V} = (lw)h$
$\mathcal{V} = (25)(18)(12) = 5400$

13. $\mathcal{V} = Bh$
$50 = 10h$
$h = 5$

14. Total area = $6e^2$
$e^3 = 6e^2$
$e^3 - 6e^2 = 0$
$e^2(e-6) = 0$
$e^2 = 0$ (reject)
$e - 6 = 0$
$e = 6$

$\mathcal{V} = B$
$\mathcal{V} = lwh$
$\mathcal{V} = e \cdot e \cdot e$
$\mathcal{V} = e^3$

15. $\mathcal{V} = Bh = 25(10) = 250$

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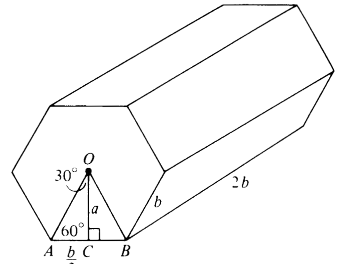
16.



We have 2 congruent bases
We have 6 congruent lateral faces
m∠ AOB = 360/6 = 360
m∠ AOC = 30

(continued)

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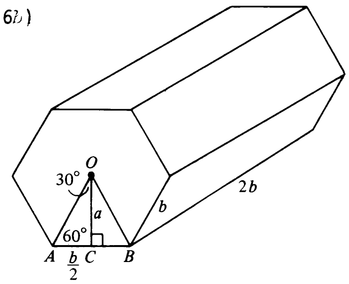
16. *continued*

$a = \frac{b}{2}\sqrt{3}$; P = 6(AB) = 6b

$\mathcal{A}_1 = \frac{1}{2}\left(\frac{b\sqrt{3}}{2}\right)(6b)$

$2\mathcal{A}_1 = 3b^2\sqrt{3}$



$\mathcal{A}_2 = 2b^2$ area of one lateral face

$6\mathcal{A}_2 = 12b^2$

Total area $= 2\mathcal{A}_1 + 6\mathcal{A}_2$

Total area $= 3b^2\sqrt{3} + 12b^2$

Total area $= 3(4 + \sqrt{3})b^2$

17. Let B = area of base

B = $\frac{1}{2}(a)(a)$

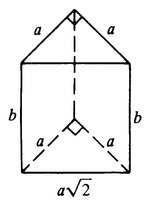
B = $\frac{a^2}{2}$

$\mathcal{V}$ = Bh

$\mathcal{V} = \frac{a^2}{2} \cdot b$

$\mathcal{V} = \frac{1}{2}a^2b$

18.



We have 2 congruent bases.

$\mathcal{A}_1 = \frac{1}{2}a \cdot a = \frac{a^2}{2}$

$2\mathcal{A}_1 = a^2$

$\mathcal{A}_2 = (a\sqrt{2})(b) = ab\sqrt{2}$

$\mathcal{A}_3 = ab$

$2\mathcal{A}_3 = 2ab$

Total area $= 2\mathcal{A}_1 + \mathcal{A}_2 + 2\mathcal{A}_3$

Total area $= a^2 + ab\sqrt{2} + 2ab$

19. Let plane P intersect parallel planes P_1 and P_2.
Planes P and P_1 intersect in line l, P and P_2 intersect in line m (Postulate 2-6); l and m lie in plane P.
Assume they intersect; then P_1 and P_2 are not parallel, contrary to what is given.
l and m are parallel (Definition 6-1).

20. Let P_1 and P_2 be parallel such that $l \perp P_1$.
Choose a point K in P_1 and not in l.
l and K determine a third plane, P (Theorem 2-5.1)
Plane P intersects P_1 and P_2 in lines m and n (Postulate 2-6)
$m \parallel n$ (Theorem 13-2.1)
$l \perp P_1$ (Definition 4-7)
$l \perp m$ (Definition 4-7)
$l \perp n$ (Corollary 6-1.1b)
Choose another point in P_1 not in the plane of K and l.
Repeating the above argument, we have a second line in P_2.
$l \perp P_2$ (Theorem 4-5.1).

21. Let planes P_1 and P_2 be perpendicular to l at R and S, respectively.
Assume P_1 and P_2 intersect in m (Postulate 2-6)
Choose any point A of m such that A is not in l.

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21. *continued*

$l \perp \overleftrightarrow{RA}$ (Definition 4-7)
$l \perp \overline{SA}$ (Definition 4-7)
There is only one perpendicular from A to l (Theorem 4-4.4).
Contradiction; $P_1 \parallel P_2$.

22. Let plane P be parallel to planes P_1 and P_2
Consider $l \perp P$.
$l \perp P_1$ (Theorem 13-2.2)
$l \perp P_2$ (Theorem 13-2.2)
Plane $P_1 \parallel$ plane P_2 (Theorem 13-2.3).

23. $l \perp P$ and $m \perp P$ (Given)
l and m are coplanar (Theorem 4-5.8)
Assume l and m intersect in A.
Then there are two lines from A perpendicular to plane P, contradicting Theorem 4-5.6.
$l \parallel m$.

24. $l \parallel m$, and plane $P \perp l$ (Given)
Consider any point M of m; through M, $n \perp P$
$l \parallel n$ (Theorem 13-2.4)
There is only one line parallel to l through M (Postulate 6-1)
m and n coincide, and $m \perp P$.

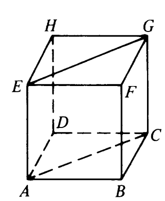
25. Plane $P_1 \parallel$ plane P_2 (Given)
Let Q and R be distinct points of P_1.
If S and T are the respective projections of Q and R onto P_2 (Definition 4-6)
$\overleftrightarrow{QS} \perp P_2$ (Definition 4-6)
$\overleftrightarrow{RT} \perp P_2$ (Definition 4-6)
$\overleftrightarrow{QS} \parallel \overleftrightarrow{RT}$ (Theorem 13-2.4)
The plane determined by $\overleftrightarrow{QS}$ and $\overleftrightarrow{RT}$ intersects P_1 and P_2 in $\overleftrightarrow{QR}$ and $\overleftrightarrow{ST}$ (Theorem 13-2.1)
$\overleftrightarrow{QR} \parallel \overleftrightarrow{ST}$ (Theorem 13-2.1)
Quadrilateral QRST is a parallelogram (Definition 7-1)
$\overline{QS} \cong \overline{RT}$ (Theorem 7-1.2).

26. The lateral edges of a prism are parallel (Theorem 13-2.6)
The plane of a right section of a prism is perpendicular to a lateral edge (Definition 13-9)
If a plane is perpendicular to one of two parallel lines, it is perpendicular to the other (Theorem 13-2.2)
The plane of a right section is perpendicular to each lateral edge (Postulate 2-1).

27. The base of a right prism is perpendicular to its lateral edges (Definition 13-10).
The base of a right prism is a right section of the prism (Definition 13-9)
Each lateral edge of a right prism is an altitude of the prism (Definition 13-8)
The lateral area of a right prism is equal to the product of the perimeter of the base and its altitude (Postulate 2-1, Theorem 13-2.7).

28. Given prism P with base ABCDE ... , and section FGHIJ... parallel to the base
$\overline{AF} \parallel \overline{GB}$ (Theorem 13-2.6)
$\overline{AB} \parallel \overline{FG}$ (Theorem 13-2.1)
Quadrilateral ABFG is a parallelogram (Definition 7-1)
$\overline{AB} \cong \overline{FG}$ (Theorem 7-1.2)
Similarly, the remaining pairs of sides of the base and section are parallel and congruent
$\overline{EB} \parallel \overline{JG}$
$\overline{EB} \cong \overline{JG}$
$\triangle EAB \cong \triangle JFG$ (SAS)
$\angle A \cong \angle F$ (Definition 3-3)
Similarly, the remaining pairs of angles are congruent
Base ABCDE ... ≅ section FGHIJ ... (Definition 3-3).

29.



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Exercises continued

29. *continued*

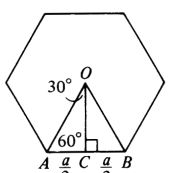
Parallelogram ABCD ≅ parallelogram EFGH (Definition 13-12)
$\triangle$EFG ≅ $\triangle$GHE (Theorem 7-1.1)
$\triangle$ABC ≅ $\triangle$CDA (Theorem 7-1.1)
Quadrilateral ACGE is a parallelogram (Theorem 7-2.2)
$\overline{EG} \parallel \overline{AC}$ (Theorem 7-1.2)
$\overline{EG} \cong \overline{AC}$ (Theorem 7-1.2)
$\triangle$ABC ≅ $\triangle$EFG (SSS)
$\triangle$ADC ≅ $\triangle$EGH (SSS)
ABCEFG and ACDEGH are prisms with congruent bases and the same altitude (Definition 13-7)
$\mathcal{V}$ prism ABCEFG = $\mathcal{V}$ prism ACDEGH (Theorem 13-2.8)

30. Consider a prism with base B and altitude h.
Construct a rectangular parallelepiped with base B' and altitude h' such that $\mathcal{A}$B = $\mathcal{A}$B', and $h = h'$.
For either figure, any section parallel to the base is congruent to the base (Corollary 13-2.7b)
They have equal volumes (Postulate 13-3)
$\mathcal{V}$ rectangular parallelepiped = $\mathcal{A}$B' • h'
$\mathcal{V}$ prism = $\mathcal{A}$B • h (Postulate 2-1).

31. A parallelepiped is a prism whose bases are parallelograms (Definition 13-12)
$\mathcal{V}$ prism = $\mathcal{A}$B • h (Theorem 13-2.9)
Any face of a parallelepiped may be the base of a prism (Definition 13-12)
$\mathcal{V}$ parallelepiped = $\mathcal{A}$ face • h (Postulate 2-1).

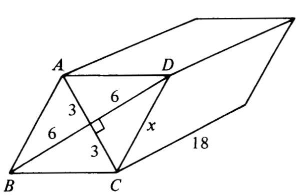
32. $\mathcal{V}$ = 10(35) = 350 cubic feet
$\mathcal{V} = Bh$
$350 = (lw)(h)$
$350 = (10)(8)h$
$h = 4\frac{3}{8}$ feet

33.



m∠ AOB = 360/6 = 60
m∠ AOC = 30
$OC = \frac{a}{2}\sqrt{3}$
P = 6(AB) = $6a$
Let $\mathcal{A}$ = area of base
$\mathcal{A} = \frac{1}{2}aP$;
$\mathcal{A} = \frac{1}{2}(\frac{a}{2}\sqrt{3})\ (6a)$
$\mathcal{A} = \frac{3a^2\sqrt{3}}{2}$
$\mathcal{V} = Bh$
$\mathcal{V} = \frac{3a^2\sqrt{3}}{2} \cdot b = \frac{3\sqrt{3}}{2}\ a^2b$

34. We have 2 congruent bases
We have 4 congruent rhombuses for lateral faces



$\mathcal{A}_1 = \frac{1}{2}(AC)(BD)$
$\mathcal{A}_1 = \frac{1}{2}(6)(12) = 36$
$2\mathcal{A}_1 = 72$
$\mathcal{V} = \mathcal{A}_1 h$
$\mathcal{V} = 36(18) = 648$
$x^2 = (6)^2 + (3)^2$; $x = 3\sqrt{5}$
$\mathcal{A}_2 = 18x = 54\sqrt{5}$
$4\mathcal{A}_2 = 216\sqrt{5}$
Total area = $2\mathcal{A}_1 + 4\mathcal{A}_2 = 72 + 216\sqrt{5}$

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35. Total area = $6e^2$
$36 = 6e^2$
$e = \sqrt{6}$
$D = e\sqrt{3}$
$D = \sqrt{18} = 3\sqrt{2}$

36. $\mathcal{V} = e^3$
$729 = e^3$
Total area = $6e^2$ = 486
$D = e\sqrt{3} = 9\sqrt{3}$

37. Refer to the figure for Exercise 29.
Parallelogram ABCD ≅ parallelogram EFGH
Plane ABCD // EFGH (Definition 13-12)
$\overline{AB} \parallel \overline{GH}$ (Corollary 6-1.1c)
$\overline{AB} \cong \overline{GH}$ (Theorem 3-1.8)
ABGH is a parallelogram (Theorem 7-2.2)
Diagonals $\overline{AG}$ and $\overline{BH}$ bisect each other (Theorem 7-1.5)
Similarly, any other pair of diagonals of the parallelepiped bisect each other.

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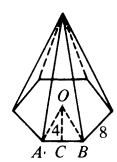
Class Exercises

1. Lateral faces are congruent isosceles triangles
Let S = slant height
Let P = perimeter of base
lateral area = $\frac{1}{2}$SP
lateral area = $\frac{1}{2}(10)(48) = 240$



m∠ AOB = 360/6 = 60
m∠ AOC = 30
$a = 4\sqrt{3}$; and P = 6(AB) = 48
$\mathcal{A}_1 = \frac{1}{2}a$P
$\mathcal{A}_1 = \frac{1}{2}(4\sqrt{3})(48) = 96\sqrt{3}$
Total area = lateral area + $\mathcal{A}_1$
Total area = $240 + 96\sqrt{3}$

2. $\mathcal{V} = \frac{1}{3}Bh$
$240 = \frac{1}{3}(120)h$
$h = 6$

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Class Exercises continued

3. We have 6 congruent pyramids, each base is a square. Each pyramid will have congruent altitudes drawn from the center of the cube, perpendicular to each congruent square base.

$\mathcal{V} = e^3$ of the cube
$\frac{\mathcal{V}}{6}$ = volume of each congruent pyramid
$\frac{\mathcal{V}}{6} = \frac{e^3}{6}$

4. There are 5 congruent isosceles triangles with base 5 and altitude 10.
$\mathcal{A} = \frac{1}{2}(5)(10) = 25$
Lateral area = $5\mathcal{A} = (5)(25) = 125$

5. Volume of pyramid = $\frac{1}{3}Bh = \frac{32}{3}$

Volume of cube = $e^3 = \frac{32}{3}$; then $e = \sqrt[3]{\frac{32}{3}} = \frac{2\sqrt[3]{36}}{3}$

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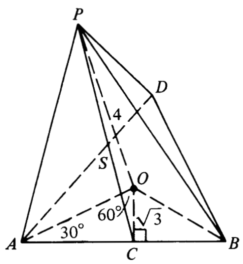
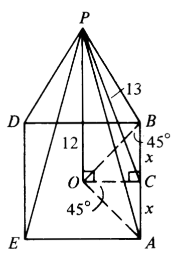
Exercises

1. 

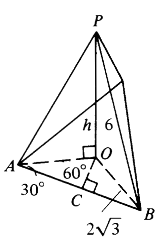
$m\angle AOB = 360/3 = 120$
$m\angle AOC = 60$
$AC = (OC)\sqrt{3}$
$3 = (OC)\sqrt{3}$
$OC = \sqrt{3}$

Let s = slant height
$s^2 = (4)^2 + (\sqrt{3})^2$
$s = \sqrt{19}$
lateral area = $\frac{1}{2} sP$; where $P = 3(AB) = 18$
lateral area = $\frac{1}{2}(\sqrt{19})(18) = 9\sqrt{19}$

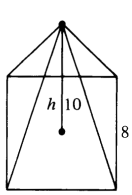
2. Let B = area of base
$B = \frac{1}{2}(3)(5) = 15/2.$
$\mathcal{V} = \frac{1}{3}Bh$
$\mathcal{V} = \frac{1}{3}(15/2)(4)$
$\mathcal{V} = 10$

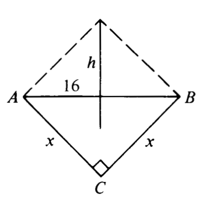
3. 

Applying Theorem 8-8.1 to $\triangle POC$, $OC = 5$.
$BC = 5$ since $\triangle OCB$ is isosceles.
Therefore $AB = 10$
The area of square base ABDE = $(10)(10) = 100$
$\mathcal{V} = \frac{1}{3}Bh = \frac{1}{3}(100)(12) = 400.$

4. 

Let O be the point of intersection of the altitudes of the base,
$m\angle OAC = 30$, and $m\angle AOC = 60$
Since $AC = CB$, and $AB = 6$, we find $CB = 3$.
Therefore in $\triangle BOC$, $OB = 2\sqrt{3}$ (Corollary 8-9.3a)
In right $\triangle POB$, $h^2 + (2\sqrt{3})^2 = 6^2$;
Therefore $h = \sqrt{24} = 2\sqrt{6}$.

5. 



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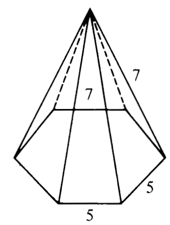
5. *continued*

Volume of square pyramid = $\frac{1}{3}Bh = \frac{1}{3}(64)(10) = 640/3$.
By Corollary 8-9.1a, $AC = 8\sqrt{2}$.
Therefore the area of the base (B) = $\frac{1}{2}(8\sqrt{2})(8\sqrt{2}) = 64$
Volume ($\mathcal{V}$) of triangular pyramid = $\frac{1}{3}Bh$
$\mathcal{V} = 640/3 = \frac{1}{3}(64)(h)$
$h = 10$

6. 

$m\angle AOB = 360/6 = 60$
$m\angle AOC = 30$
$OC = \frac{5}{2}\sqrt{3}$
$OA = 2(AC) = 5 = OB$
$\mathcal{A} = \frac{1}{2}(OC)P$; Where $P = 6(AB) = 30$
$\mathcal{A} = \frac{1}{2}\left(\frac{5\sqrt{3}}{2}\right)(30) = \frac{75\sqrt{3}}{2}$ (area of base)

In $\triangle COP$ use Theorem 8-8.1
Let s = slant height
$s^2 = (2\sqrt{6})^2 + \left(\frac{5\sqrt{3}}{2}\right)^2$

$s = \frac{3}{2}\sqrt{19}$

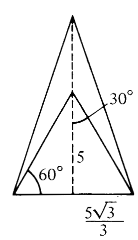
lateral area = $\frac{1}{2} sP$

$= \frac{1}{2}\left(\frac{3\sqrt{19}}{2}\right)(30) = \frac{45\sqrt{19}}{2}$

$\mathcal{V} = \frac{1}{3}Bh$
$\mathcal{V} = \frac{1}{3}\left(\frac{75\sqrt{3}}{2}\right)\cdot(2\sqrt{6}) = 25\sqrt{18} = 75\sqrt{2}$

7. See *Class Exercise* 3 on page 558. Six pyramids of equal volume are formed. Since the volume of the cube is $2^3 = 8$, the volume of one pyramid is $8/6 = 4/3$.

8. Base is a equilateral triangle with altitude 5.
Lateral faces are 3 congruent isosceles triangles with lateral edge 10.



$\mathcal{A}_1$ = area of base

$\mathcal{A}_1 = \frac{h^2\sqrt{3}}{3}$ (Theorem 12-2.7)

$\mathcal{A}_1 = \frac{25\sqrt{3}}{3}$

$\mathcal{A}_1 = \frac{s^2\sqrt{3}}{4}$ (Theorem 12-2.6)

$\frac{25\sqrt{3}}{3} = \frac{s^2\sqrt{3}}{4}$ (Transitive property)
$s^2 = 100/3$
$s = \frac{10\sqrt{3}}{3}$, a side of base edge

(continued next page)

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Exercises continued

8\. *continued*

$m\angle AOB = 360/3 = 120$
$m\angle AOC = 60$
$AC = (OC)(\sqrt{3})$
$\frac{5\sqrt{3}}{3} = (OC)(\sqrt{3})$
$OC = 5/3$
$AO = 2(OC) = 10/3 = OB$

Apply Theorem 8-8.1 to $\triangle POC$
$(10)^2 = s^2(\frac{5\sqrt{3}}{3})^2$
$s = \sqrt{\frac{275}{3}} = \frac{5}{3}\sqrt{33}$

$P = 3(AB) = 10\sqrt{3}$
lateral area $= \frac{1}{2}sP$
lateral area $= \frac{1}{2}(\frac{5\sqrt{33}}{3})(10\sqrt{3}) = 25\sqrt{11}$
Total area $= \mathcal{A}_1$ + lateral area

Total area $= \frac{25\sqrt{3}}{3} + 25\sqrt{11}$

9\. $s = \frac{5\sqrt{33}}{3}$
Apply Theorem 8-8.1 to $\triangle POC$;
$s^2 = h^2 + (\frac{5}{3})^2$
$h^2 = \frac{275}{3} - \frac{25}{9}$
$h = \frac{20}{3}\sqrt{2}$
$\mathcal{V} = \frac{1}{3}Bh$
$\mathcal{V} = \frac{1}{3}(\frac{25\sqrt{3}}{3})(\frac{20\sqrt{2}}{3})$
$\mathcal{V} = \frac{500\sqrt{6}}{27}$

10\. $\mathcal{V}_1 = \frac{1}{3}B_1h_1$; and $\mathcal{V}_2 = \frac{B_2h_2}{3}$

Let $x = h_1 = h_2$

$\mathcal{V}_1 = \frac{1}{3}(25)(x)$; $\mathcal{V}_2 = \frac{1}{3}(36)x$

$\frac{\mathcal{V}_1}{\mathcal{V}_2} = \frac{25}{36}$

11\. $\frac{\mathcal{A}_1}{\mathcal{A}_2} = \left(\frac{h_1}{h_2}\right)^2$
$\frac{\mathcal{A}_1}{48} = \left(\frac{1x}{2x}\right)^2$
$\frac{\mathcal{A}_1}{48} = \frac{1}{4}$
$\mathcal{A}_1 = 12$

12\. $h_1 = x$
$h_2 = 10$

$y = \mathcal{A}_1$

$2y = \mathcal{A}_2$

$\frac{\mathcal{A}_1}{\mathcal{A}_2} = \left(\frac{h_1}{h_2}\right)^2$

$\frac{1y}{2y} = (\frac{x}{10})^2$

$\frac{1}{2} = (\frac{x}{10})^2$

$\frac{1}{\sqrt{2}} = \frac{x}{10}$

$x = 5\sqrt{2}$

$10 - x = 10 - 5\sqrt{2}$

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13\. The triangles formed are isosceles (Theorem 13-3.1)
The sides of the base are congruent (Definition 13-14)
The isosceles triangles have congruent sides (Theorem 13-3.1)
The triangles are congruent (Postulate 1-17)

14\. Let $\mathcal{L}$ be lateral area, s be slant height, and p be perimeter of the base. The area of a face is the product of $(\frac{1}{2})s$ and a side of the base (Theorem 12-2.2). The lateral area is the sum of the areas of the faces (Postulate 12-3)
$\mathcal{L} = (\frac{1}{2})s \cdot p$ (Addition property, Distributive property).

15\. From part (1) and part (2) we know that the ratio of corresponding sides of A'B'C' ... and ABC ... is constant. It remains to show that corresponding angles are congruent.
Plane VAC cuts the section in $\overline{A'C'} \parallel \overline{AC}$
$\triangle ABC \sim \triangle A'B'C'$ (Theorem 8-5.2)
$\angle ABC \cong \angle A'B'C'$ (Theorem 8-5.1)
Similarly, the other corresponding angles are congruent;
The polygons are similar (Definition 8-4).

16\. $\mathcal{A}\triangle ABC/\mathcal{A}\triangle A'B'C' = VK^2/(VK')^2$ (Theorem 13-3.3)
$\mathcal{A}$ quadrilateral DEFG/$\mathcal{A}$ quadrilateral D'E'F'G' $= UL^2/(UL')^2$ (Theorem 13-3.3); $VK/VK' = UL/UL'$ (Division property); $VK^2/(VK')^2 = UL^2/(UL')^2$ (Multiplication property); $\mathcal{A}\triangle ABC/\mathcal{A}\triangle A'B'C' = \mathcal{A}$ quadrilateral DEFG/$\mathcal{A}$ quadrilateral D'E'F'G' (Transitive property); $\mathcal{A}\triangle A'B'C' = \mathcal{A}$ quadrilateral D'E'F'G' (Division property).

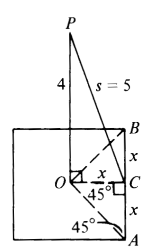
17\. Two pyramids P and P' have equal altitudes, p and p' and bases, B and B', such that $\mathcal{A}$ base B = $\mathcal{A}$ base B'.
Planes parallel to and equidistant from B and B' determine sections of equal area (Corollary 13-3.3a)
$\mathcal{V}$ pyramid P = $\mathcal{V}$ pyramid P' (Cavalieri's principle).

18\. Triangular pyramid O-ABC has volume $\mathcal{V}$, base B, and altitude a such that $\mathcal{A}B = b$.
The volume of O-ABC plus the volume of a quadrangular pyramid O-BCMN is equal to the volume of the resulting prism ABCOMN (Definition 13-1).
Plane OMC divides pyramid O-BCMN into two triangular pyramids, O-MBC and O-MCN
$\triangle MBC \cong \triangle MCN$ (Theorem 7-1.1)
$\mathcal{V}$ pyramid O-MBC = $\mathcal{V}$ pyramid O-MCN (Corollary 13-3.3b)
Pyramid O-MNC is pyramid C-MNO.
C-MNO and O-ABC have congruent bases and the same altitude (Definition 13-7).
$\mathcal{V}$ pyramid C-MNO = $\mathcal{V}$ pyramid O-ABC (Corollary 13-3.3b).
$\mathcal{V}$ prism ABCOMN = $\mathcal{A}\triangle ABC \cdot a$.
$\mathcal{V}$ pyramid O-ABC = $(\frac{1}{3})\,\mathcal{A}\triangle ABC \cdot a$ (Division property).

19\. Divide pyramid P-ABCDE ... into triangular pyramids by passing planes through P and each of the diagonals of the base drawn from the common vertex. All of the triangular pyramids have a common vertex and bases whose union is the base of pyramid P-ABCDE ...
The sum of the measures of the bases of the triangular pyramids is equal to the measure of the base of pyramid P-ABCDE ... (Postulate 12-3).
The volume of pyramid P-ABCDE ... is the sum of the volumes of the triangular pyramids (Postulate 13-1)
$\mathcal{V} = (\frac{1}{3})b_1a + (\frac{1}{3})b_2a + (\frac{1}{3})b_3a + \ldots$

$\mathcal{V} = (\frac{1}{3})a\,(b_1 + b_2 + b_3 + \ldots)$ (Commutative property, Distributive property)
$\mathcal{V} = (\frac{1}{3})ab$, where b is the measure of the base of pyramid P-ABCDE ... (Postulate 2-1).

20\.



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Exercises continued

20. *continued*

$m\angle AOB = 360/4 = 90$
$m\angle AOC = 45$
$AC = CB = x$
By Theorem 8-8.1 applied to $\triangle POC$, $OC = 5 = x$
Let B = area of base
$\mathcal{V} = \frac{1}{3}Bh$
$\mathcal{V} = \frac{1}{3}(2x)^2h$
$\mathcal{V} = \frac{1}{3}(6)^2(4) = 48$

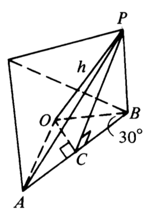
21. $\mathcal{A}_1 = \frac{s^2\sqrt{3}}{4}$ area of base or one lateral face

$64 = \frac{s^2\sqrt{3}}{4}$

$s^2 = \frac{4^4}{\sqrt{3}}$

$s = \frac{4^2}{\sqrt{\sqrt{3}}} = \frac{16}{\sqrt[4]{3}}$

$s = \frac{16}{\sqrt[4]{3}} \cdot \frac{\sqrt[4]{3^3}}{\sqrt[4]{3^3}} = \frac{16\sqrt[4]{3^3}}{3}$, base edge or lateral edge



$AB = PB = \frac{16\sqrt[4]{3^3}}{3}$
$m\angle AOB = 360/3 = 120$
$m\angle AOC = 60$
$AC = (OC)\sqrt{3}$

$\frac{8\sqrt[4]{3^3}}{3} = (OC)\sqrt{3}$

$OC = \frac{8\sqrt[4]{3^3}}{3\sqrt{3}} = \frac{8\sqrt[4]{3^3}}{3\sqrt[4]{3^2}} = \frac{8}{3}\sqrt[4]{3}$

$AO = 2(OC) = \frac{16}{3}\sqrt[4]{3} = BO$

$h^2 = \left(\frac{16\sqrt[4]{3^3}}{3}\right)^2 - \left(\frac{16\sqrt[4]{3}}{3}\right)^2$

$h^2 = \frac{768\sqrt{3} - 256\sqrt{3}}{9} = \frac{512\sqrt{3}}{9}$

$h = \sqrt{\frac{512\sqrt{3}}{9}}$ $\qquad h = \frac{2^4\sqrt{2\sqrt{3}}}{3}$
$\mathcal{V} = \frac{1}{3}Bh$

$\mathcal{V} = \frac{1}{3}(64)\left(\frac{2^4\sqrt{2\sqrt{3}}}{3}\right)$ $\qquad \frac{4^5 \cdot \sqrt[4]{12}}{3^2}$

22. There are four congruent equilateral triangles.
Three lateral faces are congruent equilateral triangles.

$\mathcal{A} = \frac{s^2\sqrt{3}}{4}$ area of base or one lateral face.
$4\mathcal{A} = 144;\ \mathcal{A} = 36$

$36 = \frac{s^2\sqrt{3}}{4}$
$s^2 = \frac{4(36)}{\sqrt{3}}$

$s = \frac{12}{\sqrt{\sqrt{3}}} = \cdot \frac{12}{\sqrt[4]{3}} \cdot \frac{\sqrt[4]{3^3}}{\sqrt[4]{3^3}} = \frac{12\sqrt[4]{3^3}}{3}$

$s = 4\sqrt[4]{3^3}$ base edge or lateral edge.

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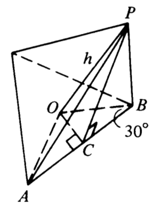
22. *continued*



$AB = PB = 4\sqrt[4]{3^3}$
$m\angle AOB = 360/3 = 120$
$m\angle AOC = 60$
$AC = (OC)\sqrt{3}$

$2\sqrt[4]{3^3} = (OC)\sqrt{3}$

$OC = \frac{2\sqrt[4]{3^3}}{\sqrt{3}} = \frac{2\sqrt[4]{3^3}}{\sqrt[4]{3^2}}$

$OC = 2 \cdot \sqrt[4]{3}$

$AO = 2(OC) = 4\sqrt[4]{3} = BO$

$h^2 = (4\sqrt[4]{3^3})^2 - (4\sqrt[4]{3})^2$

$h^2 = 48\sqrt{3} - 16\sqrt{3} = 32\sqrt{3}$

$h = \sqrt{32\sqrt{3}} = 4\sqrt{2\sqrt{3}}$

$\mathcal{V} = \frac{1}{3}Bh$
$\mathcal{V} = \frac{1}{3}(36)(4\sqrt{2\sqrt{3}})$

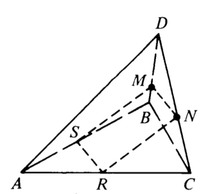
$\mathcal{V} = 48\sqrt{2\sqrt{3}}$

$\mathcal{V} = 48\sqrt{2} \cdot \sqrt{\sqrt{3}}$

$\mathcal{V} = 48\sqrt[4]{2^2} \cdot \sqrt[4]{3} = \mathcal{V} = 48\sqrt[4]{12}$

23. Refer to the figure of Exercise 29, Section 13-2.
$\overline{AB} \parallel \overline{GH}$, and $\overline{AB} \cong \overline{GH}$ (Definition 13-12)
Plane ABGH divides the parallelepiped into two congruent triangular prisms.
Continue as in Exercise 17.

24.



Refer to the figure. Plane R is parallel to $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$, and cuts $\overline{DB}$, $\overline{DC}$, $\overline{AC}$, and $\overline{AB}$ in M, N, R, and S, respectively.
Plane ADB intersects plane R in $\overleftrightarrow{MS}$, which is parallel to $\overleftrightarrow{AD}$
Plane ADC intersects plane R in $\overleftrightarrow{NR}$, which is parallel to $\overleftrightarrow{AD}$
Plane BCD intersects plane R in $\overleftrightarrow{MN}$, which is parallel to $\overleftrightarrow{BC}$; and
Plane BCD intersects plane R in $\overleftrightarrow{SR}$, which is parallel to $\overleftrightarrow{BC}$ (Definition 13-7, Definition 6-1)
$\overleftrightarrow{SM} \parallel \overleftrightarrow{RN}$ (Corollary 6-1.1c)
$\overleftrightarrow{MN} \parallel \overleftrightarrow{SR}$ (Corollary 6-1.1c)
Quadrilateral MNRS is a parallelogram (Definition 7-1).

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Exercises

1. lateral area = $2\pi rh$
 $= 2\pi(5)(8) = 80\pi$
 Total area = $2\pi r(r + h)$
 $= 2\pi(5)(5 + 8) = 130\pi$

2. $\mathcal{L}$ = lateral area
 $\mathcal{L} = 2\pi rh$
 $r = \dfrac{\mathcal{L}}{2\pi h}$; $h = \dfrac{\mathcal{L}}{2\pi r}$

3. lateral area = $2\pi rh$
 $183 = 2\pi(20)h$
 $h = \dfrac{183}{40\pi}$

 Total area = $2\pi r\ (r + h)$
 $= 2\pi\ (20)(20 + \dfrac{183}{40\pi}) = 800\pi + 183$

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4. $\mathcal{V} = \pi r^2 h$
 $\mathcal{V} = \pi(6)^2(10) = 360\pi$

5. $\mathcal{V} = \pi r^2 h$
 $108 = \pi(26)^2 h$
 $h = \dfrac{108}{(26)^2\pi} = \dfrac{108}{676\pi} = \dfrac{27}{169\pi}$

6. $\mathcal{V} = \pi r^2 h$
 $\mathcal{V} = \pi(6)^2(2) = 72\pi$

7. lateral area = πrs
 $= \pi(12)(13) = 156\pi$

8. Isosceles triangle.

9. lateral area = πrs
 $40.4 = \pi(3.2)s$
 $s = \dfrac{40.4}{3.2\pi} = \dfrac{404}{32\pi} = \dfrac{101}{8\pi}$

10. $\mathcal{V} = \frac{1}{3}Bh$
 $\mathcal{V} = \frac{1}{3}(12)(5) = 20$

11. $\mathcal{V} = \frac{1}{3}\pi r^2 h$; $r = 5/2$
 $75 = \frac{1}{3}\pi\ (\frac{5}{2})^2 h$
 $225 = \dfrac{25\pi h}{4}$
 $h = \dfrac{(225)(4)}{25\pi} = \dfrac{36}{\pi}$

12. $c = 2\pi r$
 $15 = 2\pi r$
 $r = 15/2\pi$
 $\mathcal{V} = \frac{1}{3}\pi r^2 h$
 $\mathcal{V} = \frac{1}{3}\pi\left(\dfrac{15}{2\pi}\right)^2(3)$
 $\mathcal{V} = \dfrac{225}{4\pi}$ cubic feet
 $\dfrac{225}{4\pi} \cdot \dfrac{4}{5} = \dfrac{45}{\pi}$

13. CE = 8, CD = 12
 $\triangle CEN \sim \triangle CDB$
 CE/CD = EN/DB
 $8/12 = r/5$
 $r = \dfrac{10}{3}$ = EN ; Apply Theorem 8-8.1 to $\triangle$CEN:
 $(8)^2 + r^2 = (CN)^2$
 CN = 26/3
 Apply Theorem 8-8.1 to $\triangle$CBD:
 $(CD)^2 + (DB)^2 = (BC)^2$
 $(12)^2 + (5)^2 = (BC)^2$
 BC = 13

continued

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13. *continued*

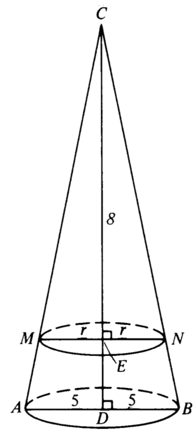
 lateral area of cone CMN = $\pi(EN)(CN) = \pi(\frac{10}{3})(\frac{26}{3}) = \frac{260\pi}{9}$
 lateral area of cone ABC = $\pi(DB)(BC) = 65\pi$
 lateral area of ABNM = $65\pi - \frac{260\pi}{9} = \frac{325\pi}{9}$

 Volume of cone ABC = $\frac{1}{3}\pi(DB)^2(CD) = 100\pi$

 Volume of cone MNC = $\frac{1}{3}\pi(CE)^1(EN)^2 = \frac{800\pi}{27}$

 Volume of ABNM = $100\pi - \frac{800\pi}{27} = \frac{1900\pi}{27}$

 $\mathcal{V}_2 = \dfrac{\pi r^2 h}{3} = \dfrac{\pi(9)(8)}{3} = 24\pi$



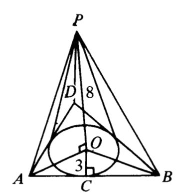
14. The locus is a cylinder surface with given line an axis of symmetry.

15. Cube / Cylinder

 Cube:
 $\mathcal{V} = e^3$
 $\mathcal{V} = (6)^3$
 $\mathcal{V} = 216$

 Cylinder:
 $\mathcal{V}' = \pi r^2 h$
 $216 = \pi\left(\frac{5}{2}\right)^2 \cdot h$
 $h = \dfrac{864}{25\pi}$

16. Pyramid - Base is an equilateral triangle, radius of cone is apothem of the base of pyramid.



 $m\angle AOB = 360/3 = 120$
 $m\angle AOC = 60$
 $AC = (OC)\sqrt{3} = 3\sqrt{3}$
 $AB = 6\sqrt{3}$
 Area of base = $\dfrac{(AB)^2\sqrt{3}}{4} = 27\sqrt{3}$
 $\mathcal{V} = \frac{1}{3}Bh$
 $\mathcal{V} = \frac{1}{3}(27\sqrt{3})(8) = 72\sqrt{3}$
 Volume of cone = $\frac{1}{3}\pi r^2 h$
 Volume of cone = $\frac{1}{3}(\pi)(9)(8) = 24\pi$

17. The section, base, and cylindrical surface determine a new cylinder (Definition 13-6).
 The section is congruent to the base (Theorem 13-4.1)

18. The area of the base is πr^2 (Definition 12-5);
 The volume is $\pi r^2 h$ (Theorem 13-4.2).

19. The altitude of the right circular cylinder is an element of the cylinder (Definition 13-17); the base is a right section (Definition 13-17); the perimeter of the right section is the circumference of the base (Postulate 2-1); the lateral area of a right circular cylinder is the product of its altitude and circumference (Postulate 13-4, Postulate 2-1).

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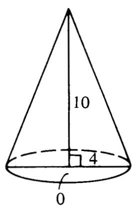
Exercises continued

20. $\mathcal{L} = 2\pi rh$ (Theorem 13-4.3)
If B is a base, $\mathcal{A}B = 2\pi r^2$ (Definition 12-5)
$\mathcal{T} = 2\pi r^2 + 2\pi rh = 2\pi r\,(r + h)$ (Addition and Distributive properties).

21. If the radius is r, the circumference is $2\pi r$, and if the slant height is s, then $\mathcal{L} = (\frac{1}{2})2\pi r \cdot s = \pi rs$ (Postulate 13-5)
The area of the base is πr^2.
$\mathcal{T} = \pi r^2 + \pi rs = \pi r\,(r + s)$ (Addition and Distributive properties).

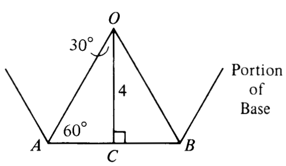
22. Follows immediately from Postulate 13-6.

23.



The radius of cone is an apothem of the base of the pyramid.

$\mathcal{V}_1 = \frac{1}{3}\pi r^2 h$
$\mathcal{V}_1 = \frac{160\pi}{3}$



m∠ AOB = 360/6 = 60
m∠ AOC = 30
$OC = (AC)\sqrt{3}$
$4 = (AC)\sqrt{3}$
$AC = \frac{4\sqrt{3}}{3}$; and $AB = \frac{8\sqrt{3}}{3}$

$P = 6(AB) = 16\sqrt{3}$
$\mathcal{A} = \frac{1}{2}\,aP$
$\mathcal{A} = \frac{1}{2}\,(4)(16\sqrt{3}), = 32\sqrt{3}$, area of base.

$\mathcal{V}_2 = \frac{1}{3}Bh$
$\mathcal{V}_2 = \frac{1}{3}\,(32\sqrt{3})\,(10)$
$\mathcal{V}_2 = \frac{320\sqrt{3}}{3}$

24. Apply Theorem 8-8.1 to find the altitude $= \sqrt{s^2 - r^2}$.

25. $\mathcal{V} = \frac{1}{3}\pi r^2 h = \frac{1}{3}(\pi)\left(\frac{1}{12}\right)^2\left(\frac{1}{6}\right) = \pi/2592$ cubic feet.
Therefore the number of cones which can be filled from 7.5 gallons of ice cream is:
$\frac{1}{7.5} \div \frac{\pi}{2592} = \frac{1728}{5\pi}$

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26. Cylinder

lateral area $= 2\pi rh$
$= 2\pi(5)(5)$
$= 50\pi$
$\mathcal{V}_1 = \pi r^2 h$
$\mathcal{V}_1 = \pi(5)^2\,(5) = 125\pi$

Cone

$\mathcal{V}_2 = \frac{1}{3}\pi r^2 h^2$
$125\pi = \frac{\pi}{3}\,(5)^2 h_2$
$h_2 = 15$
$s^2 = (h)^2 + (5)^2$
$s = 5\sqrt{10}$
lateral area $= \pi rs$
$= \pi(5)(5\sqrt{10})$
$= 25\pi\sqrt{10}$

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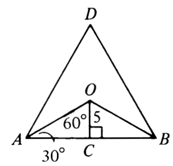
27. Cone

$\mathcal{V}_1 = \frac{1}{3}\pi r^2 h$
$\mathcal{V}_1 = 100\pi$
The radius of cone is an apothem of the base of the pyramid

Pyramid - base is an equilateral triangle

m∠ AOB = 360/3 = 120
m∠ AOC = 60
$AC = 5\sqrt{3}$
$AB = 10\sqrt{3}$
$P = 3(AB) = 30\sqrt{3}$
$\mathcal{A} = \frac{1}{2}aP$
$\mathcal{A} = \frac{1}{2}(5)(30\sqrt{3}) = 75\sqrt{3}$
$\mathcal{V}_2 = \frac{1}{3}Bh$
$\mathcal{V}_2 = \frac{1}{3}(75\sqrt{3})(12) = 300\sqrt{3}$
$\mathcal{V}_1 - \mathcal{V}_2 = 300\sqrt{3} - 100\pi = 100(3\sqrt{3} - \pi)$

28.



$CO = h$
$CO' = x$
$\triangle CO'N \sim \triangle COB$
$\frac{CO'}{CO} = \frac{O'N}{OB}$
$\frac{x}{h} = \frac{r}{R}$
$x = \frac{r}{R} \cdot h$
$x = \frac{1}{\sqrt[3]{2}} \cdot h$
$x = \frac{h}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}$
$x = \frac{\sqrt[3]{4}}{2}$

$\mathcal{V}_1$ of cone ABC $= \frac{1}{3}R^2 h = \mathcal{V}_3$ of ABNM
$\mathcal{V}_2$ of cone MNC $= \frac{1}{3}\pi r^2 x$
$\mathcal{V}_3$ of ABNM $= \mathcal{V}_1 - \mathcal{V}_2$
$\frac{1}{3}\pi r^2 x = \frac{1}{3}\pi R^2 h - \frac{1}{3}\pi r^2 x$
$2\pi r^2 x = \pi R^2 h$
$2r^2 x = R^2 h$
$2r^2 \cdot \frac{rh}{R} = R^2 h$
$2r^3 h = R^3 h$
$\frac{r^3}{R^3} = \frac{1h}{2h} = \frac{1}{2}$
$\frac{r}{R} = \frac{1}{\sqrt[3]{2}}$

Class Exercises

1. $\pi\, r_1 s_1$
2. $\pi\, r_2 s_2$
3. $(\pi\, r_1 s_1 - \pi\, r_2 s_2) = \pi(r_1 s_1 - r_2 s_2)$

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Exercises

1. $\mathcal{A} = 4\pi r^2 = 64\pi$

2. $\mathcal{A}/\mathcal{A}' = (C/C')^2$

$\frac{16\pi}{100\pi} = (C/C')^2$

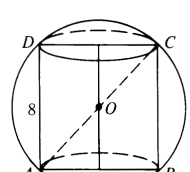
$4/10 = C/C'$

$C/C' = 2/5$

3. Diameter of Sphere = side of a square face
$d = 8$; $r = 4$
$\mathcal{A} = 4\pi r^2 = 64\pi$

4. Maximum height will approach 12.
If the height is 12, the sphere will be inscribed in the cylinder.

5.



$AC = 10$
$(AC)^2 = (AD)^2 + (DC)^2$ (Theorem 8-8.1)
$(10)^2 = (8)^2 + (DC)^2$
$DC = 6$ diameter of the cylinder

continued

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Exercises continued

5. *continued*

$r = 3$ radius of the cylinder
lateral area $= 2\pi rh$
$= \pi(3)(8) = 48\pi$

6. $\mathcal{V}_1 = \frac{4}{3}\pi R^3$ $\qquad \mathcal{V}_2 = \frac{4}{3}\pi r^2$

$\mathcal{V}_1 = \frac{4}{3}\pi(6)^3 = 288\pi \qquad \mathcal{V}_2 = \frac{4}{3}\pi(4)^3 = \frac{256\pi}{3}$

$\mathcal{V}_1 - \mathcal{V}_2 = 288\pi - \frac{256\pi}{3} = \frac{608\pi}{3}$

7. $\mathcal{V}/\mathcal{V}' = (r/r')^3$
Let $x = \mathcal{V}_1$
$3x = \mathcal{V}_2$
$1x/3x = (r/r')^3$
$1/3 = (r/r')^3$

$\frac{1}{\sqrt[3]{3}} = \frac{r_1}{r_2}$

$r_2 = \sqrt[3]{3}\, r_1$

8. Cone

$\mathcal{V}_1 = \frac{1}{3}\pi r_1^2 h$

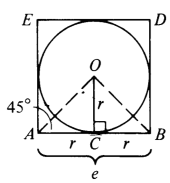
$x = r_1 = r_2$
$h = 2x$
$\mathcal{V}_1 = \frac{1}{3}\pi x^2(2x) = \frac{2\pi x^3}{3}$

Sphere

$\mathcal{V}_2 = \frac{4}{3}\pi r_2^3$

$\mathcal{V}_2 = \frac{4}{3}\pi x^3$

$\frac{\mathcal{V}_1}{\mathcal{V}_2} = \frac{\frac{2\pi x^3}{3}}{\frac{4\pi x^3}{3}} = \frac{1}{2}$

9. 

Sphere

$\mathcal{A}_1 = 4\pi r^2$
m∠ AOB = 360/4 = 90
m∠ AOC = 45

Cube

$\mathcal{A}_2 = 6e^2$
$e = 2r$
$\mathcal{A}_2 = 24r^2$

$\frac{\mathcal{A}_1}{\mathcal{A}_2} = \frac{4\pi r^2}{24 r^2} = \frac{\pi}{6}$

10. $\mathcal{V} = \frac{3}{4}\pi r^3$; and $\mathcal{A} = 4\pi r^2$

$\frac{4}{3}\pi r^3 = 4\pi r^2$
$4\pi r^3 - 12\pi r^2 = 0$
$r^3 - 3r^2 = 0$
$r^2(r - 3) = 0$
$r^2 = 0 \qquad | \qquad r - 3 = 0$
$r = 0$ (reject) $\qquad | \qquad r = 3$

11. The lateral area of a cone with radius r_1 and slant height s_1 is $\mathcal{L}_1 = \pi r_1 s_1$ (Postulate 13-5)
Similarly, the lateral area of the upper cone is
$\mathcal{L}_2 = \pi r_2 s_2$
The lateral area of the frustum is
$\mathcal{L} = \pi r_1 s_1 - \pi r_2 s_2 = \pi(r_1 s_1 - r_2 s_2)$ (Subtraction property)
The triangles formed by the slant height and radius of each cone are similar (Corollary 6-3.1a, Corollary 8-5.1a);
$s_1/s_2 = (\pi s_1/r_1)(r_1^2 - r_2^2) = (\pi s_1/r_1)(r_1 - r_2)(r_1 + r_2)$ (Distributive property)
$\mathcal{L} = (\pi s_1 - \pi s_2)(r_1 + r_2)$ (Postulate 2-1)
$\mathcal{L} = \pi s(r_1 + r_2)$, where $s = s_1 - s_2$ (Postulate 2-1, Distributive property)
$\mathcal{L} = (\frac{1}{2})s(2\pi r_1 + 2\pi r_2)$ (Distributive property).

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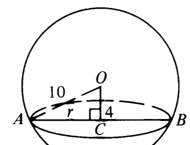
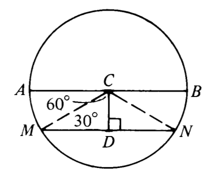
Exercises continued

12. See proof outline pages 568-569.

13. 

$\overline{AB}$ is diameter of the circle.
$(10)^2 = r^2 + (4)^2$ (Theorem 8-8.1)
$r = \sqrt{84} = 2\sqrt{21}$



$\overline{MN}$ is side of the equilateral triangle
m∠ MCN = 360/3 = 120
m∠ MCD = 60
MC = 2(CD)
$2\sqrt{21} = 2$ (CD)
CD $= \sqrt{21}$
MD $= (CD)\sqrt{3} = (\sqrt{21})(\sqrt{3}) = \sqrt{63} = 3\sqrt{7}$
MN $= 2(MD) = 6\sqrt{7}$
$\mathcal{A} = \frac{(MN)^2\sqrt{3}}{4} = 63\sqrt{3}$

14. Cube

$\mathcal{A}_2 = 6e^2$

$\mathcal{V}_2 = e^3$

Sphere

$\mathcal{A}_1 = 4\pi r^2$

$4\pi r^2 = 6e^2$
$r^2 = \frac{3e^2}{2\pi}$

$r = \sqrt{\frac{3e^2}{2\pi}} = e\sqrt{\frac{3}{2\pi}} = \frac{e}{2\pi}\sqrt{6\pi}$

$\mathcal{V}_1 = \frac{4}{3}\pi r 3$

$\mathcal{V}_1 = \frac{4}{3}\pi r^2 \cdot r$

$\mathcal{V}_1 = \frac{4\pi}{3} \cdot \frac{3e^2}{2\pi} \cdot \frac{e}{2\pi}\sqrt{6\pi} = \frac{e^3\sqrt{6\pi}}{\pi}$

$\frac{\mathcal{V}_1}{\mathcal{V}_2} = \frac{\frac{e^3\sqrt{6\pi}}{\pi}}{e^3} = \frac{\sqrt{6\pi}}{\pi}$

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Review Exercises

1. No, the sum is greater than 360°.
2. There are four congruent triangles for faces.
The base face is an equilateral triangle.
The 3 lateral faces are congruent equilateral triangles.

$\mathcal{A} = \frac{s^2\sqrt{3}}{4} = \frac{25\sqrt{3}}{4}$
Total area $= 4\mathcal{A} = 25\sqrt{3}$

3. 4
4. It is the point of concurrence of the planes parallel to the faces and the required distances from them.

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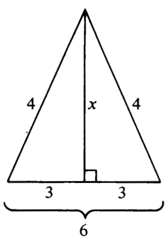
Review Exercises continued

5. Base is a square of side 3.

   P = 4(3) = 12
   lateral area = eP
   = (10)(12) = 120

6. D = $e\sqrt{3}$
   $6\sqrt{3} = e\sqrt{3}$
   $e = 6$
   Total area = $6e^2$ = 216

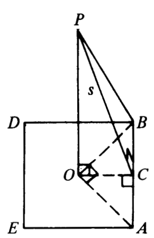
7. 

   $(4)^2 = (3)^2 + x^2$
   $x = \sqrt{7}$
   B_1 = area of base = $\frac{1}{2}(6)(x) = 3x = 3\sqrt{7}$
   P = 6 + 4 + 4 = 14.
   lateral area = Ph
   = (14)(10) = 140
   Total area = lateral area + $2B_1$ = 140 + $6\sqrt{7}$
   $\mathcal{V} = Bh$
   $\mathcal{V} = (3\sqrt{7})(10) = 30\sqrt{7}$

8. Total area = $6e^2$
   $64 = 6e^2$
   $e^2 = 32/3$
   $e = \frac{4\sqrt{6}}{3}$

   $\mathcal{V} = e^3 = e^2 \cdot e$
   $\mathcal{V} = \frac{32}{3} \cdot \frac{4\sqrt{6}}{3} = \frac{128\sqrt{6}}{9}$

9. lateral area = eP
   $360 = 13e$
   $e = 27\frac{9}{13}$

10. 

   m∠ AOB = 360/4 = 90
   m∠ AOC = 45
   OC = AC = $\frac{5}{2}$

   $s^2 = (10)^2 + \left(\frac{5}{2}\right)^2$
   $s = \sqrt{\frac{425}{4}} = \frac{5}{2}\sqrt{17}$

   P = 4(AB) = 20
   lateral area = $\frac{1}{2}s$P
   lateral area = $\frac{1}{2}\left(\frac{5\sqrt{17}}{2}\right)(20) = 25\sqrt{17}$
   B = $(AB)^2 = (5)^2$ = 25 area of the base
   Total area = lateral area + B = $25\sqrt{17}$ + 25

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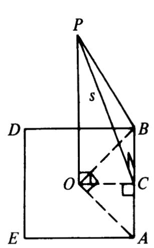
11. 

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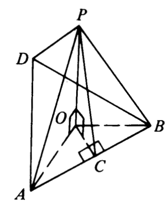
11. *continued*

   AB = $2x$
   m∠ AOB = 360/4 = 90
   m∠ AOC = 45
   OC = AC = x
   $(13)^2 = x^2 + (5)^2$, and x = 12
   $\mathcal{V} = \frac{1}{3}Bh$, B = $(AB)^2 = 4x^2$ = 576
   $\mathcal{V} = \frac{1}{3}(576)(5)$ = 960

12. The base is an equilateral triangle
   There are 3 congruent equilateral triangles for lateral faces
   lateral edge = base edge

   AB = PB = 9
   m∠ AOB = 360/3 = 120
   m∠ AOC = 60
   AC = (OC)$\sqrt{3}$
   $\frac{9}{2}$ = (OC)$\sqrt{3}$
   $\frac{3\sqrt{3}}{2}$ = OC
   AO = 2(OC) = $3\sqrt{3}$ = OB

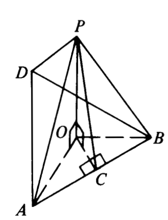
   Let PO = h
   Therefore in △POB
   $h^2 = 81 - 27$ (Theorem 8-8.1)
   $h = \sqrt{54} = 3\sqrt{6}$

13. There are 4 congruent equilateral triangles, one for base and 3 for lateral faces.

   Let AC = x
   m∠ AOB = 360/3 = 120
   m∠ AOC = 60
   AC = (OC)$\sqrt{3} = x$
   OC = $\frac{x\sqrt{3}}{3}$
   OA = 2(OC) = $\frac{2x\sqrt{3}}{3}$ = OB
   AB = PB = $2x$
   PB = $2x$ and OB = $\frac{2x\sqrt{3}}{3}$
   $(2x)^2 = 36 + \left(\frac{2\sqrt{3}x}{3}\right)^2$
   $x^2 = \frac{27}{2}$
   $x = \sqrt{\frac{27}{2}} \quad 3 = \frac{3}{2}\sqrt{6}$

   $B_1 = \frac{(AB)^2\sqrt{3}}{4} = \frac{4x^2\sqrt{3}}{4} = x^2\sqrt{3}$

   Total area = $4B_1 = 4x^2\sqrt{3} = 4(3\sqrt{6}/2)^2 \cdot \sqrt{3} = 54\sqrt{3}$

14. 

   In 30-60-90 triangle ACD
   $6 = x\sqrt{3}$ (where AC = x)
   $x = 2\sqrt{3}$
   AB = $2x = 4\sqrt{3}$ base edge
   B = area of base = $\frac{(AB)^2\sqrt{3}}{4} = 12\sqrt{3}$
   Let PC = s
   Then CB = $2\sqrt{3}$

continued

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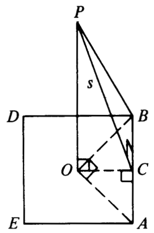
Review Exercises continued

14. *continued*

Thus in $\triangle$PCB: $s^2 = 12^2 - (2\sqrt{3})^2$
$s = 2\sqrt{33}$
$P = 3(AB) = 12\sqrt{3}$
lateral area $= \frac{1}{2}sP$;
lateral area $= \frac{1}{2}(2\sqrt{33})(12\sqrt{3}) = 36\sqrt{11}$
Total area $= B +$ lateral area $= 12\sqrt{3} + 36\sqrt{11}$

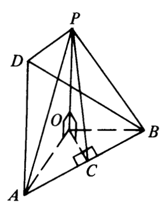
15.



$m\angle AOB = 360/4 = 90$
$m\angle AOC = 45$
$OC = AC = x$
$B =$ area of base $= (AB)^2 = 4x^2$

In right $\triangle$POC:
$x^2 + 6^2 = 8^2$ (Theorem 8-8.1)
$x = 2\sqrt{7}$
$V = \frac{1}{3}Bh$
$V = \frac{1}{3}(4x^2)(6) = 8x^2 = 224$

16. There are 4 congruent equilateral triangles



$m\angle AOB = 360/3 = 120$
$m\angle AOC = 60$
$AC = (OC)\sqrt{3}$ (use $\triangle$AOC)
$x = (OC)\sqrt{3}$
$OC = \frac{x\sqrt{3}}{3}$
$P = 3(AB) = 6x$
$B =$ area of base $= \frac{1}{2}aP$,
$= \frac{1}{2}\left(\frac{x\sqrt{3}}{3}\right)(6x) = x^2\sqrt{3}$
In right $\triangle$PCB: $x^2 + s^2 = (2x)^2$ (Theorem 8-8.1)
Therefore $s = x\sqrt{3}$
lateral area $= \frac{1}{2}sP$
$= \frac{1}{2}(x\sqrt{3})(6x) = 3x^2\sqrt{3}$
Total area = lateral area + B
$400 = 3x^2\sqrt{3} + x^2\sqrt{3}$
$400 = 4x^2\sqrt{3}$
$x^2 = \frac{100\sqrt{3}}{3}$

$x = 10\sqrt{\frac{\sqrt{3}}{3}} = (10/3)\sqrt{3\sqrt{3}}$
In right $\triangle$POC:

$PO = h$,
$h^2 = (x\sqrt{3})^2 - \left(\frac{x\sqrt{3}}{3}\right)^2$ (Theorem 8-8.1)
$h^2 = \frac{8x^2}{3}$

$h = 2x\sqrt{\frac{2}{3}} = \frac{2x}{3}\sqrt{6}$
$V = \frac{1}{3}Bh$
$V = \frac{1}{3}\left(x^2\sqrt{3}\right)\left(\frac{2x\sqrt{6}}{3}\right) = \frac{2x^3\sqrt{2}}{3}$
$V = \frac{2}{3}\sqrt{2} \cdot x^2 \cdot x$

continued

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16. *continued*

$V = \frac{2\sqrt{2}}{3} \cdot \frac{100}{\sqrt{3}} \cdot \frac{10}{\sqrt{3}} \cdot \sqrt{3\sqrt{3}} = \frac{2000\sqrt{2\sqrt{3}}}{9}$

17. lateral area $= 2\pi rh$
$= 2\pi(6)(10) = 120\pi$

total area $= 2\pi r(r + h)$
$= 2\pi(6)(16) = 192\pi$

18. $V = \pi r^2 h$
$250 = \pi(13)^2 h$
$h = \frac{250}{169\pi}$

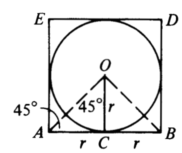
19. $V = \frac{1}{3}Bh$
$V = \frac{1}{3}(10)(16) = 20$

20.

Cube	Cylinder
$V_1 = e^3$	$V_2 = \pi r^2 h$
$V_1 = (10)^3$	$1000 = \pi(5)^2 h$
$V_1 = 1000$	$h = \frac{40}{\pi}$

21. lateral area $= \pi rs$
$64 = \pi(r)(8)$; $r = \frac{8}{\pi}$
$h = \sqrt{s^2 - r^2}$ (Theorem 8-8.1)
$h = \sqrt{64 - (64/\pi^2)} = (8/\pi)\sqrt{\pi - 1}$

22.



$m\angle AOB = 360/4 = 90$
$m\angle AOC = 45$

Cube	Sphere
$V = (AB)^3$	$A = 4\pi r^2$
$V = (2r)^3$	$A = 16\pi$
$64 = 8r^3$	
$r = 2$	

23. $V/V' = (r/r')^3$
$V/V' = \frac{1}{2} = (r/r')^3$
$r/r' = 1/\sqrt[3]{2}$

24. $V_1 = \frac{4}{3}\pi r_1^3$; $r_1 = 4$
$V_1 = \frac{256\pi}{3}$
$V_2 = \frac{4}{3}\pi r_2^3 = 5$
$V_2 = \frac{500\pi}{3}$
$V_2 - V_1 = \frac{244\pi}{3}$

25. The great circle contains the diameter of the sphere.

Sphere	Cone
$V_1 = \frac{4}{3}\pi r_1^3$; $r_1 = 6$	$V_2 = \frac{1}{3}\pi r_2^2 h$; $r_2 = 6$
$V_1 = 288\pi$	$288\pi = \frac{1}{3}\pi(36)h$
	$h = 24$

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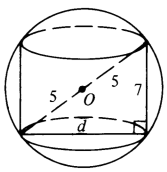
Review Exercises continued

26.

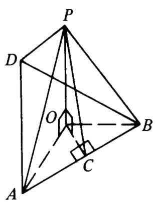


$d^2 = (10)^2 - (7)^2$
$d = \sqrt{51}$
$r = \frac{1}{2}\sqrt{51}$
lateral area $= 2\pi rh = 2\pi(\frac{1}{2}\sqrt{51})(7) = 7\pi\sqrt{51}$

Chapter Test

1. Given: 70, 100, x, $x > 0$
   $70 + 100 + x < 360$ (Theorem 13-1.2)
   $70 + 100 > x$, therefore $170 > x$ (Theorem 13-1.1)
   $70 + x > 100$, $x > 30$ (Theorem 13-1.1)
   $100 + x > 70$ true for all positive values of x.
   Therefore $30 < x < 170$.

2.



$\mathcal{A}_1$ = area of base $= \frac{(AB)^2\sqrt{3}}{4} = 25\sqrt{3}$

Total area $= 4\mathcal{A}_1 = 100\sqrt{3}$

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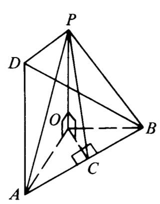
3. $P = 3 + 5 + 4 + 6 + 4 = 22$
   lateral area $= eP = (12)(22) = 264$

4. $B = \frac{1}{2}(5)(12) = 30$
   $V = Bh = 30(10) = 300$

5. Base is a square
   $B = (8)^2 = 64$
   $V = \frac{1}{3}Bh$
   $32 = \frac{1}{3}(64)h$
   $h = 3/2$

6.



Let x = OC
$m\angle$ AOB = 360/3 = 120
$m\angle$ AOC = 60
AC = (OC)$\sqrt{3}$
x = (OC)$\sqrt{3}$
OC $= \frac{x\sqrt{3}}{3}$
In right $\triangle$POC:
PO = 6, PC = s and OC $= \frac{x\sqrt{3}}{3}$
Therefore:
$s^2 = 36 + \frac{x^2}{3}$ (Theorem 8-8.1)
In right $\triangle$PCB:
$s^2 = 64 - x^2$ (Theorem 8-8.1)
Therefore:
$36 + \frac{x^2}{3} = 64 - x^2$ (Transitive property)
$x = \sqrt{21}$; $s = \sqrt{43}$

continued next page

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6. *continued*

   B = area of base $= \frac{(AB)^2\sqrt{3}}{4}$

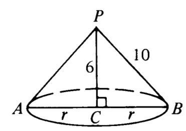
   $= \frac{4x^2\sqrt{3}}{4} = x^2\sqrt{3} = 21\sqrt{3}$

   $P = 3(AB) = 6x$
   lateral area $= \frac{1}{2}sP$
   $\frac{1}{2}(\sqrt{43})(6x) = (3\sqrt{43})(\sqrt{21}) = 3\sqrt{903}$

   Total area = B + lateral area
   $= 21\sqrt{3} + 3\sqrt{903}$

7. Total area $= 2\pi r(r + h)$
   $= 2\pi(6)(6 + 8) = 168\pi$

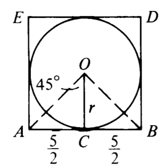
8.



$r^2 = 100 - 36$	$V = \frac{1}{3}\pi r^2 h$
$r = 8$	$V = 128\pi$

9. lateral area $= \pi rs = \pi(2)(4) = 8\pi$

10.



$m\angle$ AOB = 360/4 = 90
$m\angle$ AOC = 45
$r = \frac{5}{2}$
$V = \frac{4}{3}\pi r^3$
$V = \frac{125\pi}{6}$

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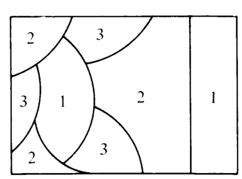
Exercises

1. 16 + 13 - 28 = 1
2. Answers will vary with each student.
3. The numbers indicate a coloring scheme to follow. There are other possible schemes. It is not possible to use fewer than three colors.



4. One strip, twice as long, with two half-twists.
5. Two strips linked together.
6. Two strips linked together, one with one-half-twist and the other, twice as long, with three half-twists.

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Exercises

1. $\overrightarrow{AB} = (1-2, 7-5) = (-1, 2)$

Exercises 2-9 are done in a way similar to Exercise 1

2. $\overrightarrow{MN} = (-9, 6)$
3. $\overrightarrow{CD} = (11, -2)$
4. $\overrightarrow{EB} = (1, 7)$
5. $\overrightarrow{EF} = (3, 9)$
6. $\overrightarrow{FB} = (-2, -2)$

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7. $\overrightarrow{AB} = (11, 1)$, $\overrightarrow{BA} = (-11, -1)$
8. $\overrightarrow{AB} = (1, 0)$, $\overrightarrow{BA} = (-1, 0)$
9. $\overrightarrow{AB} = (2, 6)$, $\overrightarrow{BA} = (-2, -6)$
10. $\overrightarrow{CD} = (-3, 4)$
11. $|\overrightarrow{AB}| = 5$
12. $\overrightarrow{CB} = (2, -3)$
13. $\overrightarrow{DA} = (2, -3)$
14. $|\overrightarrow{DA}| = \sqrt{13}$
15. $-\overrightarrow{CD} = \overrightarrow{DC} = (3, -4)$
16. C(-3, 0)
17. D(-7, 6)
18. $|\overrightarrow{AB}| = \sqrt{13}$
19. $|\overrightarrow{BA}| = \sqrt{13}$
20. M(3, -3)
21. $\overrightarrow{AB} = (-2, 2)$
22. $\overrightarrow{BC} = (-6, -4)$
23. $\overrightarrow{AC} = (-8, -2)$
24. $\overrightarrow{DA} = (4, 3)$
25. $\overrightarrow{DB} = (2, 5)$
26. $\overrightarrow{DC} = (-4, 1)$
27. The sum of the x's for $\overrightarrow{AB}$ and $\overrightarrow{BC}$ equals the x for $\overrightarrow{AC}$.
28. The sum of the y's for $\overrightarrow{AB}$ and $\overrightarrow{BC}$ equals the y for $\overrightarrow{AC}$.
29. The x-value for the vector equals the x-coordinate of the point; the y-value for the vector equals the y-coordinate of the point.

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Exercises

1. $\overrightarrow{AB} = (-7, -1)$
2. $\overrightarrow{BC} = (9, -4)$
3. $\overrightarrow{AB} + \overrightarrow{BC} = (2, -5)$
4. $\overrightarrow{CA} = (-2, 5)$
5. $\overrightarrow{ED} = (1, -1)$
6. $\overrightarrow{DE} = (-1, 1)$
7. $\overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} = (4, 4)$
8. $\overrightarrow{BE} = (4, 4)$
9. $\overrightarrow{AC} + \overrightarrow{CE} = (-7, 13)$
10. $\overrightarrow{EA} = (3, -3)$
11. Yes.
12. No, $\overrightarrow{AB} = (4, -7)$, but $\overrightarrow{DC} = (-3, -3)$.
13. Let $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$.

    $\overrightarrow{AB} = (x_2 - x_1, y_2 - y_1)$, $\overrightarrow{BC} = (x_3 - x_2, y_3 - y_2)$

    $\overrightarrow{CA} = (x_1 - x_3, y_1 - y_3)$ (Definition 14-1);

    $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = (x_2 - x_1 + x_3 - x_2 + x_1 - x_3, y_2 - y_1 + y_3 - y_2 + y_1 - y_3) = (0, 0) = 0.$
14. Suppose A, B, and C are collinear with $\overline{ABC}$; then $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ (Refer to Example 2); but $\overrightarrow{AC} = -\overrightarrow{CA}$ (Definition 14-5); $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{AC} + \overrightarrow{CA} = 0$ (Postulate 2-1)

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Class Exercises

1. $-\frac{1}{2}$
2. $-\frac{1}{2}$
3. Yes, Theorem 10-3.1.
4. (4, -2)
5. (8, -4)
6. x in $\overrightarrow{CD}$ equals $2x$ in $\overrightarrow{AB}$.
7. y in $\overrightarrow{CD}$ equals $2y$ in $\overrightarrow{AB}$.
8. Several answers possible. For example: The ratio of the sum of the y components to the sum of the x components equals the slope.

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Exercises

Answers for Exercises 1-16 will vary. Sample answers are given here.

1. (6, 14)
2. (-2, 5/2)
3. (0, 13/5)
4. (3/2, -4)
5. (-25, 0)
6. (7, 7)
7. (-6, 3/2)
8. (3, 5)
9. (-14, 8)
10. (3, 5)
11. (5/2, 1)
12. (1, -2)
13. $(-1/6, \frac{1}{2})$
14. (4, 16)
15. (-3, 0)
16. (3/5, 1/10)
17. $x = \pm\sqrt{12}$
18. $x = 0$
19. $x = \pm\sqrt{7}$
20. $x = 0$
21. $x = \pm 13$
22. $x = \mp 3$
23. (-2, -7)
24. Rectangle.
25. $\frac{3}{2}$, -11
26. Rhombus
27. Square.
28. $\overrightarrow{BC} = (-1, 1)$, $\overrightarrow{CD} = (-1, -1)$, $\overrightarrow{DA} = (1, -1)$, or $\overrightarrow{BC} = (1, -1)$ $\overrightarrow{CD} = (-1, -1)$, $\overrightarrow{DA} = (-1, 1)$

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Exercises

1. (-1, 0)
2. (6, 11)
3. (2, -8)
4. (-3, 2)
5. (4, 1)
6. (2, -6)
7. (-5, 0)
8. (0, 6)
9. $(-3, \frac{1}{3})$
10. $A + B = (5, -7)$
11. $B + A = (5, -7)$
12. $V_1 + V_2 = (0, -10)$
13. $V_2 + V_1 = (0, -10)$
14. $A + V_3 = (0, 0)$
15. $V_3 + A = (0, 0)$
16. $(A + B) + V_1 = (5, -7) + (-3, -5) = (2, -12)$
17. $A + (B + V_2) = (3, -4) + (5, -8) = (8, -12)$
18. $(V_1 + V_2) + V_3 = (0, -10) + (-3, 4) = (-3, -6)$
19. $V_1 + (V_2 + V_3) = (-3, -5) + (0, -1) = (-3, -6)$
20. $\overrightarrow{AB} = (3, 4)$, $\overrightarrow{BC} = (-8, -3)$, $\overrightarrow{AC} = (-5, 1)$ (Definition 14-1) $\overrightarrow{AB} + \overrightarrow{BC} = (-5, 1) = \overrightarrow{AC}$ (Definition 14-7, Postulate 2-1)
21. $\overrightarrow{AC} = -\overrightarrow{CA}$ (Definition 14-5) $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ (Exercise 20) Hence, $(\overrightarrow{AB} + \overrightarrow{BC}) + \overrightarrow{CA} = \overrightarrow{AC} + \overrightarrow{CA} = -\overrightarrow{CA} + \overrightarrow{CA} = 0$ (Postulate 2-1)
22. (3, 4) + (-3, -4) = 0 (Definition 14-7)
23. $\overrightarrow{AB} = (3, 4)$, $\overrightarrow{BA} = (-3, -4)$ (Definition 14-1) $\overrightarrow{AB} + \overrightarrow{BA} = 0$ (Definition 14-7)
24. Let $A = (x_1, y_1)$, $B = (x_2, y_2)$, and $C = (x_3, y_3)$.

    $(A + B) + C = (x_1 + x_2, y_1 + y_2) + (x_3, y_3) = [(x_1 + x_2) + x_3, (y_1 + y_2) + y_3]$. $A + (B + C) = (x_1, y_1) + (x_2 + x_3, y_2 + y_3) = [x_1 + (x_2 + x_3), y_1 + (y_2 + y_3)]$.

    But addition of real numbers is associative; Thus, $(A + B) + C = A + (B + C)$.

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Exercises

1. (-6, 10)
2. (-4, -3)
3. (-30, -54)
4. (0, 0)
5. (13, -39)
6. (0, 0)
7. (-2, 2)
8. (-2, 2)
9. (20, -8)
10. (20, -8)
11. (-18, -3)
12. (-18, -3)
13. (0, 0)
14. (21, 35)
15. (-6, 8)
16. (17, -4)
17. $(-\frac{1}{2}, 5/2)$
18. (60, 70)
19. (-26, -35)
20. (4, -28)
21. (-33, 10)

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Exercises continued

25. Let $V = (x_1, y_1)$. $a(bV) = a(bx_1, by_1)$ (Definition 14-3)

 $a(bx_1, by_1) = (a(bx_1), a(by_1)) = (abx_1, aby_1)$ (Definition 14-13, Multiplication of real numbers is associative)

 $(ab)V = (abx_1, aby_1)$ (Definition 14-13)

 Hence, $a(bV) = (ab)V$ (Postulate 2-1)

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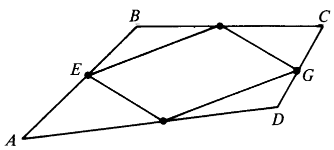
Exercises

Exercises 1-6 are student drawings.

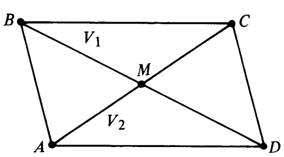
7. The third force equals 50 lb and makes an angle of $126°50'$ with the 30 lb force.

8.



$\overrightarrow{AD} + \overrightarrow{DC} = \overrightarrow{AC}$, $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ (Definition 14-6)
$\overrightarrow{AD} + \overrightarrow{DC} = \overrightarrow{AB} + \overrightarrow{BC}$ (Transitive property)
$\overrightarrow{EH} = (\frac{1}{2})\overrightarrow{AD} - (\frac{1}{2})\overrightarrow{AB}$ (Definition of Vector Subtraction)
$\overrightarrow{FG} = (\frac{1}{2})\overrightarrow{CD} - (\frac{1}{2})\overrightarrow{CB}$ (Definition of Vector Subtraction)
$\overrightarrow{EH} = \overrightarrow{FG}$ (Postulate 2-1)
$\overline{EH} \parallel \overline{FG}$ (Definition 14-9)
Similarly, $\overline{EF} \parallel \overline{HG}$
EFGH is a parallelogram (Definition 7-1)

9.



Consider quadrilateral ABCD with $\overrightarrow{BM} = \overrightarrow{MD} = V_1$

$\overrightarrow{AM} = \overrightarrow{MC} = V_2$. $\overrightarrow{BC} = V_2 + V_1$ and $\overrightarrow{AD} = V_1 + V_2$ (Definition of vector subtraction, Definition 14-5);

Hence, $\overline{BC} \parallel \overline{AD}$ (Definition 14-9)

Similarly, $\overrightarrow{AB} = -V_1 + V_2$ and $\overrightarrow{DC} = V_2 - V_1$.

$\overline{AB} \parallel \overline{DC}$ (Definition 14-9)
ABCD is a parallelogram (Definition 7-1).

10. Draw $\triangle ABC$ as required, and let P be any point.
 $\overrightarrow{PC} - \overrightarrow{PA'} = \overrightarrow{A'C}$, and $\overrightarrow{PA'} - \overrightarrow{PB} = \overrightarrow{BA'}$ (Vector subtraction)
 $\overrightarrow{PC} + \overrightarrow{PB} = 2\overrightarrow{PA'}$ (Simplification)
 Similarly, $\overrightarrow{PB} + \overrightarrow{PA} = 2\overrightarrow{PC'}$, and $\overrightarrow{PA} + \overrightarrow{PC} = 2\overrightarrow{PB}$
 Then $2\overrightarrow{PA'} + 2\overrightarrow{PB} + 2\overrightarrow{PC'} = \overrightarrow{PC} + \overrightarrow{PB} + \overrightarrow{PA} + \overrightarrow{PC} + \overrightarrow{PB} + \overrightarrow{PA}$ (Addition property)
 $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = \overrightarrow{PA'} + \overrightarrow{PB'} + \overrightarrow{PC'}$ (Division property)

11. Let $V = (x,y)$.
 $0 \cdot V = 0 \cdot (x,y) = (0 \cdot x, 0 \cdot y)$ (Definition 14-3)
 $(0 \cdot x, 0 \cdot y) = (0, 0) = O$ (Definition 14-12)
 Suppose O_1 exists such that $V + O_1 = V$ for any vector V
 $V + O = V$ (Property of additive identity vector)
 $V + O = V + O_1$ (Transitive property)
 V' is the additive inverse of V, so $(V' + V) + O = (V' + V) + O_1$, or $O + O = O + O_1$
 That is, $O = O_1$ (Property of additive identity vector).

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Exercises continued

12. $\overrightarrow{DX} = \overrightarrow{AX} - \overrightarrow{AD}$ (Definition of vector subtraction)
 $\overrightarrow{YB} = \overrightarrow{CB} - \overrightarrow{CY}$ (Definition of vector subtraction)
 $\overrightarrow{AE} - \overrightarrow{AD} = \overrightarrow{DE}$, and $\overrightarrow{CF} - \overrightarrow{CB} = \overrightarrow{BF}$ (Definition of vector subtraction)
 $\overrightarrow{AD} = -\overrightarrow{CB}$ (Definition 14-5, Theorem 7-1.3)
 $\overrightarrow{CF} = -\overrightarrow{AE}$ (Definition 14-5, Given)
 $\overrightarrow{DE} = -\overrightarrow{BF}$ (Postulate 2-1)
 $\overrightarrow{DE} = \overrightarrow{FB}$ (Definition 14-5)
 Hence, $\overleftrightarrow{DX} \parallel \overleftrightarrow{BY}$ (Definition 6-1)

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Review Exercises

1. (-8, 3)	2. (-2, 7)	3. (5, 2)
4. (1, -9)	5. (-5, 3)	6. (-5, 4)

7. $\sqrt{x^2 + y^2} = \sqrt{(-x)^2 + (-y)^2}$

8. (-2, -2)	9. (5, 6)	10. (-3, 8)
11. (3, -8)	12. (3, -8)	13. (12, 7)

14. Any side of $\triangle ABC$ can be a diagonal of the parallelogram. (6, 1) and (0, 3) are the other two.

15. 30/7	16. 9/2	17. $x = 0$
18. $x = \pm\sqrt{249}$	19. $x = \pm 7$	20. (-3, 1)
21. (-5, -3)	22. (2, 9)	23. (9, 4)
24. (1, -1)	25. (-6, 4)	26. (1, -1)
27. (6, 6)	28. (6, 6)	

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29. (3, -2)
30. $(x,y) + (0, 0) = (x + 0, y + 0) = (0 + x, 0 + y) = (0, 0) + (x,y) = (x,y)$

31. (-10, 15)	32. (-24, 8)	33. (0, 0)
34. (0, 0)	35. (12, -12)	36. (-35, -56)
37. (0, 0)	38. (24, 36)	39. (0, 15)
40. (-6, -15)	41. (-14, 1)	42. (-11, -1)
43. $\left(\frac{20}{3}, -\frac{14}{3}\right)$	44. (-15, 11)	45. 30, -24)
46. (30, -24)		

47. $A - C$ is the vector with initial point at the terminal point of C, and terminal point at the terminal point of A.

48. $B + C$ is the diagonal of the parallelogram whose sides are congruent to B and C.

49. $B - C$ is the vector with initial point at the terminal point of C, and terminal point at the terminal point of B.

50. $C - A$ is the vector with initial point at the terminal point of A, and terminal point at the terminal point of C.

51. Draw rectangle ABCD such that $\overrightarrow{AB} = (x,y)$, $\overrightarrow{DC} = (x,y)$, $\overrightarrow{BC} = (-ky, kx)$, and $\overrightarrow{AD} = (-ky, kx)$.

 $|\overrightarrow{AC}| = \sqrt{(x - ky)^2 + (y + kx)^2} = \sqrt{(x^2 + y^2)(k^2 + 1)}$
 (Definition 14-4)
 $|\overrightarrow{DB}| = \sqrt{(ky + x)^2 + (kx + y)^2} = \sqrt{(x^2 + y^2)(k^2 + 1)}$
 (Definition 14-4)
 The fact that $\overline{AC}$ and $\overline{DB}$ bisect each other follows from the first vector proof presented in this section.

Chapter Test

1. A vector is a directed segment.
2. Any member of a set of equal directed segments.
3. D (0, 1)
4. $2\sqrt{5}$ 5. 5 6. $\sqrt{5}$
7. $V = (12, 6)$
8. $\overrightarrow{AB} + \overrightarrow{BC} = (-4, 3) + (2, -7) = (-2, 4)$; $\overrightarrow{CA} = (2, -4)$
 $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = (-2 + 2, 4 - 4) = (0, 0)$.

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Chapter Test continued

9. (-17, 14) 10. (6, 18) 11. See page 582
12. See page 590 13. No, it could be kite shaped.

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Exercises

1. Let $V_1 = (x_1, x_2), V_2 = (y_1, y_2)$.

   $V_1 \cdot V_2 = x_1y_1 + x_2y_2$ (Definition of dot product);

   $x_1y_1 + x_2y_2 = y_1x_1 + y_2x_2$ (Multiplication of real numbers is commutative)

   $V_2 \cdot V_1 \quad y_1x_1 = y_2x_2$ (Definition of dot product)

   $V_1 \cdot V_2 = V_2 \cdot V_1$ (Transitive property).

2. $V_1 \cdot (V_2 + V_3) = V_1 \cdot (0, 1, 0) = 1 \cdot 0 + 3 \cdot 1 + 5 \cdot 0$

   $= 3$ (Definition 14-7, Definition of dot product);

   $V_1 \cdot V_2 + V_1 \cdot V_3 = [1 \cdot 2 + 3 \cdot 0 + 5 \cdot 3] + [1 \cdot (-2) + 3 \cdot 1 + 5 \cdot (-3)] = 17 + (-14) = 3$ (Definition of dot product); $V_1 \cdot (V_2 + V_3) = V_1 \cdot V_2 + V_1 \cdot V_3$ (Transitive property).

3. $\sqrt{29}$

4. $V_1 \cdot V_2 = 1 \cdot 4 + 4 \cdot 2 + 3 \cdot (-4) = 12 - 12 = 0$ (Definition of dot product); since neither V_1 nor V_2 is equivalent to the zero vector, $V_1 \perp V_2$ (Definition of perpendicular vectors).

5. Not necessarily (due to zero divisors).

6. Draw △ABC with A(0, 0), B(a,0) and C(b,c).
   Let $V_1 = (a,0)$, $V_2 = (b-a, c)$, $V_3 = (-b, -c)$, $V_4 = (b, 0)$, and $V_5 = (-kc, k(b-a))$.

   V_4 and V_5 intersect in $(b, [(a-b) \cdot b]/c)$.

   The vector V_6 from $(a, 0)$ to $(b, [(a-b) \cdot b]/c)$ is $(b-a, [(a-b) \cdot b]/c)$. But this is the altitude to the opposite side if and only if $V_3 \cdot V_6 = 0$. Thus, $(-b, -c) \cdot (b-a, [(a-b) \cdot b]/c) = -b(b-a) + [-c \cdot (a-b) \cdot b]/c = 0$.

7. Let $V_1 = (a_1, a_2)$, $V_2 = (b_1, b_2)$, $V_3 = (c_1, c_2)$.

   $V_1 \cdot (V_2 + V_3) = (a_1, a_2) \cdot (b_1 + c_1, b_2 + c_2)$ (Definition 14-7);

   $V_1 \cdot (V_2 + V_3) = a_1(b_1 + c_1) + a_2(b_2 + c_2)$ (Definition of dot product);

   $V_1 \cdot (V_2 + V_3) = a_1b_1 + a_1c_1 + a_2b_2 + a_2c_2$ (Distributive property)
   $V_1 \cdot V_2 = a_1b_1 + a_2b_2$, $V_1 \cdot V_3 = a_1c_1 + a_2c_2$ (Definition of dot product);

   $V_1 \cdot V_2 + V_1 \cdot V_3 = a_1b_1 + a_2b_2 + a_1c_1 + a_2c_2$ (Addition property)

   $V_1 \cdot V_2 + V_1 \cdot V_3 = a_1b_1 + a_1c_1 + a_2b_2 + a_2c_2$ (Commutative property of addition)
   $V_1 \cdot V_2 + V_1 \cdot V_3 = V_1 \cdot (V_2 + V_3)$ (Transitive property).

DEFINITIONS

Definition 1-1 If every element of set A is also an element of set B, then A is a *subset* of B. We denote this by $A \subseteq B$.

Definition 1-2 If set A is a subset of set B, but set B contains at least one element not in set A, then A is a *proper subset* of B. We denote this by $A \subset B$.

Definition 1-3 Set A and set B are *equal sets* if and only if each is a subset of the other. A = B if and only if $A \subseteq B$ and $B \subseteq A$.

Definition 1-4 Set A and set B are *equivalent sets* if and only if the two sets have exactly the same number of elements.

Definition 1-5 The *intersection* of sets A and B is the set whose elements are elements of both A and B. We denote the intersection of A and B by $A \cap B$.

Definition 1-6 The *union* of sets A and B is the set whose elements are elements of at least one of the sets, A and B. We denote the union of A and B by $A \cup B$.

Definition 1-7 A set that has no elements is the *null* set, $\emptyset$.

Definition 1-8 Two sets whose intersection is the null set are *disjoint* sets.

Definition 1-9 The *distance* between two points A and B is the unique number corresponding to them. We denote the distance from A to B by AB.

Definition 1-10 If points A, B, and C are distinct points of a line such that AC + CB = AB then point C lies *between* points A and B.

Definition 1-11 A set of points is *collinear* if all the points lie in a line.

Definition 1-12 A set of points is *coplanar* if all the points lie in the same plane.

Definition 1-13 A *line segment* is determined by two points, A and B. All points between A and B are points of the segment, and A and B are its endpoints. We denote line segment AB by $\overline{AB}$.

Definition 1-14 AB is the *measure* of the distance between A and B. We denote the measure of $\overline{AB}$ by AB.

Definition 1-15 Point C is the *midpoint* of $\overline{AB}$ if C lies between A and B, and AC = CB.

Definition 1-16 A point may be considered a segment whose measure is zero. We may call A *a zero segment.*

Definition 1-17 The sets of points described in the line separation postulate are *half-lines,* with P as the endpoint of each half-line. We denote the half-line from P through B by $\overset{\circ}{PB}$.

Definition 1-18 A *ray* is the union of a half-line and its endpoint. We denote the ray PQ by $\overrightarrow{PQ}$.

Definition 1-19 An *angle* is the union of two noncollinear rays with a common endpoint. The common endpoint of the rays is the vertex of the angle. The two rays are the sides of the angle.

Definition 1-20 The sets described in the Plane Separation Postulate are *half planes,* with *m* the edge of each half plane.

Definition 1-21 The *measure of an angle* is the number given in the Angle Measure Postulate. We denote the measure of $\angle$ A by m$\angle$ A.

Definition 1-22 The *interior of an angle* is the intersection of the two half planes. Each half plane has for an edge the line containing one of the sides of the angle, and contains the other side.

Definition 1-23 *Adjacent angles* are two angles with a common vertex and a common side, but no common interior points.

Definition 1-24 An angle whose measure is between 0 and 90 is called an *acute angle.*
An angle whose measure is between 90 and 180 is called an *obtuse angle.*
An angle whose measure is 90 is called a *right angle.*

Definition 1-25 If when $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ intersect, they form adjacent angles of equal measure then the lines are *perpendicular.* To indicate that $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{AC}$ we write $\overleftrightarrow{AB} \perp \overleftrightarrow{AC}$.

Definition 1-26 If the two noncommon sides of adjacent angles are collinear - that is, form opposite rays - we say that the angles are a *linear pair.*

Definition 1-27 If the sum of the measures of two angles is 90, the angles are *complementary angles.* Each angle is the complement of the other.

Definition 1-28 If the sum of the measures of two angles is 180, the angles are *supplementary angles.* Each angle is the supplement of the other.

Definition 1-29 If point P lies in the interior of $\angle$ BAC, such that m$\angle$ BAP = m$\angle$ PAC, then $\overrightarrow{AP}$ is the *bisector* of $\angle$ BAC.

Definition 1-30 A straight angle is an angle whose measure is 180. Treated as an angle, a single ray is an angle whose measure is 0.

Definition 1-31 A *triangle* is a three-sided polygon.

Definition 1-32 An *acute triangle* is a triangle with three acute angles.
An *obtuse triangle* is a triangle with one obtuse angle.
A *right triangle* is a triangle with one right angle.

Definition 1-33 A *quadrilateral* is a four-sided polygon. The quadrilateral has four sides and four vertices. The endpoints of a side are *consecutive vertices.* Vertices that are not consecutive in a quadrilateral are *opposite vertices.* Sides with a common endpoint are *adjacent sides.* In a quadrilateral, sides that are not adjacent are *opposite sides.*

Definition 1-34 *Space* is the set of all points.

Definition 1-35 The two sets described in the Space Separation Postulate are *half-spaces* with plane P as the *face* of each half-space.

Definition 1-36 A *dihedral angle* is an angle formed by the union of a line and two noncoplanar half-planes that share the line as edge. The half-planes are the faces of the dihedral angle.

Definition 2-1 If the sides of two angles form two pairs of opposite rays then the angles are *vertical angles.*

Definition 3-1 *Congruent segments* are line segments which have equal measure. We denote their congruence by $\overline{AB} \cong \overline{CD}$.

Definition 3-2 *Congruent angles* are angles which have equal measure. We denote their congruence by $\angle$ YCX $\cong$ $\angle$ QDP.

Definition 3-3 If there is a correspondence ABC $\leftrightarrow$ XYZ such that the sides and angles of $\triangle$ABC are congruent to the corresponding sides and angles of $\triangle$XYZ, then ABC $\leftrightarrow$ XYZ is a congruence, and the triangles are said to be *congruent triangles.*

Definition 3-4 In a triangle, *an angle is included* by the sides of the triangles that lie in the sides of the angle. In a triangle, *a side is included* by the angles whose vertices are endpoints of the segment.

Definition 3-5 By the *SAS correspondence,* two sides and the included angle of one triangle are congruent to the corresponding parts of a second triangle.

Definition 3-6 By the *ASA correspondence,* two angles and the included side of one triangle are congruent to the corresponding parts of a second triangle.

Definition 3-7 By the *SSS correspondence*, the three sides of one triangle are congruent to the corresponding sides of a second triangle.

DEFINITIONS

Definition 3-8 An *angle bisector of a triangle* is a segment that lies on the ray bisector of an angle of the triangle and has its endpoints at the angle's vertex and at a point of the side opposite the angle.

Definition 3-9 A *median* of a triangle is a segment whose endpoints are a vertex of the triangle and the midpoint of the side opposite that vertex.

Definition 3-10 An *altitude* of a triangle is a perpendicular segment whose endpoints are a vertex of the triangle and a point of the line containing the opposite side.

Definition 3-11 A triangle is *equiangular* if all its angles are congruent.

Definition 3-12 A *scalene* triangle has no congruent sides. An *isosceles* triangle has two congruent sides. An *equilateral* triangle has all three sides congruent.

Definition 4-1 The *converse* of $p \to q$ is the implication $q \to p$ formed by interchanging the hypothesis p and the conclusion q.

Definition 4-2 The *inverse* of $p \to q$ is the implication $\sim p \to \sim q$, formed by negating each statement of $p \to q$.

Definition 4-3 The *contrapositive* of $p \to q$ is the implication $\sim q \to \sim p$, formed by negating the statements of the converse, $q \to p$.

Definition 4-4 The *perpendicular bisector* of a segment is the line perpendicular to the segment at its midpoint.

Definition 4-5 The *distance to a line from an external point* is the measure of the perpendicular segment from the point to the line.

Definition 4-6 The *projection* of an external point onto a line in a plane is the foot of the perpendicular segment from the point to the line.

Definition 4-7 *A line and a plane are perpendicular* if and only if they intersect and all the lines in the plane which pass through the point of intersection are perpendicular to the given line.

Definition 4-8 The *distance to a plane from an external point* is the measure of the perpendicular segment from the point to the plane.

Definition 4-9 The *perpendicular bisecting plane* of a segment is the plane perpendicular to the segment at its midpoint.

Definition 4-10 *Two planes are perpendicular to each other* if and only if one plane contains a line perpendicular to the second plane.

Definition 4-11 The *projection onto a plane of a segment* $\overline{AB}$ is the set of points in the plane which are the projections of the points of $\overline{AB}$.

Definition 4-12 If a plane is perpendicular to the edge of a given dihedral angle, the intersection is called a *plane angle* of the dihedral angle.

Definition 4-13 The *measure of a dihedral angle* is the same as the measure of any of its plane angles.

Definition 5-1 An *exterior angle of a triangle* is an angle that forms a linear pair with one of the interior angles of the triangle.

Definition 5-2 In a triangle, the two interior angles which do not form a linear pair with an exterior angle are the *remote interior angles* of that exterior angle.

Definition 6-1 Two distinct lines are *parallel* if and only if they are coplanar and do not intersect.

Definition 6-2 A line is a *transversal* of two or more coplanar lines if and only if it intersects each of these lines in different points.

Definition 6-3 Let $\{P_1, P_2, \ldots, P_n\}$ be a set of n distinct points in a plane, where $n \geq 3$. Let the n segments, $\overline{P_1 P_2}, \overline{P_2 P_3}, \ldots, \overline{P_{n-1} P_n}, \overline{P_n P_1}$, have the following properties:

1. No two segments intersect except at their endpoints.
2. No two segments with a common endpoint are collinear.

The union of such segments is called a *polygon*.
The *consecutive vertices of a polygon* are the endpoints of a side of the polygon.
The *consecutive angles of a polygon* are angles of a polygon at consecutive vertices.
A *diagonal of a polygon* is a line segment joining any two nonconsecutive vertices.

Definition 6-4 A *convex polygon* is a polygon whose points all lie on one side of each line containing a side of the polygon.

Definition 6-5 A *regular polygon* is a polygon with all angles congruent and all sides congruent.

Definition 7-1 A *parallelogram* is a quadrilateral in which both pairs of opposite sides are parallel.

Definition 7-2 A pair of *consecutive angles* of a parallelogram is formed by two angles that have their vertices in the endpoints of the same side of the parallelogram.

Definition 7-3 The *distance between two parallel lines* is the length of the perpendicular segment from any point of one line to the other line.

Definition 7-4 An *altitude of a parallelogram* is the perpendicular segment from any point of a line containing one side of the parallelogram to the line containing the opposite side of the parallelogram.

Definition 7-5 A *rectangle* is a parallelogram with one right angle.

Definition 7-6 A *rhombus* is a parallelogram with two adjacent sides congruent.

Definition 7-7 A *square* is a parallelogram with one right angle and two adjacent sides congruent.

Definition 7-8 If a transversal intersects two lines m and n in points A and B, then lines m and n *intercept* $\overline{AB}$ on the transversal.

Definition 7-9 A ***midline*** of a triangle is the line segment joining the midpoints of two sides of the triangle.

Definition 7-10 A quadrilateral is a *trapezoid* if the sides of exactly one opposite pair are parallel.

Definition 7-11 The *median of a trapezoid* is the line segment joining the midpoints of the nonparallel sides.

Definition 7-12 An *altitude of a trapezoid* is the perpendicular segment from any point in the line containing one base of the trapezoid to the line containing the other base.

Definition 7-13 An *isosceles trapezoid* is a trapezoid whose nonparallel sides are congruent.

Definition 8-1 For any two positive real numbers a and b, $b \neq 0$, the *ratio* of a to b is the quotient $\frac{a}{b}$.

Definition 8-2 An equation expressing the equality of two ratios is called a *proportion*.

Definition 8-3 An *exterior angle bisector of a triangle* is a segment that bisects an exterior angle of a triangle, and has its endpoints at the vertex of the bisected angle and in the line containing the side of the triangle opposite this angle.

Definition 8-4 *Similar polygons* are polygons whose corresponding angles are congruent and whose corresponding sides are proportional.

DEFINITIONS

Definition 8-5 A *ratio of similitude* of two similar polygons is the ratio of the measure of any pair of corresponding sides.

Definition 8-6 *Similar triangles* are triangles whose corresponding angles are congruent and whose corresponding sides are proportional.

Definition 8-7

The *sine of* $\angle A$ = $\frac{\text{length of side opposite } \angle A}{\text{length of hypotenuse}} = \frac{a}{c}$

Definition 8-8

The *cosine of* $\angle A$ = $\frac{\text{length of side adjacent to}}{\text{length of hypotenuse}} = \frac{b}{c}$

Definition 8-9

The *tangent of* $\angle A$ = $\frac{\text{length of side opposite } \angle A}{\text{length of side adjacent } \angle A} = \frac{a}{b}$

Definition 9-1 A *circle* is the set of all points in a plane that are the same distance from a given point in the plane.

Definition 9-2 A *sphere* is the set of all points in space that are the same distance from a given point.

Definition 9-3 A *chord of a circle* is a line segment whose endpoints are points of the circle.
A *secant* of the circle is a line that contains a chord.
A *secant ray* is a ray that contains a chord of the circle and whose endpoint is one of the endpoints of the chord.

Definition 9-4 A *diameter* of a circle or sphere is any chord containing the center of the circle or sphere, or the length of such a chord.

Definition 9-5 Two or more circles or spheres are *concentric* if they have a common center.

Definition 9-6 *Circles are congruent* if and only if their radii are congruent.

Definition 9-7 The *common chord* of two intersecting circles is the segment whose endpoints are the points of intersection of the two circles.

Definition 9-8 An *inscribed polygon* is a polygon whose vertices are all points of a circle.

Definition 9-9 A *tangent* to a circle is a line in the plane of the circle that intersects the circle in exactly one point.

Definition 9-10 A *common tangent* of two circles is a line that is tangent to two coplanar circles.

Definition 9-11 *Tangent circles* are two coplanar circles that intersect in exactly one point.

Definition 9-12 A *tangent plane* of a sphere is a plane that contains exactly one point of the sphere.

Definition 9-13 A *central angle* of a circle is an angle whose vertex is at the center of the circle.

Definition 9-14 A *minor arc* of a circle is the part of the circle intersected by a central angle and included in the angle's interior.

Definition 9-15 A *major arc* of a circle is the part of the circle intersected by a central angle and included in the angle's exterior.

Definition 9-16 A *semicircle* is an arc of a circle whose endpoints are the endpoints of a diameter of the circle.

Definition 9-17 The *degree measure of a minor arc* is the measure of its corresponding central angle.
The *degree measure of a semicricle* is 180.
The *degree measure of a major arc* is equal to 360 minus the degree measure of its corresponding minor arc.

Definition 9-18 *Congruent arcs* are arcs of the same or congruent circles whose degree measures are equal.

Definition 9-19 The *chord of an arc* of a circle is the chord of the circle whose endpoints are the same as those of the arc.

Definition 9-20 An angle *intercepts* an arc if:

1. each ray of the angle contains exactly one endpoint of the arc;
2. all other points of the arc lie in the interior of the angle.

This arc is called the *intercepted arc* of the angle.

Definition 9-21 An *inscribed angle* of a circle is an angle whose vertex is a point of the circle and whose rays contain two other points of the circle.

Definition 9-22 A *cyclic quadrilateral* is a quadrilateral whose vertices are concyclic.

Definition 9-23 A *tangent segment* is a segment of a tangent to a circle, one of whose endpoints is the point of tangency.

Definition 9-24 A *circle is inscribed* in a polygon if it is tangent to each side of the polygon. The polygon is a *circumscribed polygon* and the circle is an *inscribed circle.*

Definition 9-25 A *secant segment* is a segment that intersects a circle in two distinct points, exactly one of which is an endpoint of the segment.

Definition 9-26 The *circumference* of a circle is the limit of the perimeters of the inscribed regular polygons as the number of sides of the polygons increases without bound.

Definition 10-1 The *Cartesian product* of X and Y is the set of all ordered pairs (x, y) where x belongs to X and y belongs to Y.

Definition 10-2 To each point in a plane, there corresponds a unique ordered pair of real numbers - the abscissa and the ordinate of the point. The *abscissa* is the coordinate of the projection of the point onto the x-axis. The *ordinate* is the coordinate of the projection of the point onto the y-axis.

Definition 10-3 The distance between P (x_1, y_1) and Q (x_2, y_1), two points of a line parallel to the x-axis, is PQ, where PQ = $|x_2 - x_1|$ or $|x_1 - x_2|$.

Definition 10-4 The distance between P (x_1, y_1) and Q(x_1, y_2), two points of a line parallel to the y-axis, is PQ, where PQ = $|y_2 - y_1|$ or $|y_1 - y_2|$.

Definition 10-5 *Given* P_1 (x_1, y_1) and $P_2(x_2, y_2)$ such that $x_2 \neq x_1$, the slope of the segment $P_1 P_2$ is the number m where $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$.

Definition 10-6 The slope of the line determined by (x_1, y_1), and (x_2, y_2), such that $x_1 \neq x_2$, is the same as m, the slope of any segment of the line.

Definition 11-1 A *locus* is the set of points, and only those points, that satisfy a given condition.

Definition 11-2 The *center of a regular polygon* is the center of its circumscribed or inscribed circle. Segments drawn from the center of the polygon to the vertices of the polygon are radii of the circumscribed circle. Segments drawn from the center of the polygon perpendicular to the sides of the polygon are radii of the inscribed circle.

Definition 11-3 If a single point is common to three or more lines, then the lines are *concurrent*. The common point is the *point of concurrence.*

Definition 12-1 A *triangular region* is the union of a triangle and its interior.

Definition 12-2 A *polygon region* is the union of a finite number of coplanar triangular regions that intersect in either a line segment or a point.

Definition 12-3 The *center of a regular polygon* is the center of its circumscribed (or inscribed) circle.

DEFINITIONS

Definition 12-4 A segment joining any vertex of a regular polygon with the center of that polygon is a *radius of the polygon*.

Definition 12-5 The *apothem* of a regular polygon is the segment from the center of the polygon perpendicular to a side of the polygon.

Definition 12-6 A *circular region* is the union of a circle and its interior.

Definition 12-7 The *area of a circular region* is the limit of the areas of the regular polygons inscribed in the circle, as the number of sides increases without bound.

Definition 12-8 A *sector of a circle* is a region bounded by an arc of the circle and the two radii which contain the endpoints of the arc.

Definition 12-9 A *segment of a circle* is a region bounded by a minor arc of a circle and the chord containing the endpoints of the arc.

Definition 13-1 A *polyhedral angle* is the figure formed by three or more planes that intersect in one point.

Definition 13-2 A *polyhedral region* is a solid completely bounded by portions of intersecting planes.

Definition 13-3 A *polyhedron* is the union of the bounding plane regions of a polyhedral region.

Definition 13-4 A *diagonal of a polyhedron* is a segment joining two vertices that are not in the same face.

Definition 13-5 A polyhedron is a *regular polyhedron* if and only if all its faces are congruent regular polygons and all its polyhedral angles are congruent.

Definition 13-6 The *total area of a polyhedron* is the sum of the areas of all its faces.

Definition 13-7 Two *planes*, or a line and a plane, are *parallel* if they do not intersect.

Definition 13-8 A *prism* is a polyhedron whose faces consist of two parallel and congruent polygons, called bases, and the parallelograms, called lateral faces, formed by connecting pairs of corresponding vertices of the parallel polygons.

Definition 13-9 The *altitude of a prism* is the perpendicular segment between the parallel planes of the bases, or the length of that segment.

Definition 13-10 The polygonal region formed by the intersection of a polyhedron and a plane passing through it is a *section of the polyhedron*. A *right section of a prism* is a section formed by a plane which cuts all the lateral edges of the prism and is perpendicular to one of them.

Definition 13-11 A *right prism* is a prism whose lateral edges are perpendicular to the bases of the prism. A prism that is not a right prism is an *oblique prism*.

Definition 13-12 The *lateral area of a prism* is the sum of the areas of the lateral faces.

Definition 13-13 A *parallelepiped* is a prism in which the bases are parallelograms.

Definition 13-14 A *pyramid* is a pyramid is a polyhedron formed by joining each point in the sides of a polygonal region to a common point not in the plane of the polygonal region.

Definition 13-15 A *regular pyramid* is a pyramid the sides of whose base form a regular polygon whose center coincides with the projection of the vertex onto the base.

Definition 13-16 A *cylindrical surface* is the set of all lines parallel to a given line and intersecting a given curve in a plane that does not contain the given line. The given curve in the definition is called the *directrix* of the cylindrical surface. Each line in the set of lines referred to in the definition is called an *element of the cylindrical surface*.

Definition 13-17 A *cylinder* is that portion of a closed cylindrical surface between two parallel planes, together with the portions of the planes enclosed by the surface.

Definition 13-18 A *right circular cylinder* is the portion of a circular cylindrical surface lying between two parallel planes that are perpendicular to the elements of the surface, together with the two circular regions of the planes enclosed by the surface.

Definition 13-19 Two *figures are congruent* if and only if every dimension of one is congruent to the corresponding dimension of the other.

Definition 13-20 A *conical surface* is the set of all lines intersecting a given plane curve and passing through a fixed point that is not in the plane of the curve.

Definition 13-21 If a plane intersects one nappe of a closed conical surface, then that part of the surface between the vertex and the plane, together with the region of the plane enclosed by the surface, is a *cone*.

Definition 13-22 A *right circular cone* is a circular cone whose axis is perpendicular to the plane of the base.

Definition 13-23 The *frustum of a cone* is the figure formed by the base of the cone, a section of the cone parallel to the base, and the surface of the cone between the base and the section.

Definition 14-1 *Vector* AB is the directed segment from A to B.

Definition 14-2 *Equal vectors* are vectors having equal lengths and the same direction.

Definition 14-3 The set of all vectors equal to a particular vector is called a *class of equal vectors*. The ordered pair of numbers (x, y) defines this class.

Definition 14-4 The *length of a vector* $\overrightarrow{AB}$ belonging to the class defined by (x, y) equals $\sqrt{x^2 + y^2}$.

Definition 14-5 *Opposite vectors* are vectors having equal lengths, but opposite directions.

Definition 14-6 The *sum* $A + B$ *of two vectors* A *and* B is the vector from initial point of A to the terminal point of B, when the initial point of B is at the terminal point of A.

Definition 14-7 The sum $A + B$ of $A = (x_1, y_1)$ and $B = (x_2 , y_2)$ is the vector $(x_1 + x_2 , y_1 + y_2)$.

Definition 14-8 Two *vectors are parallel* if and only if they lie in the same line or in parallel lines.

Definition 14-9 $A = (x_1 , y_1)$ and $B = (x_2 , y_2)$ are parallel vectors if and only if $x_2 = kx_1$ and $y_2 = ky_1$, where $k \neq 0$.

Definition 14-10 *Perpendicular vectors* are of the form (x_1 , y_1) and $(- ky_1, kx_1)$, where $k \neq 0$, or of the form $(x , 0)$ and $(0, y)$.

Definition 14-11 The absolute values of $A = (x_1, y_1)$ and $B = (x_2, y_2)$ are equal if and only if $\sqrt{x_1^2 + y_1^2} = \sqrt{x_2^2 + y_2^2}$. We denote the equality of the absolute values of A and B by $|A| = |B|$.

Definition 14-12 The *zero vector*, 0, equals (0, 0).

Definition 14-13 If $V_1 = (x_1, y_1)$ and a is a scalar, then $aV_1 = (ax_1, ay_1)$.

POSTULATES

Postulate 1-1 *The Distance Postulate* To every pair of distinct points there corresponds a unique positive number.

Postulate 1-2 *The Ruler Postulate* A correspondence between the real numbers and the points of a line can be made such that:

1. to every real number there corresponds exactly one point of the line;
2. to every point of the line there corresponds exactly one real number;
3. the distance between any two points is the absolute value of the difference of the coordinates of the points.

Postulate 1-3 *The Line Separation Postulate* Any point P of a line separates the line into two distinct sets of points, one set on each side of P.

Postulate 1-4 *The Plane Separation Postulate* Any line m of plane P separates the points of P that are not an m into two sets such that

1. for any two points of A and B of a set, $\overline{AB}$ lies entirely in the set;
2. if A is in one set and B is in the other set, then $\overline{AB}$ intersects m.

Postulate 1-5 *The Angle Measure Postulate* To every angle there corresponds a real number between, but not including 0 and 180.

Postulate 1-6 *The Supplementary Angles Postulate* Two angles that form a linear pair are supplementary.

Postulate 1-7 *The Space Separation Postulate* Any plane P separates the points in space that are not in plane P into two distinct sets such that:

1. for any two points A and B of a set, $\overline{AB}$ lies entirely in that set;
2. if A is in one set and B is in the other, then $\overline{AB}$ intersects plane P.

Postulate 2-1 *The Substitution Postulate* If $a = b$ then either a or b may be replaced by the other in any statement without changing the truth or falsity of the statement.

Postulate 2-2 *The Point Uniqueness Postulate* If n is any positive number, then there is exactly one point N of $\overrightarrow{PQ}$ such that PN = n.
If XY = n, then the Point Uniqueness Postulate states that there is one and only one point R of $\overrightarrow{PQ}$ such that PR = XY.

Postulate 2-3 *The Line Postulate* Any two distinct points determine exactly one line that contains both points.

Postulate 2-4 *The Point Betweenness Postulate* Between any two points, there is at least one point of the line determined by the two points. That is, if P is between A and B, then AP + PB = AB.

Postulate 2-5 *The Plane Postulate* Any three noncollinear points determine exactly one plane that contains the three points.

Postulate 2-6 *The Plane Intersection Postulate* The intersection of two distinct planes is a line.

Postulate 2-7 *The Points-in-a-Plane Postulate* If two distinct points of a line lie in a plane, then the line lies in that plane.

Postulate 2-8 *The Space Postulate* Space contains at least four noncoplanar points.

Postulate 2-9 *The Angle Uniqueness Postulate* Given $\overrightarrow{PQ}$ on the edge of half-plane R: For any real number n, where $0 < n < 180$, there is one and only one ray $\overrightarrow{PB}$, where B is in R, such that m∠ QPB = n.

Postulate 2-10 *The Angle Sum Postulate* If B is in the interior of ∠ APQ, then m∠ APQ = m∠ APB + m∠ BPQ.

Postulate 2-11 *The Angle Difference Postulate* If A is in the interior of ∠ DBC and in the same half-plane, for edge $\overleftrightarrow{BC}$, as D, then m∠ ABD = m∠ ABC - m∠ DBC.

Postulate 3-1 *The SAS Postulate* Any SAS correspondence is a congruence.

Postulate 3-2 *The ASA Postulate* Any ASA correspondence is a congruence.

Postulate 3-3 *The SSS Postulate* Any SSS correspondence is a congruence.

Postulate 6-1 *The Parallel Postulate* Through a given point not contained in a given line, there exists only one line parallel to the given line.

Postulate 8-1 *Proportional Line Segments Postulate* Three or more parallel lines intercept proportional segments on two or more transversals.

Postulate 9-1 *The Arc Addition Postulate* If P is a point of $\widehat{AB}$, distinct from A and B, then m $\widehat{APB}$ = m $\widehat{AP}$ + m $\widehat{PB}$.

Postulate 11-1 *Two-Circle Postulate* Given circle A with radius a, circle B with radius b, and the length of their line-segment of centers $\overline{AB}$, such that AB = c; if each of the numbers a, b, c is less than the sum of the other two, then the circles intersect in exactly two points which lie on opposite sides of $\overline{AB}$.

Postulate 12-1 *The Area Postulate* To each polygonal region there corresponds a unique positive real number.

Postulate 12-2 *The Congruence Postulate for Areas* If two polygons are congruent, then their polygonal regions have equal areas.

Postulate 12-3 *The Area Addition Postulate* If a polygonal region R is the union of nonoverlapping polygonal regions R_1 and R_2 then $\mathcal{A}R = \mathcal{A}R_1 + \mathcal{A}R_2$.

Postulate 12-4 *Area of a Rectangle Postulate* The area of a rectangle equals the product of the length of its base and its altitude. $\mathcal{A}$ rectangle = $b \cdot h$.

Postulate 13-1 *The Volume Postulate* To each polyhedral region there corresponds a unique positive real number.

Postulate 13-2 *The Volume Postulate for Rectangle Solids* The volume of a rectangular solid (that is, the volume of a rectangular parallelpiped) equals the product of its length, width, and height. $V = lwh$.

Postulate 13-3 *Cavalieri's Principle* If two solid regions have equal altitudes and if sections made by planes parallel to the base of each solid and at the same distance from each base are always equal in area, then the volumes of the solid regions are equal.

Postulate 13-4 *The Lateral Area Postulate* The lateral area of a circular cylinder is equal to the product of an element and the perimeter of a right section.

Postulate 13-5 *The Area of a Cone Postulate* The lateral area of a right circular cone is equal to one-half the product of its slant height and the circumference of its base.

Postulate 13-6 *The Volume of a Cone Postulate* The volume of a circular cone is equal to one-third the product of the area of its base and its altitude. $V = \frac{1}{3}ba$.

Postulate 13-7 *The Area Postulate for Spheres* The area of a sphere is the product of 2π, the diameter, and the radius. $\mathcal{L} = 2\pi \cdot 2r \cdot r = 4\pi r^2$.

Postulate 13-8 *The Volume Postulate for Spheres* The volume V of a sphere with radius r is $4/3\ \pi r^3$.

THEOREMS AND COROLLARIES

Theorem 2-5.1 If a point does not lie in a given line, then there is exactly one plane containing both the point and the line.

Theorem 2-5.2 If two distinct lines intersect, then they intersect in exactly one point.

Theorem 2-5.3 If two lines intersect, then there is exactly one plane containing them.

Theorem 2-5.4 *The Midpoint Uniqueness Theorem* Any line segment has exactly one midpoint.

Theorem 2-5.5 All right angles are equal in measure.

Theorem 2-5.6 Two supplementary angles of equal measure are right angles.

Theorem 2-6.1 Complements of angles of equal measure, or of the same angle, have the same measure.

Theorem 2-6.2 Supplements of angles of equal measure, or of the same angle, have the same measure.

Theorem 2-6.3 *The Vertical Angle Theorem* Vertical angles have the same measure.

Theorem 2-6.4 If two intersecting lines form one right angle, then the lines form four right angles.

Theorem 2-6.5 If two intersecting lines are perpendicular, they form right angles.

Theorem 2-6.6 If two intersecting lines form a right angle, then they are perpendicular.

Theorem 3-1.1 All right angles are congruent.

Theorem 3-1.2 Two congruent supplementary angles are right angles.

Theorem 3-1.3 Complements of congruent angles, or of the same angle, are congruent.

Theorem 3-1.4 Supplements of congruent angles, or of the same angle, are congruent.

Theorem 3-1.5 Vertical angles are congruent.

Theorem 3-1.6 *The Identity Theorem for Segments* Every segment is congruent to itself.

Theorem 3-1.7 *The Identity Theorem for Angles* Every angle is congruent to itself.

Theorem 3-1.8 Both segment congruence and angle congruence are equivalence relations.

Theorem 3-2.1 Triangle congruence is an equivalence relation.

Theorem 3-4.1 *Angle Bisector Theorem* Every angle has exactly one bisector.

Theorem 3-4.2 *Isosceles Triangle Theorem* The base angles of an isosceles triangle are congruent.

Corollary 3-4.2a Every equilateral triangle is equiangular.

Theorem 3-4.3 In a triangle, if two angles are congruent, then the sides opposite these angles are congruent.

Corollary 3-4.3a Every equiangular triangle is equilateral.

Theorem 3-5.1 The angle bisectors of the base angles of an isosceles triangle are congruent.

Theorem 4-2.1 If a plane and a line not in the plane intersect, the intersection is only one point.

Theorem 4-4.1 Two lines are perpendicular if and only if they form right angles.

Theorem 4-4.2 *Perpendicular Uniqueness Theorem* In a plane, there is one and only one line perpendicular to a given line through a given point on the line.

Theorem 4-4.3 *The Perpendicular Bisector Theorem* A line is the perpendicular bisector of a segment if and only if it is the set of all points in the plane equidistant from the endpoints of the segment.

Corollary 4-4.3a If two distinct points are both equidistant from the endpoints of a segment, the two points determine the perpendicular bisector of the segment.

Theorem 4-4.4 If a point is not on a line, then there is a line through the point perpendicular to the given line.

Theorem 4-4.5 Through a point external to a line, there is at most one line perpendicular to the given line.

Theorem 4-4.6 Through a point external to a line, there is one and only one line perpendicular to the given line.

Corollary 4-4.6a No triangle has two right angles.

Theorem 4-5.1 If a line is perpendicular to each of two intersecting lines at their point of intersection, then the line is perpendicular to the plane determined by them.

Theorem 4-5.2 If a line is perpendicular to a plane, then any line perpendicular to the given line, at its point of intersection with the given plane, is in the given plane.

Theorem 4-5.3 Through a point in a given line, there passes one and only one plane perpendicular to the given line.

Theorem 4-5.4 Through a point in a given plane there passes one and only one line perpendicular to the given plane.

Theorem 4-5.5 Through a given point there passes one and only one plane perpendicular to a given line.

Theorem 4-5.6 Through a given point there passes one and only one line perpendicular to a given plane.

Theorem 4-5.7 The perpendicular bisecting plane of a segment is the set of all points equidistant from the endpoints of the segment.

Theorem 4-5.8 Two lines perpendicular to the same plane are coplanar.

Theorem 4-5.9 If a line is perpendicular to a plane, then every plane containing the line is perpendicular to the given plane.

Theorem 5-1.1 For any numbers k, m, and n, if $n = m + k$, and $k > 0$, then $n > m$.

Corollary 5-1.1a If P is a point of $\overline{AB}$ between A and B, then $AB > AP$ and $AB > BP$.

Corollary 5-1.1b If P is a point in the interior of $\angle ABC$, then $m\angle ABC > m\angle ABP$ and $m\angle ABC > m\angle CBP$.

Theorem 5-2.1 *The Exterior-Angle Theorem* The measure of an exterior angle of a triangle is greater than the measure of either remote interior angle.

Corollary 5-2.1a If a triangle has one right angle, then the other two angles must be acute.

Theorem 5-3.1 If two sides of a triangle are not congruent, then the angles opposite those sides are not congruent, the angle with the greater measure being opposite the longer side.

Theorem 5-3.2 If two angles of a triangle are not congruent, then the sides opposite those angles are not congruent, the longer side being opposite the angle with the greater measure.

Theorem 5-4.1 *The Triangle Inequality Theorem* The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Theorem 5-4.2 The shortest segment joining a line with an external point is the perpendicular segment from the point to the line.

THEOREMS AND COROLLARIES

Theorem 5-5.1 If two sides of a triangle are congruent respectively to two sides of a second triangle, and the measure of the included angle of the first triangle is greater than the measure of the included angle of the second triangle, then the measure of the third side of the first triangle is greater than the measure of the third side of the second triangle.

Theorem 5-5.2 If two sides of one triangle are congruent respectively to two sides of a second triangle, and the measure of the third side of the first triangle is greater than the measure of the third side of the second triangle, then the measure of the included angle of the first triangle is greater than the measure of the included angle of the second triangle.

Theorem 6-1.1 If two distinct lines in the same plane are perpendicular to the same line, then they are parallel.

Corollary 6-1.1a Parallel lines exist in any given plane.

Corollary 6-1.1b In a plane, if a line is perpendicular to one of two parallel lines, then it is also perpendicular to the other line.

Corollary 6-1.1c If each of two lines is parallel to a third line, then they are parallel to each other.

Corollary 6-1.1d In a plane, if a line is perpendicular to one of two parallel lines and if another line is perpendicular to the second of the two parallel lines, then these two perpendicular lines are parallel to each other.

Theorem 6-2.1 If two lines are cut by a transversal so that the alternate interior angles are congruent, then the lines are parallel.

Corollary 6-2.1a If two lines are cut by a transversal so that the corresponding angles are congruent, then the lines are parallel.

Corollary 6-2.1b If two lines are cut by a transversal so that the interior angles on the same side of the transversal are supplementary, then the lines are parallel.

Theorem 6-3.1 If two parallel lines are cut by a transversal, then the alternate interior angles are congruent.

Corollary 6-3.1a If two parallel lines are cut by a transversal, then the corresponding angles are congruent.

Corollary 6-3.1b If two parallel lines are cut by a transversal, then the interior angles on the same side of the transversal are supplementary.

Theorem 6-4.1 The measure of the exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

Theorem 6-4.2 The sum of the measures of the angles of any triangle is 180.

Corollary 6-4.2a If two angles of one triangle are congruent to two angles of another triangle, then the remaining angle of the first triangle is congruent to the remaining angle of the second triangle.

Corollary 6-4.2b The acute angles of a right triangle are complementary.

Theorem 6-5.1 The sum of the measures of the interior angles of a convex polygon of n sides equals $(n - 2)\,180$.

Corollary 6-5.1a The measure of each interior angle of a regular polygon of n sides is $\frac{(n-2)\,180}{n}$.

Theorem 6-5.2 The sum of the measures of the exterior angles of any convex polygon is 360.

Corollary 6-5.2a The measure of each exterior angle of a regular polygon of n sides is $\frac{360}{n}$.

Corollary 6-5.2b The measure of each interior angle of a regular polygon of n sides is $180 - \frac{360}{n}$.

Theorem 6-6.1 *AAS Congruence Theorem* If two angles and a nonincluded side of one triangle are congruent to two angles and a nonincluded side of another triangle, then the two triangles are congruent.

Theorem 6-6.2 *HL Congruence Theorem* If the hypotenuse and one leg of a right triangle are congruent to the corresponding hypotenuse and leg of another right triangle, then the two triangles are congruent.

Theorem 7-1.1 A diagonal of a parallelogram divides the parallelogram into two congruent triangles.

Theorem 7-1.2 The opposite sides of a parallelogram are congruent.

Theorem 7-1.3 The opposite angles of a parallelogram are congruent.

Theorem 7-1.4 Any two consecutive angles of a parallelogram are supplementary.

Theorem 7-1.5 The diagonals of a parallelogram bisect each other.

Theorem 7-1.6 If two lines are parallel, then all the points of each line are equidistant from the other line.

Theorem 7-2.1 A quadrilateral is a parallelogram if both pairs of opposite sides are congruent.

Theorem 7-2.2 A quadrilateral is a parallelogram if two of its sides are both congruent and parallel.

Theorem 7-2.3 A quadrilateral is a parallelogram if the opposite angles are congruent.

Theorem 7-2.4 A quadrilateral is a parallelogram if the angles of either opposite pair are congruent and the sides of either opposite pair are parallel.

Theorem 7-2.5 A quadrilateral is a parallelogram if the angles of either opposite pair are congruent and the sides of either opposite pair are congruent.

Theorem 7-2.6 If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Theorem 7-3.1 A rectangle has four right angles.

Theorem 7-3.2 The diagonals of a rectangle are congruent.

Theorem 7-3.3 If a quadrilateral has four right angles, then it is a rectangle.

Theorem 7-3.4 If a parallelogram has congruent diagonals, then it is a rectangle.

Theorem 7-3.5 The median to the hypotenuse of a right triangle is half as long as the hypotenuse.

Theorem 7-4.1 The four sides of a rhombus are congruent.

Theorem 7-4.2 The diagonals of a rhombus bisect its angles.

Theorem 7-4.3 The diagonals of a rhombus are perpendicular to each other.

Theorem 7-4.4 If a quadrilateral has four congruent sides, then it is a rhombus.

Theorem 7-4.5 If a parallelogram has perpendicular diagonals, then it is a rhombus.

Theorem 7-4.6 If a diagonal of a parallelogram bisects an angle of the parallelogram, then the parallelogram is a rhombus.

THEOREMS AND COROLLARIES

Theorem 7-6.1 If three or more parallel lines intercept congruent segments on one transversal, then they intercept congruent segments on any other transversal.

Theorem 7-6.2 The midline of a triangle is parallel to the third side of the triangle.

Theorem 7-6.3 The midline of a triangle is half as long as the third side of the triangle.

Theorem 7-6.4 If a line containing the midpoint of one side of a triangle is parallel to a second side of the triangle, then it also contains the midpoint of the third side of the triangle.

Theorem 7-7.1 The median of a trapezoid is parallel to the bases.

Theorem 7-7.2 The length of the median of a trapezoid is half the sum of the lengths of the bases.

Theorem 7-7.3 The angles in each pair of base angles of an isosceles trapezoid are congruent.

Theorem 7-7.4 The diagonals of an isosceles trapezoid are congruent.

Theorem 7-7.5 The opposite angles of an isosceles trapezoid are supplementary.

Theorem 7-7.6 A trapezoid is isosceles if the angles in one pair of base angles are congruent.

Theorem 7-7.7 A trapezoid is isosceles if the angles in one pair of opposite angles are supplementary.

Theorem 7-7.8 A trapezoid is isosceles if its diagonals are congruent.

Theorem 8-1.1 For any positive real numbers a, b, c, and d, $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$.

Corollary 8-1.1a *The Inversion Corollary* For any positive real numbers a, b, c, and d, $\frac{a}{b} = \frac{c}{d}$ if and only if $\frac{b}{a} = \frac{d}{c}$.

Corollary 8-1.1b *The Alternation Corollary* For any positive real numbers a, b, c, and d, $\frac{a}{b} = \frac{c}{d}$ if and only if $\frac{a}{c} = \frac{b}{d}$.

Theorem 8-1.2 For any positive real numbers a, b, c, and d, $\frac{a}{b} = \frac{c}{d}$ if and only if $\frac{a+b}{b} = \frac{c+d}{d}$.

Theorem 8-1.3 For any positive real numbers a, b, c, and d, $\frac{a}{b} = \frac{c}{d}$ if and only if $\frac{a-b}{b} + \frac{c-d}{d}$.

Theorem 8-1.4 For any positive real numbers a, b, c, d, e, and f, if $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ then $\frac{a+c+e}{b+d+f} = \frac{a}{b}$.

Theorem 8-1.5 For any positive real numbers a, b, c, d, and e, if $\frac{a}{b} = \frac{c}{d}$ and $\frac{a}{b} = \frac{c}{e}$, then $d = e$.

Theorem 8-2.1 If a line segment parallel to one side of a triangle intersects the other two sides, then it divides these two sides into proportional segments.

Corollary 8-2.1a If a line parallel to one side of a triangle intersects the other two sides, then it divides these two sides proportionally.

Theorem 8-2.2 If a line divides two sides of a triangle into proportional segments, then the line is parallel to the remaining side of the triangle.

Corollary 8-2.2a If a line divides two sides of a triangle proportionally, then the line is parallel to the remaining side of the triangle.

Theorem 8-3.1 An angle bisector of any triangle divides the side of the triangle opposite the angle into segments proportional to the adjacent sides.

Corollary 8-3.1a An exterior angle bisector of a triangle determines with each of the other vertices segments along the line containing the opposite side of the triangle which are proportional to the two remaining sides.

Theorem 8-4.1 If two triangles are similar to the same triangle, or to similar triangles, then the triangles are similar to each other.

Corollary 8-4.1a If a given triangle is similar to a triangle that is congruent to a third triangle, then the given triangle is also similar to the third triangle.

Theorem 8-5.1 *AAA Similarity Theorem* If the corresponding angles of two triangles are congruent, then the two triangles are similar.

Corollary 8-5.1a *AA Similarity Corollary* If two pairs of corresponding angles of two triangles are congruent, then the triangles are similar.

Corollary 8-5.1b If two right triangles have a congruent pair of corresponding acute angles, then the triangles are similar.

Corollary 8-5.1c If a line parallel to one side of a triangle intersects the other two sides, then it cuts off a triangle similar to the original triangle.

Theorem 8-6.1 *SAS Similarity Theorem* If two sides of one triangle are proportional to two sides of another triangle, and the angles included by those sides are congruent, then the triangles are similar.

Theorem 8-6.2 *SSS Similarity Theorem* If the corresponding sides of two triangles are proportional, then the two triangles are similar.

Theorem 8-7.1 The altitude to the hypotenuse of a right triangle separates the triangle into two triangles that are similar to each other and to the original triangle.

Corollary 8-7.1a The altitude to the hypotenuse of a right triangle divides the hypotenuse so that either leg is the mean proportional between the hypotenuse and the segment of the hypotenuse adjacent to that leg.

Corollary 8-7.1b The altitude to the hypotenuse of a right triangle is the mean proportional between the segments of the hypotenuse.

Theorem 8-8.1 *The Pythagorean Theorem* The sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse.

Theorem 8-8.2 If the sum of the squares of the lengths of two sides of a triangle equals the square of the length of the third side, then the angle opposite this third side is a right angle.

Theorem 8-9.1 In an isosceles right triangle, the hypotenuse is $\sqrt{2}$ times as long as a leg.

Corollary 8-9.1a In an isosceles right triangle, either leg is $\frac{\sqrt{2}}{2}$ times as long as the hypotenuse.

Theorem 8-9.2 In a 30-60-90 triangle, the side opposite the angle of measure 30 is half as long as the hypotenuse.

Theorem 8-9.3 In a 30-60-90 triangle, the side opposite the angle of measure 60 is $\frac{\sqrt{3}}{2}$ times as long as the hypotenuse.

Corollary 8-9.3a In a 30-60-90 triangle, the hypotenuse is $\frac{2\sqrt{3}}{3}$ times as long as the side opposite the angle of measure 60.

THEOREMS AND COROLLARIES

Corollary 8-9.3b In a 30-60-90 triangle,the longer leg is $\sqrt{3}$ times as long as the shorter leg.

Theorem 9-2.1 The line containing the center of a circle and the midpoint of a chord of the circle that is not a diameter is perpendicular to the chord.

Theorem 9-2.2 The line containing the center of a circle and perpendicular to a chord of the circle bisects the chord.

Theorem 9-2.3 In the plane of a circle, the perpendicular bisector of a chord of the circle contains the center of the circle.

Corollary 9-2.3a No circle contains three collinear points.

Theorem 9-2.4 In the same circle, or in congruent circles, chords are congruent if and only if they are equidistant from the center of the circle.

Theorem 9-2.5 In the same circle, or in congruent circles, if two chords are not congruent, then the longer chord is nearer the center of the circle than the shorter chord.

Corollary 9-2.5a The diameter of a circle is the longest chord of the circle.

Theorem 9-2.6 In the same circle or in congruent circles, if two chords are not congruent, then the chord nearer the center of the circle is the longer of the two chords.

Theorem 9-3.1 A line perpendicular to a radius at the endpoint on the circle is tangent to the circle.

Theorem 9-3.2 The radius of a circle is perpendicular to a tangent at the point of tangency.

Theorem 9-3.3 A line perpendicular to a tangent of a circle at the point of tangency contains the center of the circle.

Theorem 9-4.1 The intersection of a sphere with a plane containing the center of the sphere is a circle whose center and radius are the same as those of the sphere.

Theorem 9-4.2 The intersection of a sphere with a plane that contains points in the interior of the sphere is a circle of the sphere.

Theorem 9-4.3 If a line contains the center of a sphere and the center of the circle of intersection of the sphere with a plane that does not contain the center of the sphere, then the line is perpendicular to the intersecting plane.

Theorem 9-4.4 If a line contains the center of a sphere and is perpendicular to a plane that intersects the sphere and contains interior points of the sphere other than the center, then the line also contains the center of the circle of intersection.

Theorem 9-4.5 If a line contains the center of the circle of intersection of a sphere with a plane that does not contain its center and if the line is perpendicular to the plane, then the line also contains the center of the sphere.

Theorem 9-4.6 A plane perpendicular to a radius of a sphere at its intersection with the sphere is tangent to the sphere.

Theorem 9-4.7 The radius of a sphere is perpendicular to a tangent plane at the point of tangency.

Theorem 9-4.8 A line perpendicular to a tangent plane of a sphere at the point of tangency contains the center of the sphere.

Theorem 9-5.1 In the same circle or in congruent circles, two arcs are congruent if and only if their corresponding central angles are congruent.

Theorem 9-5.2 In the same circle, or in congruent circles, if two chords are congruent, then their arcs are congruent.

Theorem 9-5.3 In the same circle, or in congruent circles, if two arcs are congruent, then their chords are congruent.

Theorem 9-6.1 The measure of an inscribed angle is one-half the measure of its intercepted arc.

Corollary 9-6.1a Angles inscribed in the same arc are congruent.

Corollary 9-6.1b If two inscribed angles intercept congruent arcs, then the angles are congruent.

Corollary 9-6.1c An angle inscribed in a semicircle is a right angle.

Theorem 9-6.2 The opposite angles of an inscribed quadrilateral are supplementary.

Theorem 9-6.3 In any circle, parallel chords intercept congruent arcs.

Theorem 9-6.4 In any circle, a tangent and a chord parallel to it intercept congruent arcs.

Theorem 9-7.1 If one side of a quadrilateral subtends congruent angles at the two nonadjacent vertices, then the quadrilateral is cyclic.

Theorem 9-7.2 If a pair of opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.

Theorem 9-8.1 The measure of the angle formed by a tangent and a chord of a circle is one-half the measure of its intercepted arc.

Theorem 9-8.2 The measure of an angle formed by two chords intersecting in a point in the interior of a circle is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

Theorem 9-8.3 The measure of an angle formed by two secants of a circle intersecting in a point in the exterior of the circle is equal to one-half the difference of the measures of the intercepted arcs.

Theorem 9-8.4 The measure of an angle formed by a secant and a tangent to a circle intersecting in a point in the exterior of the circle is equal to one-half the difference of the measures of the intercepted arcs.

Theorem 9-8.5 The measure of an angle formed by two tangents to a circle is equal to one-half the difference of the measures of the intercepted arcs.

Corollary 9-8.5a The sum of the measure of an angle formed by two tangents to a circle and the measure of the closer intercepted arcs is 180.

Theorem 9-9.1 Two tangent segments that have the same endpoint in the exterior of the circle to which they are tangent are congruent.

Corollary 9-9.1a Two tangents to a circle that intersect in an exterior point of the circle form congruent angles with the line containing both the exterior point and the center of the circle.

Theorem 9-9.2 If a secant segment and a tangent segment to the same circle share an endpoint in the exterior of the circle, then the length of the tangent segment is the mean proportional between the length of the secant segment and the length of its external segment.

Corollary 9-9.2a If a secant segment and a tangent segment to the same circle share an endpoint in the exterior of the circle, then the square of the length of the tangent segment equals the product of the lengths of the secant segment and its external segment.

Theorem 9-9.3 If two secant segments of the same circle share an endpoint in the exterior of the circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.

Theorem 9-9.4 If two chords intersect in the interior of a circle, thus determining two segments in each chord, the product of the lengths of the segments of one chord equals the product of the lengths of the segments of the other chord.

THEOREMS AND COROLLARIES

Theorem 9-10.1 The ratio, π, of the circumference of a circle to its diameter is the same for all circles.

Corollary 9-10.1a The circumferences of any two circles are proportional to their radii.

Theorem 9-10.2 If an arc of a circle of radius r has measure n, then the length of the arc is $\frac{n}{360} \cdot 2\pi r$.

Theorem 10-2.1 *The Distance Formula* The distance PQ between any two points, $P(x_1, y_1)$ and $Q(x_2, y_2)$, is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ or $\sqrt{(\Delta x)^2 + (\Delta y)^2}$

Theorem 10-2.2 *The Midpoint Formula* The midpoint of the segment determined by points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is the point $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Theorem 10-3.1 Two nonvertical lines are parallel if and only if they have equal slopes.

Theorem 10-3.2 Two nonvertical lines with slopes m_1 and m_2 are perpendicular if and only if $m_1 = \frac{1}{m_2}$; that is, $m_1 \cdot m_2 = -1$.

Theorem 10-4.1 *The Point-Slope Theorem* An equation of the line that contains the point (x_1, y_1) and has a slope m is $y - y_1 = m(x - x_1)$, where (x, y) is any other point of the line.

Corollary 10-4.1a *The Slope-Intercept Theorem* An equation of the line with a slope m and y-intercept b is $y = mx + b$, where (x, y) is any other point of the line.

Theorem 10-4.2 *The Two-Intercept Theorem* An equation of the line with x-intercept a and y-intercept b is $\frac{x}{a} + \frac{y}{b} = 1$.

Theorem 11-1.1 The set of points in the interior of an angle and equidistant from the sides is the bisector of the angle, exclusive of the vertex.

Theorem 11-7.1 *The Perpendicular Bisector Theorem for Concurrence* The perpendicular bisectors of the sides of a triangle are concurrent at a point equidistant from the vertices of the triangle.

Corollary 11-7.1a Any three noncollinear points are points of one and only one circle.

Corollary 11-7.1b Two nonconcentric circles intersect in at most two points.

Theorem 11-7.2 *The Altitude Theorem for Concurrence* The lines containing the three altitudes of a triangle are concurrent.

Theorem 11-7.3 *The Median Theorem for Concurrence* The medians of a triangle are concurrent at a point of each median located two-thirds of the way from the vertex to the opposite side.

Theorem 11-7.4 *The Angle Bisector Theorem for Concurrence* The angle bisectors of a triangle are concurrent at a point equidistant from the sides of the triangle.

Theorem 12.1.1 The area of a square equals the square of the length of a side. $\mathcal{A}$ square $= s^2$.

Corollary 12-1.1a The area of a square equals one-half the square of the length of one of its diagonals. $\mathcal{A}$ square $= \frac{1}{2}d^2$.

Theorem 12-2.1 The area of a right triangle equals one-half the product of the lengths of its legs. $\mathcal{A}$ right $\Delta = \frac{1}{2}(l_2 \cdot l_2)$.

Theorem 12-2.2 The area of any triangle equals one-half the product of the lengths of its base and the altitude to that base. $\mathcal{A}$ triangle $= \frac{1}{2} b \cdot h$.

Corollary 12-2.2a Two triangles have equal areas if their bases have the same length and the altitudes to their bases have the same length.

Corollary 12-2.2b Triangles that share the same base and have their vertices in a line parallel to the base have equal areas.

Theorem 12-2.3 If two triangles have congruent bases, then the ratio of their areas equals the ratio of the lengths of their altitudes.

Theorem 12-2.4 If two triangles have congruent altitudes, then the ratio of their areas equals the ratio of the lengths of their bases.

Theorem 12-2.5 The area of any triangle equals the product of the lengths of any two sides multiplied by the sine of the included $\angle$ C. $\mathcal{A}$ triangle $= \frac{1}{2} ab \cdot \sin \angle C$.

Theorem 12-2.6 The area of an equilateral triangle equals $\frac{\sqrt{3}}{4}$ times the square of the length of a side. $\mathcal{A}$ equilateral triangle $= \frac{s^2\sqrt{3}}{4}$.

Theorem 12-2.7 The area of an equilateral triangle equals $\frac{\sqrt{3}}{3}$ times the square of the length of an altitude. $\mathcal{A}$ equilateral triangle $\frac{h^2\sqrt{3}}{3}$.

Theorem 12-3.1 The area of a parallelogram equals the product of the lengths of a base and the altitude to that base. $\mathcal{A}$ parallelogram $= b \cdot h$.

Theorem 12-3.2 The area of a rhombus equals one-half the product of the lengths of its diagonals. $\mathcal{A}$ rhombus $= \frac{1}{2}(d_1 \cdot d_2)$.

Theorem 12-3.3 The area of a trapezoid equals one-half the product of the length of the altitude and the sum of the lengths of the bases. $\mathcal{A}$ trapezoid $= \frac{1}{2} h (b_1 + b_2)$.

Theorem 12-4.1 The area of a regular polygon equals one-half the product of the lengths of the apothem and the perimeter. $\mathcal{A}$ regular polygon $= \frac{1}{2} a \cdot p$.

Theorem 12-5.1 The area of a circle with radius r equals πr^2.

Theorem 12-5.2 The area of a sector with radius r and a central angle of measure n equals $\frac{n}{360} \cdot \pi r^2$.

Theorem 12-6.1 The ratio of the areas of two similar triangles equals the square of their ratio of similitude.

Corollary 12-6.1a The ratio of similitude of any pair of similar triangles equals the square root of the ratio of their areas.

Theorem 12-6.2 The ratio of the areas of two similar polygons equals the square of their ratio of similitude.

Corollary 12-6.2a The ratio of similitude of any pair of similar polygons equals the square root of the ratio of their areas.

Corollary 12-6.2b The ratio of the areas of two circles equals the square of their ratio of similitude.

Corollary 12-6.2c The ratio of similitude of two circles equals the square root of the ratio of their areas.

Theorem 13-1.1 The sum of the measures of any two face angles of a trihedral angle is greater than the measure of the third face angle.

Theorem 13-1.2 The sum of the measures of the face angles of any convex polyhedral angle is less than 360.

Theorem 13-2.1 If a plane intersects two parallel planes, then it intersects them in two parallel lines.

Theorem 13-2.2 If a line is perpendicular to one of two parallel planes, then it is perpendicular to the other.

Theorem 13-2.3 If two planes are perpendicular to the same line, then they are parallel.

THEOREMS AND COROLLARIES

Corollary 13-2.3a If two planes are both parallel to a third plane, then they are parallel to each other.

Theorem 13-2.4 If two lines are perpendicular to the same plane, then the lines are parallel.

Corollary 13-2.4a If a plane is perpendicular to one of two parallel lines, then the plane is perpendicular to the other line.

Theorem 13-2.5 Parallel planes are everywhere equidistant.

Theorem 13-2.6 The lateral edges of a prism are congruent and parallel.

Corollary 13-2.6a The plane of a right section of a prism is perpendicular to all its lateral edges.

Theorem 13-2.7 The lateral area of a prism is equal to the product of the perimeter of a right section and the length of a lateral edge.

Corollary 13-2.7a The lateral area of a right prism is equal to the product of the perimeter of one of its bases and its altitude.

Corollary 13-2.7b Every section of a prism made by a plane parallel to the bases is congruent to the bases.

Theorem 13-2.8 Two prisms have equal volumes if their bases have equal areas and their altitudes are equal.

Corollary 13-2.8a The plane passing through two diagonally opposite edges of a parallelepiped divides it into two triangular prisms of equal volume.

Theorem 13-2.9 The volume of a prism is the product of the area of a base and the altitude. $V = \mathcal{A}\, B\, h$.

Corollary 13-2.9a The volume of a parallelepiped is the product of the area of any face and the length of the altitude to that face.

Theorem 13-3.1 The lateral edges of a regular pyramid are congruent.

Corollary 13-3.1a The lateral edges of a regular pyramid form congruent isosceles triangles.

Theorem 13-3.2 The lateral area of a regular pyramid is equal to one-half the product of its siant height and the perimeter of its base. $\mathcal{L} = \frac{1}{2} s \cdot p$

Theorem 13-3.3 If a pyramid is cut by a plane parallel to its base, the section is a polygon similar to the base, and the lateral edges and altitude are divided proportionally, with the ratio of their lengths to the lengths of the segments cut off between the section and the vertex equal to the ratio of similitude of the base and the section.

Corollary 13-3.3a If two pyramids have congruent altitudes and bases with equal areas, sections parallel to the bases at equal distances from the vertices have equal areas.

Corollary 13-3.3b If two pyramids have congruent altitudes and bases with equal areas, then they have equal volumes.

Theorem 13-3.4 The volume of a triangular pyramid equals one-third the product of the area of its base and the altitude. $V = \frac{1}{3} b \cdot a$.

Theorem 13-3.5 The volume of any pyramid is equal to one-third the product of the area of its base and altitude.

Theorem 13-4.1 The bases of a cylinder are congruent.

Corollary 13-4.1a Every section of a cylinder made by a plane parallel to the bases is congruent to the bases.

Theorem 13-4.2 The volume of a cylinder is the product of the area of the base and the altitude. $V = \mathcal{A}B \cdot a$.

Corollary 13-4.2a The volume V of a right circular cylinder with radius of base r and altitude h equals $\pi r^2 h$. $V = \pi r^2 h$.

Theorem 13-4.3 The lateral area of a right circular cylinder equals the product of its altitude and circumference.

Corollary 13-4.3a The total area, T, of a right circular cylinder with altitude h and radius of base r is $2\pi r^2 + 2\pi rh$ or $2\pi r(r + h)$.

Theorem 13-4.4 If the lateral area of a right circular cone is L, the total area is T, the radius of the base is r, and the slant height is s, then $\mathcal{L} = \pi rs$ and $T = \pi r^2 + \pi rs = \pi r\ (r + s)$.

Theorem 13-4.5 The volume V of a circular cone whose altitude is h and whose base has radius r equals $\frac{1}{3}\pi r^2 h$.

Theorem 13-5.1 The lateral area of a frustum of a right circular cone is one-half the product of the slant height and the sum of the circumferences of two bases.

Theorem 13-5.2 The area of the surface generated by a line segment revolving about an axis in its plane, but not perpendicular to it nor crossing it, is equal to the product of the projection of the segment onto the axis and the circumference of the circle whose radius is the perpendicular to the segment drawn from its midpoint to the axis.

Theorem 14-4.1 Addition of vectors is commutative. $(V_1 + V_2 = V_2 + V_1)$

Theorem 14-4.2 Addition of vectors is associative. $(V_1 + V_2) + V_3 = V_1 + (V_2 + V_3)$

Theorem 14-5.1 If a is a scalar and V_1 and V_2 are vectors, then $a\ (V_1 + V_2) = aV_1 + aV_2$

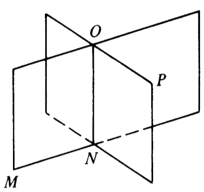
Theorem 14-5.2 If a and b are scalars and V is a vector, then $(a + b)V = aV + bV$.

Theorem 14-5.3 If a and b are scalars and V is a vector, then $a(bV) = (ab)V$.

Test 1

Exercises

1. True　2 False　3. False　4. True
5. False　6. True　7. True　8 False
9. True　10. False　11. True　12. True
13. True　14. True　15. False　16. True
17. False　18. False　20. "if" true and "then" false
21. of measure 45　22. acute　23. scalene
24. supplementary　25. octagon　26. 44
27. BCHI　28. ∅　29. IH　30. ∅
31. No　32. Answers vary any three points
33. $\overleftrightarrow{BE}$　34. ∠ADC　35. ∠BCE　36. 6
37. ⊥　38. bisector　39. exterior　40. diagonal
41. opposite　42. $\overline{TC}$, $\overline{RA}$ (or $\overline{TA}$, $\overline{RC}$)
43. 110
44.



Test 2

Exercises

1. False　2. True　3. False　4. False
5. False　6. False　7. True　8. True
9. False　10. False
11. Two angles are vertical angles　12. skew
13. postulate
14. Intersecting lines are perpendicular
15. Angles of equal measure are right angles
16. may not　17. undefined terms　18. are collinear
19. False　20. False　21. True
22. More information than necessary
23. Not sufficiently restrictive
24. Too much information
25. a) False　b) False　c) True
26. transitive
27. *Given* ∠1 and ∠2 vertical angles
∠1 and ∠3 complementary angles
Prove ∠2 and ∠3 complementary angles

Statements	*Reasons*
1. ∠1 and ∠2 vertical ∠s	1. Given
2. m ∠ 1 = m ∠ 2	2. Vertical angles have equal measure
3. ∠1 and ∠3 are supplementary	3. Given
4. M ∠ 1 + m ∠ 3 = 90	4. Def. of complementary angles
5. m ∠ 2 + m ∠ 3 = 90	5. Substitution (2) in (4)
6. ∠ 2 and ∠ 3 are complementary	6. Def. of complementary angles

Test 3

Exercises

1. False　2. True　3. False　4. False
5. True　6. False　7. True　8. False
9. True　10. False　11. False　12. False
13. b　14. c　15. b　16. d
17. △ TSP ≅ △ WRQ;
TSRW and PSRQ (a three dimensional drawing)
18. ∠SPT = ∠RQW
19. isosceles
20. 1. EC = BD (addition)
2. ∠ABC ≅ ∠ACB (Theorem 3 - 4.2)
3. $\overline{BC}$ ≅ $\overline{BC}$
4. △ BCE = △ CBD (SAS)

22. 1. BC = BC
2. AC = DB (addition)
3. △ ABC = △ DQB (SAS)
4. ∠PCB = ∠QBC (Definition 3 - 3)

23. 1. Draw $\overline{RT}$
2. $\overline{RT}$ ≅ $\overline{RT}$
3. △ STR = △ WRT (SAS)
4. ∠SRT = ∠WTR (Def. 3 - 3)
5. ∠STW - ∠WRS (addition)

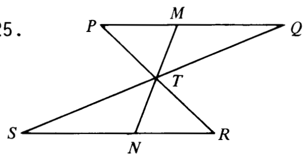
Test 4

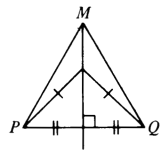
Exercises

1. If △ APE is a right triangle, then m∠A = 90.
2. contrapositive
3. If 3 + 4 ≠ 7, then 12 ∠̸ 5
4. perpendicular　5. indirect
6. False
7. The line is the perpendicular bisector
8. True　9. True
10. The conclusion false
11. $\overline{ET}$　12. projection
13. is not　14. indirect
15. one　16. Theorems 4-4.4 and 4-4.5
17. $\overline{OP}$　18. e.g. ∠A - BC - 0
19. $\overline{AP}$ = $\overline{CP}$　20. Fig. 1
21. True; Theorem 4-4.3
22. See the proofs of Theorem 4-4.5 and 4-4.6 (Page 154-155)
23. Prove the contrapositive (Theorem 3-4.2)
24. No. It is true that if two triangles are congruent then the corresponding angles are congruent. The converse of this statement is <u>not</u> true

Test 4

25. $\overline{PT} \cong \overline{TR}$ and $\overline{QT} \cong \overline{TS}$
$\angle PTQ \cong \angle RTS$ (vertical angles)
$\triangle PTQ \cong \triangle RTS$ (SAS)
$\angle P \cong \angle R$ (Def. 3 - 3)
$\angle PTM \cong \angle RTN$ (vertical angles)
$\triangle PMT \cong \triangle RNT$ (ASA)
$\overline{NT} \cong \overline{MT}$ (Def. 3 - 3)

25. 

20. 

Test 5

Exercises

1. Multiplication property of order
2. Addition property of order
3. Trichotomy property
4. Exterior Angle Theorem
5. QM < LP
6. >
7. opposite $\overline{LM}$
8. opposite $\angle P$
9. $\angle B$, $\angle H$, $\angle J$
10. $4 < x < 10$
11. is not
12. $\overline{DC}$
13. BC
14. DC + DG
15. $\angle CDE$
16. $\angle ACF$
17. $m\angle ACF > m\angle ABC$ (T5-2.1), $m\angle ABC > m\angle BEG$ (T5-2.1)
$m\angle ACF > m\angle BEG$ (Transitive property).
18. Suppose it has a right angle; then, C5-2.1a, the other two angles are acute. But, by C3-4.2a, all three angles must be congruent. Since an acute angle cannot be a right angle (D1-24), we have a contradiction. Thus, no equilateral triangle has a right angle.
19. Use the subtraction property of order
20. $\angle NFP \cong \angle NPF$ (T3-4.2), $m\angle NPF = m\angle NPA + m\angle APF$ (P2-10) $m\angle NPF > m\angle APF$ and $m\angle APF < m\angle NFP$ (Subtraction prop. and P2-1) AP > AF (T5-3.2)
21. AP + PB > AB, AP + PC > AC, PB + PC > BC (T5-4.1)
2 (AP + PB + PC) > AB + AC + BC (Addition property, distributive property), AP + PB + PC > $\frac{1}{2}$ (AB + AC + BC).

Test 6

Exercises

1. l and m
2. cannot
3. is
4. alternate interior angles
5. Corresponding angles
6. interior angles on the same side of the transversal
7. transversal
8. 150
9. 115
10. 180 - 8x or 45 - 2x
11. 30 - 60 - 90
12. 1080
13. $\frac{5 \cdot 180}{7}$ or $\approx$ 129
14. 360
15. 14 (Refer to Section 6-5, Exercise 38 for a quick solution)

Test 6

16. cannot
17. The smallest possible angle-measure is 60, which is the measure of each angle of a regular 3-gon.
18. Refer to page 234.
19. Refer to page 234.
20. $\angle 1 \cong \angle 8$ implies $\angle 4 \cong \angle 5$ (T2-6.3), $l \parallel m$ (T6-2.1).
21. $\angle 1 \cong \angle 4$, $\angle 2 \cong \angle 3$ (T3-4.2), $\angle 2 \cong \angle 1$ (P2-1)
$AB \parallel DE$ (C6-2.1a)
22. Draw $\triangle AOW$ such that $\overline{EAWB}$ and $\overline{JOA}$.
$m\angle OAW + m\angle AWO + m\angle AOW = 180$ (T6-4.2)
$m\angle JOW + m\angle AOW = 180$, $m\angle BWO + m\angle DWA = 180$,
$m\angle EAO + m\angle WAO = 180$ (D1-28). $m\angle JOW + \angle BWO$ $+ m\angle EAO + m\angle AOW + m\angle OWA + m\angle WAO = 3(180)$ or
$m\angle JOW + m\angle BWO + m\angle EAO = w(180) = 360$.
23. 70
24. AC = AD (T3-4.3), $\angle BAC = \angle EAD$ (P2-11),
$\angle ABC = \angle AED$ (AAS)
25. $\angle C \cong \angle F$ (T6-3.1), $m\angle F = 90$ (P2-1), CB = FE (Subtraction), $\triangle ABC \cong \triangle DEF$ (HL).

Test 7

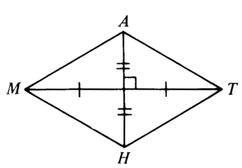
Exercises

1. 144
2. 51
3. $S \subset R \subset H \subset Q$
4. 18
5. 12
6. a. 2 b. 8
7. a. 149 b. 31
8. The figures are designated by the following abbreviations; P: parallelogram, R: rectangle Rh: rhombus, S: square, T: trapezoid, IT: isosceles trapezoid.
Opposite Sides Parallel (Both pairs) P, R, Rh, S
Exactly one pair parallel sides T, IT
Opposite Sides Congruent (Both pairs) P, R, Rh, S
Exactly one pair opposite congruent sides IT
All Sides Congruent Rh, S
Exactly One Pair Adjacent Congruent Angles None
All Angles Congruent R, S
Opposite Angles Supplementary IT
Diagonals Bisect Angles Rh, S
9. Quadrilateral NPEM is a parallelogram (T7-2.2)
NPEM is a rectangel (T7-3.4)
10. $\triangle APE \cong \triangle CQE$ (ASA), AB = DC (T7-1.2)
AP = CQ (D3-3); BP = DQ (Subtraction)
11. $\angle FDE \cong \angle FED$ (T3-4.2); $\angle DEF \cong \angle DAF$, $\angle FDE \cong \angle FCE$ (T7-1.3)
$\angle DAF \cong \angle FCE$ (P2-1), $\triangle ABC$ is isosceles (T3-4.3).
12. $\angle A \cong \angle C$ implies $\angle DAE \cong \angle BCF$ and $\angle EAF \cong \angle FCE$:
$\angle B \cong \angle D$ (T7-1.3), AD = CB (T7-1.2);
$\triangle ADE \cong \triangle CBF$ (ASA); $\angle DEA \cong \angle BPC$ (D3-3)
$\angle AFC \cong \angle CEA$ (T2-6.2); quadrilateral AFCE is a parallelogram (T7-2.3)
13. Using $\overline{ME}$ as a transversal for $\overline{GM}$ and $\overline{EO}$, $\angle GME \cong \angle QFM$;
$\angle MRG \cong \angle ORE$ (T2-6.3); MG = OE (T7-1.2)
$\triangle MRG \cong \triangle ORE$ (AAS).
14. $\triangle QKM \cong \triangle PMK$ (SAS); $\angle MQK \cong \angle KPM$ (D3-3)
quadrilateral KPMQ is a parallelogram (T7-2.5)
15. $\angle TSR \cong \angle TRS$ and each has measure 45 (see the example in Section 7-5), $m\angle STR = 90$ (T6-4.2), $m\angle STN = 67\frac{1}{2}$ (T3-4.2 and T6-4.2) $m\angle NTR = 90 - 67\frac{1}{2} = 22\frac{1}{2} = 1/3$ $m\angle STN$ (Subtraction).

MIDYEAR TEST

Exercises

1. 55
2. octagon
3. isosceles
4. obtuse
5. is not
6. true
7. If 3, 4, and 5 may represent the sides of a triangle, then 3 + 4 > 5.
8. a) No b) not sufficiently restrictive
9. is not
10. $\sim p \rightarrow \sim q$
11. a) K b) $\overline{UK}$
12. If two triangles are not congruent then corresponding angles are not congruent.
13. may not
14. $\overline{CX}$
15. right angle
16. $\angle$
17. J
18. Quadrilateral ABCD is a rhombus (T7-4.4), $\overline{AC}$ and $\overline{DB}$ are diagonals of rhombus ABCD and $\overline{AC}$ bisects $\overline{DB}$ (T7-1.5) hence J is the midpoint of $\overline{DB}$ (D1-15).
19. Draw $\overline{JC}$. $\angle HCJ \cong \angle GCJ$ (T7-4.2), HC = GC (Given), and JC = JC (Identity), so $\angle HCJ \cong \angle GCJ$ (SAS). Thus $\angle JHC \cong \angle JGC$ (D3-3).
20. 45
21. interior angles on the same side of the transversal
22. $\angle OSM \cong \angle LME$ (C6-2.1a), $\angle OMS \cong \angle LEM$ (C6-2.1a), $\angle SOM \cong \angle MLE$ (T6-2.1), SM = ME (Given), $\triangle SOM \cong \triangle MLE$ (AAS), OS = LM (D3-3) and $\overline{OS} \parallel \overline{LM}$ (Given) Quadrilateral OSML is a parallelogram (T7-2.2). Hence, $\overline{SM} \parallel \overline{OL}$ (D7-1).
23.



24. Def. of projection (T4-5.6)
25. $\overline{AB} \parallel \overline{NW}$ and AB = NW (D7-1, given), Quadrilateral ABWN is a parallelogram (T7-2.2), $\angle NAS$ is a right angle (T7-3.1), ▱ABWN is a rectangle (D7-5).
26. $\overline{TE} \parallel \overline{SP}$ (transitive property), FE = EP (given), $\overline{TE}$ is a midline of $\triangle SFP$ (T7-6.4): 2 TE = SP (T7-6.3).
27. DA = EG (T7-4.1), $\angle ADL \cong \angle EGL$ (T7-1.2, T7-4.2), DL = LG (T7-1.5), $\triangle DLA \cong \triangle GLE$ (SAS). The result can also be established using AAS.

Test 8

1. 32
2. $\frac{AB \cdot AD}{CD}$
3. $\sqrt{21}$
4. 4.8
5. 2
6. is not
7. 4.8
8. 3
9. 25
10. $\frac{UP}{PR}$
11. $\angle R$
12. $\sqrt{23.04}$
13. $\frac{3}{5}$
14. 31 inches
15. 36
16. 4
17. $\frac{1}{2}$
18. 45°
19. 60°
20. $\triangle ADE \sim \triangle BCE$ (AAA), $\frac{AD}{AE} = \frac{BC}{EB}$ so $AD \cdot EB = AE \cdot BC$
21. $\frac{PQ}{QS} = \frac{PS}{RQ} = \frac{QS}{SR}$ (Given)

    $\triangle PSQ \sim \triangle QRS$ (T8-6.2)

    $\angle PQS \cong \angle QSR$ (Def. similar triangles)

    $\overline{PQ} \parallel \overline{SR}$ (T6-2.1)
22. $AB^2 = BD^2 + AD^2$ (T8-8.1), but BD = ½AD so $3/4AB^2 = AD^2$, or $3(AB)^2 = 4\ (AD)^2$
23. 48 + 24 3 feet

Test 9

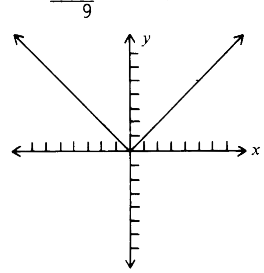
Exercises

1. False
2. True
3. True
4. False
5. True
6. False
7. False
8. False
9. True
10. great circle
11. 60
12. minor
13. 40
14. $\angle BAC$
15. =
16. 135
17. a) 90 b) 17 c) 73
18. $\sqrt{17a^2 + 8a + 1}$
19. $\frac{20}{\pi}$ inches
20. 24
21. $\frac{4}{\pi}$.
22. $\sqrt{73}$
23. $m\angle ABD = m\angle ACD = 90$ (C9-6.1c), $\triangle ABD \cong \triangle ACD$ (HL), $\overline{BD} \cong \overline{CD}$ (D3-3), or use (T9-5.3).
24. Use D4-7 and T9-4.6
25. $m\angle B + m\angle D = 180$ (T9-6.2), $\angle B \cong \angle EFC$ (C6-3.1a) $m\angle EFC + m\angle D = 180$ (P2-1), quadrilateral EFCD is cyclic (T9-7.2)

Test 10

Exercises

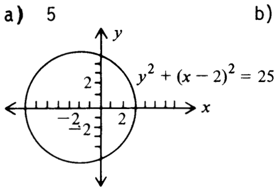
1. IV
2. 0
3. $\{2, 1, 0, -1\}$
4. 10
5. (8, 1)
6. are not
7. (7, 4)
8. $5 + (3 + \sqrt{2})\sqrt{5}$
9. $-\frac{5}{2}$
10. $\frac{3}{5}$
11. $-\frac{2}{5}$
12. 0
13. $y = -4x - 1$
14. $\frac{x}{5} + \frac{y}{8} = 1$
15. $y = \frac{17}{8}x + \frac{31}{8}$
16. $y = -\frac{7}{2}x + 6$
17. $(x + 1)^2 + (y + 3)^2 = 25$
18. The slope of the line through (-7, 6) and (5, 18) is 1, but the slope of the line through (-7, 6) and (2,-11) is $-\frac{17}{9}$. The points are not collinear.
19.



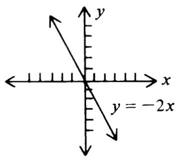
Test 10

Exercises

20. a) 5 b) $y^2 + (x - 2)^2 = 25$



21.



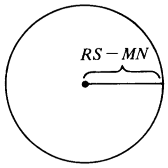
22. Locate △ ABC in a coordinate plan such that A (0, 0), B (2a, 0), and C (2b, 2c). Find the midpoint of each side and determine an equation of each median. Solve the three equations simultaneously to show they have a point in common.

Test 11

Exercises

1. The locus is a circle with center (0, 0) and radius 7.
2. It is a line segment on the line $y = \frac{1}{3}x$ with endpoints $\left(\frac{3\sqrt{10}}{5}, \frac{\sqrt{10}}{5}\right)$ and $\left(\frac{-3\sqrt{10}}{5}, \frac{-\sqrt{10}}{5}\right)$

3.



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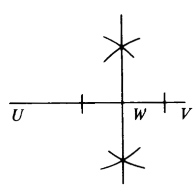
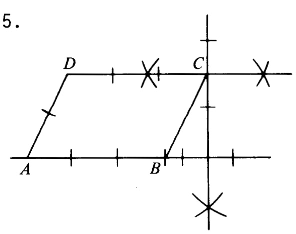
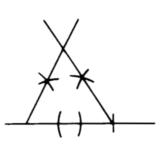


5.

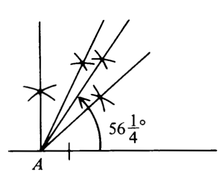


6. $y - 7 = \frac{14}{5}\left(x - \frac{5}{2}\right)$

7.



8.



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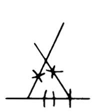

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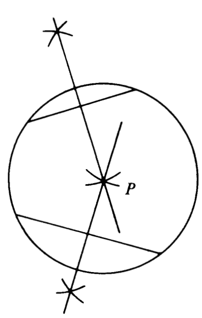
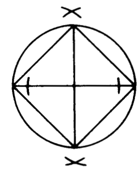


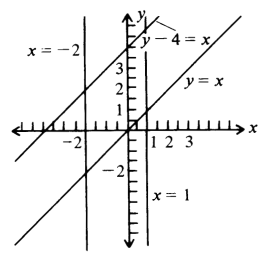
Test 11

11.



12. $\dfrac{5 + \frac{\sqrt{205}}{3}}{20}$, $1 + \dfrac{5 + \frac{\sqrt{205}}{3}}{-10}$

13.



Test 12

Exercises

1. $\frac{15}{4}$	2. $\frac{49}{2}$	3. 4
4. $\frac{1}{9}$	5. $h = \frac{25}{4}$	6. 50
7. 20	8. $12\sqrt{3}$	9. 10
10. $24\sqrt{3}$	11. $32\sqrt{3}$	12. 49π
13. $\frac{1}{\pi}$	14. 36π	15. sixteen
16. $12\sqrt{2}$	17. $64 - 4\,(4\pi) = 64 - 16\pi$	
18. $\frac{1}{9}$	19. $9\,(1+\sqrt{3})$	20. 30
21. $\frac{5}{6}$		

22. $\mathcal{A}$ ▱ ABCD = MR · DC, and $\mathcal{A}$ quadrilateral AMCN = MR · NC (T12-3.1). But, DC = 2NC (D1-15). Thus, $\mathcal{A}$ quadrilateral AMCN = MR · ½DC = ½ $\mathcal{A}$ ▱ ABCD (P2-1).

Test 13

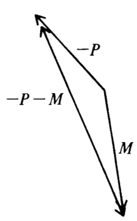
Exercises

1. may not, (T13-1.2)	2. 4
3. 96 sq. in.	4. 177
5. volume is doubled	6. 9
7. $\sqrt{433}$	8. $96\sqrt{3}$
9. 48.9	10. 9
11. 3.6	12. 96π
13. 70π	14. 147π
15. 176π	16. 3 : 5
17. pentagonal or oblique	18. 12
19. $18\sqrt{3}$	20. 2
21. $\frac{30 - 10\sqrt{3}}{3}$	22. 1:1

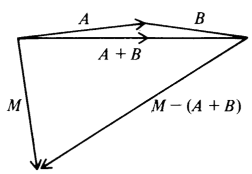
Test 14

Exercises

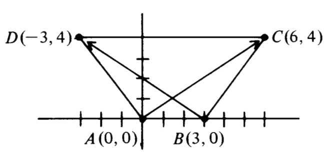
1. (-4, -11)
2. (-7, 5)
3. $\sqrt{40}$ or $2\sqrt{10}$
4. (-4, 6)
5. (k, 6k), k $\neq$ 0
6. x = $\pm$ 3
7. (3, -27)
8. (-17, -5)
9. (o, 6.5) where R = -2
10. is not
11. cannot
12. Locate **P** and **W** in the coordinate plane so that the initial point of each vector is the origin. Then, **W** - **P** will lie in quadrants I and IV, while **P** - **W** will lie in quadrants II and III.
13. Again, locate **R** and **B** with the initial point of each at the origin.
14.



15.

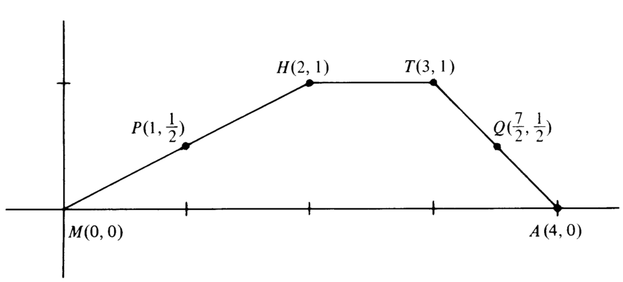


16. is not
17. In quadrilateral ABCD, let A (0, 0), B (3, 0), C (6, 4), D(-3, 4). Then, $|\overrightarrow{AC}| = |\overrightarrow{BD}|$ but $\overline{AD} \nparallel \overline{BC}$. Quadrilateral ABCD is not a parallelogram so it cannot be a rectangle.



18. In trapezoid MATH, M (0, 0), A (4, 0), T (3, 1) H (2, 1). $\overrightarrow{MA}$ = (4, 0) and $\overrightarrow{HT}$ = (1, 0). $\frac{1}{4}\overrightarrow{MA} = \overrightarrow{HT}$ so $\overrightarrow{MA} \parallel \overrightarrow{HT}$ (D14-9). If P and Q are the midpoints of $\overline{MH}$ and $\overline{AT}$, respectively, then P (1, $\frac{1}{2}$) and Q $(\frac{7}{2}, \frac{1}{2})$. $\overrightarrow{PQ} = (\frac{5}{2}, 0)$ and $\overrightarrow{PQ} = \frac{5}{2}$ HT so

$\overrightarrow{PQ} \parallel \overrightarrow{MA} \parallel \overrightarrow{HT}$ (D14-9), transitive property. $\overrightarrow{MA} + \overrightarrow{HT}$ = (5, 0) (D14.7). Thus, $\overrightarrow{PQ} = \frac{1}{2}(\overrightarrow{MA} + \overrightarrow{HT})$



Final Test

Exercises

1. one
2. may not
3. If 7 + 5 = 12, then the earth is a sphere
4. subject, predicate
5. inverse
6. are
7. is not
8. always
9. greatest
10. is
11. is
12. CF = BE (Subtraction property), $\angle$ABC $\cong$ $\angle$DCB (T6-3.1), $\triangle$ ABE = $\triangle$DCF (SAS), $\angle$CFD $\cong$ $\angle$BEA (D3-3) $\angle$DFE $\cong$ $\angle$AEF (P2-11), $\overline{AE} \parallel \overline{FD}$ (T6-2.1)
13. $\overline{MN} \parallel \overline{AB}$ (D7-9, T7-6.3), so $\overline{RQ} \parallel \overline{MN}$ (T7-6.2) BQ = QN (given), BP = PM (T7-6.4).
14. EB = AC (diagonals of a regular polygon), AF = FB (given) EF = CF (substraction property), AE = BC (given), $\triangle$ AFE $\cong$ $\triangle$ BFC (SSS).
15. $\triangle$ AED and $\triangle$ BAE are congruent (SSS) and isosceles (D3-12), $\angle$EAR $\cong$ $\angle$AER (T3-4.2, D3-3 and transitive property), $\triangle$ ARE is isosceles (T3-4.2)

16. may not 17. $\sqrt{117}$ 18. 24
19. outside 20. $6(\sqrt{3} - \sqrt{2})$ 21. 40
22. 90 23 $\overline{AE}$ 24.
25. halved 26. $y = \frac{1}{3}(x - 4)$ 27. is not
28. y = 3 29. the center of the square
30. $\frac{1}{2}$a 31. $6\sqrt{3}$ 32. $3\sqrt{2} - 1$
33. 9π - 18 34. See construction 4, page 473.
35. See construction 6, page 476.
36. See construction 8, page 476.
37. See construction 10, page 479.
38. 144 39. 40. 106
41. (-19, -2) 42. (-21, -20)
43. $\angle$A $\cong$ $\angle$D and $\angle$B $\cong$ $\angle$C (T6-2.1), $\angle$AEB $\cong$ $\angle$CED (T2-6.3) $\triangle$ AEB $\sim$ $\triangle$ DEC (AAA)
44. Let $\overline{RO}$ and $\overline{HM}$ intersect in T, and let RT = x, HT = y. Then, A $\triangle$ RHM = $\frac{1}{2}$x $\cdot$ 2y = xy, A $\triangle$ RHO = $\frac{1}{2}$y $\cdot$ 2x = xy. However, RO $\neq$ MH, so $\triangle$ RHM $\ncong$ $\triangle$ RHO.
45. Use T9-8,4. T9-6.3 and C9-6.1b

1 2 3 4 5 6 7 8 9 10 EBEB 86 85 84 83 82 81 80 79 78 77

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