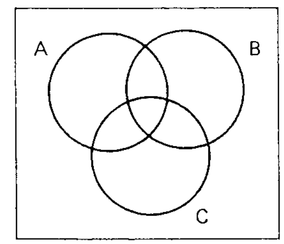
INTRODUCTION TO PROBABILITY

INSTRUCTOR'S MANUAL

John E. Freund
Arizona State University



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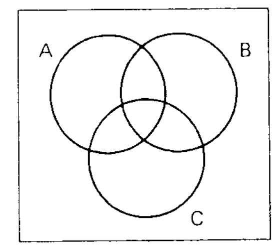

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FOREWORD

This manual is meant to supplement the hints to the exercises given directly in the text and the answers to the odd-numbered exercises given (with some details) at the end of the book. It does not contain detailed solutions of exercises for which the answers can be read off diagrams or tables, but it covers all the exercises which may cause some difficulties to the beginner, and all those which deal with generalizations of the material in the text.

Separately, on pages 32 through 39, this manual contains the answers to the even-numbered exercises, so that they can, perhaps, be duplicated for student use.

JOHN E. FREUND

CHAPTER 1

POSSIBILITIES

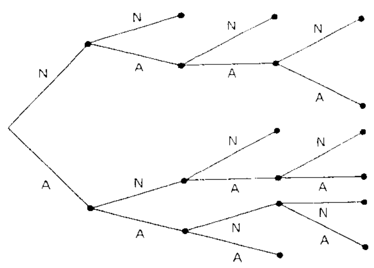
NOTES AND COMMENTS

We cannot very well predict which television program will get the best rating unless we know at least what shows are on the air, and we cannot very well predict the winner of an election unless we know at least the names of the candidates. More generally, we cannot make intelligent predictions or decisions unless we know at least what is possible; in other words, we must know what is possible before we can judge what is probable. Thus, the first chapter of this book is devoted to problems relating to possibilities, as a first step which, hopefully, will lead to the determination of corresponding probabilities. It should be kept in mind, however, that the "jump" from possibilities to probabilities is a big one (a controversial one, in fact), which will be discussed in later chapters. For the time being, we will not be able to go beyond possibilities, and the purpose of the extra questions asked in Exercises 1, 2, 4, and 5 of the first set is to remind the student of this fact.

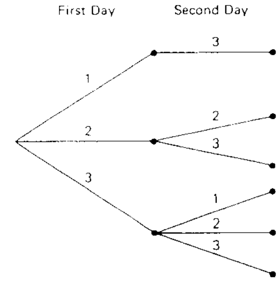
DISCUSSION OF EXERCISES ON PAGES 10 THROUGH 14

Exercises 1 through 8 deal with tree diagrams, Exercises 9 through 12, 17, and 22 require the multiplication rule on page 8, while the remaining exercises require also the multiplication rule on page 10.

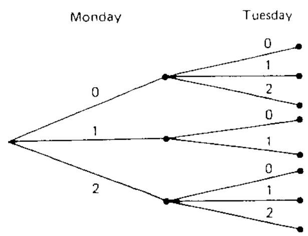
1. The tree diagram is similar to that of Figure 1.4, except that for the first choice there are only three possibilities (branches) instead of five, and there are altogether nine possibilities; (a) 2 (Morris and Mason, Mathews and Mason); (b) 2 (Brown and Adams, Brown and Perkins). We cannot conclude anything about the likelihood of these choices without having additional information or without making assumptions.
2. The tree diagram is similar to that of Figure 1.4, except that for the first choice there are only four possibilities (branches) instead of five, and there are altogether 12 possibilities; (a) 3 of the 12 possibilities, or 25% (Routes A and A, A and B, A and D); (b) 3 of the possibilities, or 25% (Routes A and A, B and B, D and D). We cannot conclude anything about the likelihood of these choices; for all we know, the businessman might always take Route A to work and Route B on the way home.
3. At first there are two branches corresponding to whether or not a violator is properly licensed, from each of these emanate two branches corresponding to whether the violation is major or minor, and from each of these emanate two branches corresponding to whether or not there was a previous violation.
4. The tree diagram is shown on the next page. Without making further assumptions, there is nothing we can conclude about the likelihood that there will be a seventh game.
5. With reference to Figure 1.3, the choices for the boy and his brother are the same, but for the third step the number of possibilities (branches) depends on how many pieces of candy are left.



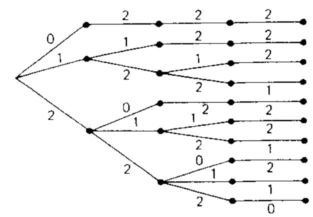
Tree diagram for Exercise 4



Tree diagram for Exercise 6



Tree diagram for Exercise 7



Tree diagram for Exercise 8

6. The tree diagram is shown above.
7. The tree diagram is shown above. Concinuing the branches, we find that there are 3 + 2 + 3 + 2 + 3 + 3 + 2 + 3 = 21 possibilities for the three days.
8. The tree diagram is shown above.
9. 4·8 = 32 and (4 - 2)·(8 - 3) = 2·5 = 10.
10. (a) 4·15 = 60; (b) 2·6 = 12 of 60, or 20%; (c) 2·9 = 18 of 60, or 30%.
11. (b) 4·4 = 16; (c) 4·3 = 12.
12. (a) 5·4 = 20; (b) 4·3 = 12 of 20, or 60%; (c) 3·2 = 6 of 20, or 30%.
13. (a) 2·3 = 6; (b) 3·2·3 = 18.
14. (a) 10·10 = 100; (b) 7·6 = 42; (c) 3·7·4·6 = 504.
15. 5·2·2·2 = 40.
16. 8·12·9 = 864.
17. (a) 9·8 = 72; (b) 3·2 = 6; (c) 72 - 6 = 66. Note also that in 6·6 = 36 of the possibilities Amsterdam will not be visited in either year, and hence that Amsterdam will be visited exactly once in 72 - 6 - 36 = 30 of the possibilities. This last result can also be obtained by arguing that Amsterdam can be visited in the first year but not the second year in 3·6 = 18 ways, in the second year but not in the first year in 6·2 = 12 ways, and hence that there are 18 + 12 = 30 possibilities in which Amsterdam is visited only once.
18. 3·3·3·3·3·3·3·3 = 6,561.
19. (a) 2^{15} = 32,768; (b) $15 \cdot 2^{14}$ = 245,760, since there 15 ways of choosing the question which the student does not answer, and then two ways of answering each of the other 14 questions.
20. 2·2·2·2·2 - 1 = 31, since there are two possibilities for each coin but we have to subtract 1 for the case where he does not take any of

the coins.
21. $2^{10} - 1$ = 1,023.
22. (a) 10·8 = 80; (b) 9·9 = 81.

DISCUSSION OF EXERCISES ON PAGES 20 THROUGH 22

Exercises 1 through 9 deal with permutations, Exercises 10 through 14 with permutations of indistinguishable objects, and Exercise 15 with factorials.

1. 8·7·6 = 336.
2. (a) 16·15·14·13 = 43,680; (b) 10·9·8·7 = 5,040; (c) 10·15·14·13 = 27,300.
3. 9·8·7·6·5·4 = 60,480.
4. (a) 8! = 40,320; (b) 5!·3!·2 = 1,440, where 5! is the number of ways he can arrange the business texts, 3! is the number of ways he can arrange the foreign language texts, and we multiply by 2 since either set of books can be on the left or on the right.
5. 9! = 362,880; 8! = 40,320 of 362,880, or 1/9.
6. 24·23·22 = 12,144; 3·23·22 = 1,518 of 12,144, which is 12.5%.
7. (a) 8! = 40,320; (b) 4! = 24; (c) 4!·2·2·2·2 = 384; (d) 4!·4!·2 = 1,152; (e) 4!·4!·2 = 1,152, since the men can be arranged in 4! ways, the women in 4! ways, and the first person on the left can be a man or a woman.
8. (a) 4! = 24; (b) 3! = 6, since we can arbitrarily seat one of the four persons and then seat the other three in 3! = 6 ways.
9. (n - 1)!; (a) 7! = 5,040; (b) 7!/2 = 2,520.
10. (a) 4! = 24; (b) 5!/2 = 60; (c) 7!/3! = 840; (d) $\frac{6!}{2!2!}$ = 180.
11. (a) $\frac{8!}{3!}$ = 6,720; (b) $\frac{6!}{2!2!}$ = 180; (c) $\frac{8!}{4!}$ = 1,680; (d) $\frac{8!}{3!3!}$ = 1,120

(e) $\frac{8!}{2!3!}$ = 3,360.
12. $\frac{9!}{3!3!2!}$ = 5,040.
13. (a) 8! = 40,320; (b) $\frac{8!}{4!4!}$ = 70; (c) $\frac{8!}{2!2!4!}$ = 420.
14. (a) 8·7·6·5·4 = 6,720; (b) 5·4·3 = 60; (c) 7·6·5·4 = 840; (d) 8·7·6·5 = 1,680.
15. (a) 6! = 6·5·(4·3·2·1) = 6·5·4!; (b) $\frac{10!}{10\cdot 9\cdot 8} = \frac{10\cdot 9\cdot 8\cdot 7!}{10\cdot 9\cdot 8}$ = 7! and not 7; (c) $\frac{1}{2!} + \frac{1}{2!} = \frac{1}{2} + \frac{1}{2} = 1$; 10! + 3! = 3,628,800 + 6, which does not equal 13! = 6,227,020,800 (see Table I).

DISCUSSION OF EXERCISES ON PAGES 29 THROUGH 32

Exercises 1 through 7 deal with combinations, Exercises 8 through 14 are combination problems which also involve the multiplication rules on pages 8 and 10, Exercises 15, 16, 23, and 24 involve sums of binomial coefficients or sums of expressions involving binomial coefficients, and Exercises 20 through 22 deal with binomial coefficients.

1. $\frac{11\cdot 10}{2!}$ = 55.
2. $\frac{10\cdot 9\cdot 8}{3!}$ = 120; $\frac{9\cdot 8}{2!}$ = 36 of 120 is 30%.
3. $\frac{12!}{6!6!}$ = 924.
4. $\frac{8!}{4!4!}$ = 70.
5. 19·18·17·16·15/5! = 11,628; the principal will not be included in

$18\cdot17\cdot16\cdot15\cdot14/5! = 8{,}568$ of the 11,628 cases, which is 73.68%.

6. $\frac{15!}{11!\cdot4!} = 1{,}365$

7. (a) $\binom{15}{3} = 455$; (b) $\binom{15}{6} = 5{,}005$; (c) $\binom{15}{12} = 455$.

8. $\binom{8}{2}\cdot\binom{5}{3} = 28\cdot10 = 280$.

9. $\binom{6}{3}\cdot\binom{4}{2} = 20\cdot6 = 120$.

10. (a) $\binom{12}{3}\cdot3^9 = 4{,}330{,}260$ since the three questions which are answered correctly can be chosen in $\binom{12}{3}$ways, and each of the other questions can be answered in 3 ways; (b) $\binom{12}{8}\cdot3^4 = 40{,}095$; (c) there are $\binom{12}{5}\cdot3^7 = 1{,}732{,}104$ for five correct answers and $\binom{12}{6}\cdot3^6 = 673{,}596$ for six correct answers; thus, there are more possibilities for five correct answers.

11. $\binom{9}{5}\cdot\binom{5}{2}\cdot\binom{11}{3}= 207{,}900$.

12. (a) $\binom{13}{4}= 715$; (b) $\binom{13}{3} = 286$; (c) $\frac{286}{715 + 286}\cdot100 = 28.57\%$.

13. (a) $\binom{10}{3}= 120$; $\binom{10}{1} = 10$; (c) $\binom{10}{2}\cdot\binom{2}{1} = 90$, since there are $\binom{10}{2}$ ways of selecting two good ones and $\binom{2}{1}$ ways of selecting a bad one; (d) $\frac{90 + 10}{220}\cdot100 = 45.45\%$, since there are $\binom{12}{3} = 220$ possibilities altogether; (e) $\frac{10}{220}\cdot100 = 4.55\%$.

14. (a) $\binom{20}{4} = 4{,}845$; (b) $\binom{8}{2}\binom{8}{2} = 28\cdot28 = 784$; (c) $\binom{8}{1}\binom{8}{1}\binom{4}{2} = 8\cdot8\cdot6 = 384$.

15. (a) 6 possibilities depending whether all the dice come up 1, 2, 3, 4, 5, or 6; (b) $6\cdot5 = 30$, since there are six possibilities for the two dice which come up with the same number of points and then five possibilities for the other die; (c) $\binom{6}{3} = 20$; (d) $6 + 30 + 20 = 56$.

16. When zero speeding cases are included there are $\binom{3}{3} = 1$ possibilities, since all three of the other cases must be selected; when one speeding case is included there are $\binom{3}{2} = 3$ possibilities; when two speeding cases are included there are $\binom{3}{1} = 3$ possibilities; when three speeding cases are included there are $\binom{3}{0} = 1$ possibilities; and altogether there are, therefore, $1 + 3 + 3 + 1 = 8$ possibilities.

17. There are $\binom{6}{2} = 15$ ways of placing the r's, then $\binom{4}{2} = 6$ ways of placing the e's, and then 2 ways of placing the a and the d; there are, thus, altogether $15\cdot6\cdot2 = 180$ permutations.

18. (d) $\binom{8}{3}\cdot\binom{5}{3}\cdot2 = 56\cdot10\cdot2 = 1{,}120$; (e) $\binom{8}{3}\cdot\binom{8}{2}\cdot3! = 56\cdot10\cdot6 = 3{,}360$.

19. $\binom{9}{3}\cdot\binom{6}{3}\cdot\binom{3}{2} = 84\cdot20\cdot3 = 5{,}040$.

20. and 21. Look up the binomial coefficients in Table II and substitute

their values in the equations.

22. (a) $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n!}{r!(n-r)!} = \frac{n!}{n-(n-r)!(n-r)!} = \binom{n}{n-r}$;

(b) $\binom{n-1}{r-1} + \binom{n-1}{r} = \frac{(n-1)!}{(n-r)!(r-1)!} + \frac{(n-1)!}{(n-r-1)!r!}$

$= \frac{r(n-1)! + (n-r)(n-1)!}{(n-r)!r!}$

$= \frac{n(n-1)!}{(n-r)!r!} = \frac{n!}{(n-r)!r!} = \binom{n}{r}$:

(c) $\frac{n}{n-r}\cdot\binom{n-1}{r} = \frac{n(n-1)!}{(n-r)(n-r-1)!r!} = \frac{n!}{(n-r)!r!} = \binom{n}{r}$.

23. $\binom{5}{1} + \binom{5}{2} + \binom{5}{3} + \binom{5}{4} + \binom{5}{5} = 5 + 10 + 10 + 5 + 1 = 31$.

24. For $a = 1$ and $b = 1$ the binomial theorem yields $(1 + 1)^n = 2^n = \binom{n}{0} + \binom{n}{1} + \dots + \binom{n}{n}$; (a) $2^{12} = 4{,}096$; (c) $2^{24} = 16{,}777{,}216$.

CHAPTER 2

PROBABILITIES

NOTES AND COMMENTS

The fact that in many elementary text questions concerning the meaning of probability statements are avoided, is really an insult to the intelligence of the intended audience. Of course, the subject is controversial, but it does not require the I.Q. of a genius to appreciate some of the advantages and some of the weaknesses of each interpretation. The <u>frequency concept</u>, introduced on pages [illegible] through 51, is probably easiest to understand and most widely used. The main point to get across is that we must always refer to "what happens in the long run to similar events." For instance, if somebody wants to know the probability that a certain painting is authentic, this would somehow have to reflect the percentage of the time that art experts are right in authenticating paintings of this kind. Like Mrs. Brown and her broken wrist on page 50, this may be of no comfort to someone who wants to buy one particular painting; and yet, what else can anyone really say. The <u>classical approach</u> also has a good deal of intuitive appeal, certainly with regard to games of chance, but as it applies only when the outcomes are all equiprobable, its range of applicability is limited. Some authors of books on probability define a <u>subjective probability</u>, discussed on pages 51 through 53, as the "strength of one's belief" and let it go at that. After all, we all know what is meant by the "strength of a belief," or do we? In practice, we must know how the strength of a belief is to be determined, and the usual answer is that such probabilities are determined by referring to risk-taking situations. Theoretically, this is fine, but it would hardly make sense to ask the weatherman to place a bet each time he predicts the chances for rain.

A strong case for the frequency concept is presented by H. Reichenbach in the book listed in the Bibliography on page 224; a treatment of the subjective approach may be found in R. C. Jeffery's "The Logic of Decision" (McGraw-Hill Book Co., 1965).

DISCUSSION OF EXERCISES ON PAGES 43 THROUGH 47

Exercises 1 through 7 and 9 are straightforward applications of the formula s/n for equiprobable outcomes, Exercises 8 and 10 through 23 are similar problems requiring some of the combinatorial methods of the last chapter, and Exercises 24 and 25 are similar problems dealing with odds. Exercises 26 through 29 concern the converting of probabilities to odds and vice versa, and Exercises 30 through 34 are theoretical or discussion problems.

1. through 7. and 9. require only the counting of the number of favorable cases and the total number of possibilities, so there is no need to go into details.

8. (a) the number of favorable cases is $\binom{5}{2} = 10$ and the total number of possibilities is $\binom{12}{2} = 66$, so that the probability is $\frac{10}{66} = \frac{5}{33}$;
(b) the number of favorable cases is $\binom{5}{1}\binom{7}{1} = 5\cdot 7 = 35$, so that the probability is $\frac{35}{66}$.

10. The number of favorable cases for parts (a), (b), and (c) are, respectively, $\binom{4}{2} = 6$, $\binom{13}{2} = 78$, and $\binom{4}{1}\binom{4}{1} = 16$, the total number of possibilities is $\binom{52}{2} = 1{,}326$, and the corresponding probabilities are $\frac{6}{1326} = \frac{1}{221}$, $\frac{78}{1326} = \frac{3}{51}$, and $\frac{16}{1326} = \frac{8}{663}$.

11. The number of favorable cases is $13\cdot 13\cdot 13\cdot 13$ and the total number of possibilities is $\binom{52}{4}$.

12. The number of favorable cases for zero, one, and three 3's are, respectively, $5\cdot 5\cdot 5 = 125$, $\binom{3}{1}\cdot 5\cdot 5 = 75$, and 1, and the total number of possibilities is $6\cdot 6\cdot 6 = 216$.

13. The number of favorable cases for parts (a) and (b) are $\binom{18}{4}$ and $\binom{17}{4}$, and the total number of possibilities is $\binom{20}{4}$.

14. The number of favorable cases are, respectively, $\binom{11}{3}$ and $\binom{10}{1}$, and the total number of possibilities is 220, as in the text.

15. The number of favorable cases is $6\cdot 2\cdot 6\cdot 5!$ since there are six positions for the three books (first, second, third, or second, third, fourth, etc.), two ways in which the business books can be arranged, six ways of choosing the book that goes between the two business books, and 5! ways of arranging the other five books; the total number of possibilities is 8!.

16. (b) The number of favorable cases for the three parts are, respectively, $\binom{120}{2}$, $\binom{60}{2}$, and $\binom{60}{1}\binom{120}{1}$, and the total number of possibilities is $\binom{180}{2}$.

17. The number of favorable cases for the three parts are, respectively, $\binom{14}{2}$, $\binom{11}{2}$, $\binom{14}{1}\binom{11}{1}$, and the total number of possibilities is $\binom{25}{2}$.

18. The number of favorable cases for the four parts are, respectively, 1, 2^{10}, $\binom{10}{3}\cdot 2^7$, and $\binom{10}{5}\cdot 2^5$; the total number of possibilities is 3^{10}.

19. The number of favorable cases for the four parts are, respectively, 1, $\binom{15}{5}$, $\binom{15}{7}$, and 1; the total number of possibilities is 2^{15}.

20. The number of favorable cases is $\binom{7}{3}$ and the total number of possibilities is $\binom{8}{3}$.

21. The number of favorable cases for the three parts are, respectively, $\binom{5}{2}$, $\binom{3}{2}$, and $\binom{5}{1}\binom{3}{1}$; the total number of possibilities is $\binom{8}{2}$.

22. The number of favorable cases for the three parts are, respectively, $\binom{11}{7}$, $\binom{10}{6}$, and $\binom{11}{8}$; the total number of possibilities is $\binom{12}{8}$; it is assumed that each of the 12 cities has the same chance of being chosen.

23. The number of favorable cases is $\binom{4}{1}\binom{3}{1}$ and the total number of possibilities is $\binom{6}{3}\binom{4}{2}$.

24. The number of favorable cases for the three parts are, respectively, $\binom{10}{3}$, $\binom{10}{1}$, $\binom{2}{1}\binom{10}{2}$; the total number of possibilities is $\binom{12}{3}$.

25. The number of favorable cases for the first two parts are $\binom{8}{2}\binom{8}{2}$ and $\binom{4}{4}$; the total number of possibilities is $\binom{20}{4} = 4,845$; for the third part we subtract $\binom{12}{4}/\binom{20}{4}$ from 1.

26. (a) The odds are $\frac{15}{64}$ to $1 - \frac{15}{64} = \frac{49}{64}$, or 15 to 49; the other parts are done the same way.

27. (a) The odds are $\frac{1}{20}$ to $1 - \frac{1}{20} = \frac{19}{20}$, or 1 to 19; the other parts are done the same way. If it is desired to check on the probabilities, in (a) there are $\binom{6}{3} = 20$ ways of choosing the letters and only one of these is a success; for (b) the number of favorable cases is $\binom{7}{1}$ and the total number of possibilities is $\binom{8}{2}$; in (c) the probability is obtained by subtracting from 1 the probability that the bills are all $1 bills, namely, by subtracting from 1 the quantity $\binom{8}{3}/\binom{14}{3}$.

28. (a) The probability of not rolling "7 or 11" is $\frac{7}{7+2} = \frac{7}{9}$; the other parts are done the same way.

29. (a) The probability that she will make a mistake is $\frac{13}{13+2} = \frac{13}{15}$;
(b) The probability that two will get the right coat and two a wrong coat is $\frac{1}{1+3} = \frac{1}{4}$, and the probability that at least one of them will get the right coat is $\frac{5}{5+3} = \frac{5}{8}$; (c) the probability of getting a meaningful word is $\frac{1}{1+5} = \frac{1}{6}$; incidentally, among the 24 possibilities only "nest," "tens," "sent," and "nets"are meaningful words.

30. $a(1 - p) = bp$, $a - ap = bp$, $a = ap + bp$, $a = p(a + b)$, $p = \frac{a}{a+b}$.

31. When p = 1 the odds for success are 1 to 0, which is undefined since we cannot divide by zero; when p = 0 the odds for failure are undefined for the same reason, and we therefore do not speak of the odds for success either.

32. For example, the instructor may be known to favor certain of the novels.

33. It would be unreasonable to assume that the 12 cheeses are equally popular.

34. Using the principle of equal ignorance we can arrive at any figure we want. For instance, we could get 1/3 instead of 1/2 in our example by saying that either there is human life elsewhere in the universe, other life forms but no human life exists elsewhere in the universe, or no life forms whatsoever exist elsewhere in the universe.

DISCUSSION OF EXERCISES ON PAGES 54 THROUGH 57

Exercise 1 through 8 relate to the frequency interpretation, Exercises 9 through 18 deal with subjective probabilities, Exercises 19 through 25 are discussion questions, and Exercises 26 through 28 are experiments designed to illustrate the Law of Large Numbers.

2. (b) Insurance companies have certain health requirements which make their policy holders healthier than average; also, persons having insurance may be in an economic bracket exposing them to better health care and fewer risks.

5. (a) 24/60 = 0.40; (b) 0.40 to 1 - 0.40 = 0.60, or 2 to 3; (c) 3 to 2; (d) Odds of 3 to 2 would be fair, and the person should, therefore, bet only 15 cents against our dime; as stated, the bet favors us.

6. (a) 38/114 = 1/3; (d) since we estimate the probability as 1/3, we should really give two to one odds and are, therefore, favored by the bet.

7. (a) Since 856 - 214 = 642 drivers had their seatbelts fastened, we estimate the probability as 642/856 = 0.75; (d) since the odds are 3 to 1, we should really bet $12 against the person's $4 and are, therefore, favored by the bet.

8. (a) Since 360 - 54 = 306 of the students are against the requirement we estimate the probability as 306/360 = 0.85; (d) since the odds are 0.85 to 0.15 or 17 to 3, and the person offers us odds of 12 to 2 (or 18 to 3), we would be favored.

13. Since he is willing to bet, he considers the odds at least fair, and hence the probability to be at least 7/9.

14. Since the stockbroker feels that he should get better than 3 to 1 odds, this means that he considers the probability to be less than 1/4.

15. Since he is willing to bet at odds of 5 to 1, he feels that the probability is at least 5/6; since he is not willing to bet at odds of 6 to 1, he feels that the probability is less than 6/7.

16. Since he is willing to bet at odds of 8 to 2, he feels that the probability is at least 4/5; since he is not willing to bet at odds of 13 to 2, he feels that the probability is less than 13/15.

17. and 18. are like 15. and 16.

19. The more information we have, the more certain we are that the value of a probability is correct, or close, but the probability, itself, can be large or small.

20. So far as the frequency interpretation is concerned, the outcome of a single event cannot prove a probability statement right or wrong; so far as subjective probabilities are concerned, they are descriptive of a person's strength of his belief, and whether this is really his belief cannot be determined by the outcome of the event.

21. (a) We must refer to the reliability, or thruthfulness, of witnesses (the same ones or other ones) in similar situations; (b) we might ask ourselves at what odds we would be willing to bet that the testimony is true.

22. For example, one might be based on first-year sales, another might be based on the reviews of critics, another might be based on previous successes of the author, and another might be based on previous successes of the publisher.

24. If we base our call on the flip of a coin, there is a fifty-fifty chance of being right regardless of how many red beads and how many white beads there are in the box, and there is no way in which these odds can be improved without some further information about the beads. Nevertheless, many persons seem to be more willing to bet under condition (b).

25. Draw a tree diagram showing the six possibilities of first choosing one of the bóxes and then one of the coins. Since the coin drawn is a penny this eliminates three of the possibilities, and among the remaining possibilities the other coin is a penny in two of thre

CHAPTER 3
EXPECTATIONS

NOTES AND COMMENTS

In recent years, mathematical expectations have been playing an ever increasing role in decision making, particularly, in operations research, including the quantitative methods used in business and the social sciences. This is based on the premise that it is "rational" to select whichever alternative has the "most promising" mathematical expectation: the one which maximizes expected profits, minimizes expected costs, maximizes expected sales, minimizes expected losses, and so on.

As is explained in the text, though, this approach also entails many difficulties, including the determination of values of probabilities, assigning "cash values" to the consequences of correct or incorrect decisions, and questions of marginal utility.

DISCUSSION OF EXERCISES ON PAGES 64 THROUGH 67

All the exercises in this set are fairly straightforward applications of the formula for mathematical expectations.

2. $2.60\cdot\frac{4}{52} = \$0.20$.
3. Expectation is $5\cdot\frac{1}{20} = \$0.25$ for Box A, and $13\cdot\frac{1}{50} = \$0.26$ for Box B; although the difference is small, a drawing from Box B should be preferable.
4. (a) for two tickets $E = 500\cdot\frac{2}{1000} = \$1.00$; (b) for three tickets $E = 500\cdot\frac{3}{1000} = \$1.50$.
5. $E = 5000\cdot\frac{1}{20,000} = \$0.25$, and it is not worthwhile to spend the $0.30.
8. (a) when they are evenly matched $E = 15,000\cdot\frac{1}{2} + 9,000\cdot\frac{1}{2} = \$12,000$; (b) for the better player $E = 15,000\cdot\frac{2}{3} + 9,000\cdot\frac{1}{3} = \$13,000$, and for the poorer player $E = 15,000\cdot\frac{1}{3} + 9,000\cdot\frac{2}{3} = \$11,000$.
9. His expectation is $8,000\cdot\frac{1}{5} = \$1,600$, and he would be better off if he agreed to take $2,000 regardless of who wins.
10. If the amount we pay is A, then $2\cdot\frac{1}{6} + (-A)\cdot\frac{5}{6} = 0$, and $A = \$0.40$, where we substitute -A into the formula for an expectation because it represents money we pay rather than money we receive.
11. If the amount we pay is x, then $1\cdot\frac{1}{6} + 2\cdot\frac{1}{6} + 3\cdot\frac{1}{6} + (-x)\cdot\frac{3}{6} = 0$, and $x = \$2.00$.
12. (a) $E = 4\cdot\frac{1}{8} + 5\cdot\frac{1}{4} + 6\cdot\frac{5}{16} + 7\cdot\frac{5}{16} = \frac{93}{16} = 5\frac{13}{16}$ games; (b) the probability that he will see zero games is $\frac{1}{8}+\frac{1}{4} = \frac{3}{8}$, and the expectation is $0\cdot\frac{3}{8} + 1\cdot\frac{5}{16} + 2\cdot\frac{5}{16} = \frac{15}{16}$ games.
14. The player who originally has $5 will either win $3 with the probability p or lose $5 with the probability 1 - p. Thus, his expectation is 3p + (-5)(1 - p) or 3p - 5(1 - p), and since the game is fair, the expectation must equal 0. Solving for p the equation 3p - 5(1 - p) = 0 yields 3p - 5 + 5p = 0, 8p = 5, and p = 5/8. If the amounts are a and b instead of 3 and 5, we get bp - a(1 - p) = 0, and $p = \frac{a}{a + b}$. This is the famous problem of <u>gambler's ruin</u>.
15. Since the expectation is $7\cdot\frac{20}{45} + 12\cdot\frac{20}{45} + 14\cdot\frac{5}{45} = \$0.10$, it would be worthwhile to pay 8 cents, an even deal to pay 10 cents, and not worthwhile to pay 12 cents.
16. E = 5,400(0.40) + (-3,500)(0.60) = $60, which is a positive expected profit, but whether this makes it worthwhile for him to bid on the job is another matter. He may want a higher return on his investment of time and equipment.
18. A customer can expect to pay 40(0.58) + 50(0.23) + 30(0.19) = 40.4 cents on a sandwich, 25(0.52) + 20(0.35) = 20.0 cents for a drink, 30(0.28) = 8.4 cents for french fries, and hence altogether 40.4 + 20.0 + 8.4 = 68.8 cents.
19. He can expect to sell the shipment for 6,000(0.25) + 5,500(0.46) + 5,000(0.19) + 4,500(0.10) = $5,430, so that the expected profit is $5,430 - $5,000 = $430.
20. The corresponding probabilities are $\frac{3}{3+1} = \frac{3}{4}$, $\frac{3}{3+17} = \frac{3}{20}$, and $\frac{1}{1+9} = \frac{1}{10}$, and the expectation is $50\cdot\frac{3}{4} + 25\cdot\frac{3}{20} + 0\cdot\frac{1}{10} = 41\frac{1}{4}$ cents.
21. 0(0.26) + 1(0.33) + 2(0.28) + 3(0.09) + 4(0.04) = 1.32 purchases.
22. 0(0.22) + 1(0.34) + 2(0.25) + 3(0.13) + 4(0.05) + 5(0.01) = 1.48 fires.

DISCUSSION OF EXERCISES ON PAGES 71 THROUGH 74

Exercises 1 through 8 and 10 deal with the determination of subjective probabilities, while Exercises 9 and 11 through 15 deal with the determination of utilities; Exercise 16 is a discussion problem.

2. The expectation for the $12,900 job is 12,900p, and since this is less than $8,600, we get $12,900p < 8,600$ and $p < 2/3$.
5. The lawyer's expected contingency fee is 1,200p; (a) $400 > 1,200p$ and $p < \frac{1}{3}$; (b) $1,200p > 400$ and $p > \frac{1}{3}$; (c) $1,200p = 400$ and $p = \frac{1}{3}$.
7. For the second contractor the expected cost of the repair job is 15,000 - 5,000p; (a) he will give the job to the first contractor if $12,000 is less than 15,000 - 5,000p dollars, namely, if p is less than 3/5; (b) he will give the job to the second contractor if 15,000 - 5,000p is less than 12,000, namely, if p is greater than 3/5; (c) the expectation equals 12,000 when p = 3/5. Note that he will give the job to the first contractor when p is small and he is unlikely to get the penalty deduction.
8. If p is the probability that a customer will ask for double his money back, the manufacturer's expectation is 1.20 - 2.40p per can.
9. 13 = 2.50(0.25) + U(0.75) and U = $16.50.
10. If the probability of his having a good time is p, his expectation is 50p - 20(1 - p) if he goes to the party.
13. If U is the utility he assigns to $300, than $U\cdot\frac{13}{52} = 1$ and U = 4.
14. The values read off the graph are approximately 1.8, -2.4, 2.25, and -0.4.
15. The values read off the graph are approximately 0.9, 1.4, 1.85, and

2.25.

16. See answers to even-numbered exercises.

DISCUSSION OF EXERCISES ON PAGES 80 THROUGH 85

Exercises 4, 5, 12, and 13 introduce variations into the example discussed in the text. Exercises 1, 2, and 3 pertain to one and the same decision problem, and so do Exercises 6, 7, and 8, and Exercises 9, 10, and 11. Exercises 3 and 16 illustrate situations where none of the standard criteria can be used.

1. (a) The table is shown on page 238 of the text; (b) the expected profit is $2,050,000\cdot\frac{1}{3} - 500,000\cdot\frac{2}{3}$ = \$350,000 if they build the arena and $1,000,000\cdot\frac{1}{3} + 100,000\cdot\frac{2}{3}$ = \$400,000 if they do not build the arena, so that it would be preferable not to build the arena; (c) the expected profit is $2,050,000\cdot\frac{2}{5} - 500,000\cdot\frac{3}{5}$ = \$520,000 if they build the arena and $1,000,000\cdot\frac{2}{5} + 100,000\cdot\frac{3}{5}$ = \$460,000 is they do not build the arena, so that it would be preferable to build the arena; (d) to avoid the possible loss of \$500,000, he would vote against putting up the funds for the arena; (e) hoping to attain the profit of \$2,050,000, he would for putting up the funds for the arena; (f) the table is shown on page 238 of the text, and it can be seen that the greatest possible opportunity loss is least if they decide to build the new arena; (g) the expected opportunity loss is $0\cdot\frac{1}{3} + 600,000\cdot\frac{2}{3}$ = \$400,000 if they build the new arena, but only $1,050,000\cdot\frac{1}{3} + 0\cdot\frac{2}{3}$ = \$350,000 if they do not build the arena.
2. (a) $2,050,000\cdot\frac{1}{2} + 1,000,000\cdot\frac{1}{2}$ = \$1,525,000; (b) $-500,000\cdot\frac{1}{2} + 100,000\cdot\frac{1}{2}$ = -\$200,000.
3. (a) Build the arena, as this is the only way they can even hope to make a profit of at least \$1,200,000; (b) They may very well decide not to build as this would assure their not going out of business.
4. (a) The expected profit is $4,000,000\cdot\frac{1}{2} - 1,200,000\cdot\frac{1}{2}$ = \$1,400,000 if they make the tires and (as in the text) -\$210,000 if they do not make the tires, so that Mr. Green's decision would not be changed; (b) the expected profit is $4,000,000\cdot\frac{1}{4} - 1,200,000\cdot\frac{3}{4}$ = \$100,000 if they make the tires and (as in the text) -\$95,000 if they do not make the tires, so that Mr. Green's decision would be reversed.
6. (a)

	Friends at L.J.	Friends at M.B.
Drive to L.J.	11	15
Drive to M.B.	13	9

(b) the expectations are $11\cdot\frac{5}{6} + 15\cdot\frac{1}{6} = \frac{70}{6}$ and $13\cdot\frac{5}{6} + 9\cdot\frac{1}{6} = \frac{74}{6}$, and since the first one is smaller he should drive to La Jolla. (c) the expectations are $11\cdot\frac{2}{3} + 15\cdot\frac{1}{3} = \frac{37}{3}$ and $13\cdot\frac{2}{3} + 9\cdot\frac{1}{3} = \frac{35}{3}$, and since the second is smaller he should drive to Mission Beach;

(d) the expectations are $11\cdot\frac{3}{4} + 15\cdot\frac{1}{4} = 12$ and $13\cdot\frac{3}{4} + 9\cdot\frac{1}{4} = 12$, and it does not matter where he tries first; (e) he goes to Mission Beach to avoid the possibility of having to go 15 miles; (f) he goes to Mission Beach hoping that it will take altogether only 9 miles; (g) the entries of the first row of the table are 0 and 6 and those of the second row are 2 and 0, and since 2 is less than 6, he should go to Mission Beach.

7. (a) the expectation is $11\cdot\frac{3}{4} + 13\cdot\frac{1}{4} = 11.5$; (b) the expectation is $15\cdot\frac{3}{4} + 9\cdot\frac{1}{4} = 13.5$.
8. The expectations are $11\cdot\frac{1}{2} + 13\cdot\frac{1}{2} = 12$ and $15\cdot\frac{1}{2} + 9\cdot\frac{1}{2} = 12$.
9. (a) the two expected inconveniences are $-10\cdot\frac{1}{7} + 6\cdot\frac{6}{7} = \frac{26}{7}$ and $20\cdot\frac{1}{7} + 0\cdot\frac{6}{7} = \frac{20}{7}$, and since the second is smaller she should not take the raincoat; (b) the expected inconveniences are $-10\cdot\frac{1}{4} + 6\cdot\frac{3}{4} = 2$ and $20\cdot\frac{1}{4} + 0\cdot\frac{3}{4} = 5$, and since the first is smaller she should take the raincoat; (c) the expected inconveniences are $-10\cdot\frac{1}{6} + 6\cdot\frac{5}{6} = \frac{20}{6}$ and $20\cdot\frac{1}{6} + 0\cdot\frac{5}{6} = \frac{20}{6}$, so that it does not matter whether she takes the raincoat; (d) since the entries of the first row are 0 and 6 while those of the second row are 30 and 0, she would minimize the greatest possible loss of opportunity by taking the raincoat.
10. (a) $-10\cdot\frac{1}{2} + 20\cdot\frac{1}{2} = 5$; (b) $6\cdot\frac{1}{2} + 0\cdot\frac{1}{2} = 3$.
11. (a) $-10\cdot\frac{5}{9} + 20\cdot\frac{4}{9} = \frac{10}{3}$; (b) $6\cdot\frac{5}{9} + 0\cdot\frac{4}{9} = \frac{10}{3}$; since $\frac{10}{3} = 3\frac{1}{3}$ is less than 5, the larger of the two values obtained in Exercise 10, the gambling scheme of this exercise is preferable to the one of Exercise 10.
12. The two expected gains are 2,000,000p - 440,000(1 - p) and -1,200,000p + 20,000(1 - p), and they are equal when 2,000,000p - 440,000(1 - p) = -1,200,000p + 20,000(1 - p), 3,200,000p = 460,000(1 - p), 3,200,000p + 460,000p = 460,000, 3,660,000p = 460,000, and p = 460,000/3,660,000 = 23/183; the corresponding expected gain is $2,000,000\cdot\frac{23}{183} - 440,000\cdot\frac{160}{183} = -133,333\frac{1}{3}$, and even though it is negative, it is preferable to the expected gain of -\$590,000 to which he is exposed when he uses the coin or the possible expected gain of -\$996,667 to which he is exposed when he uses the die (see page 79 in the text).
13. (a) The expected value of perfect information is $2,050,000\cdot\frac{1}{3} + 100,000\cdot\frac{2}{3}$ = \$750,000, and since this exceeds both \$400,000 and \$350,000 by more than \$50,000, it would be worthwhile to spend the money to get the information; (b) the expected value of perfect information is $11\cdot\frac{2}{3} + 9\cdot\frac{1}{3} = \frac{31}{3}$ miles times 3 cents, namely, 31 cents, and since we find that neither $\frac{37}{3}\cdot 3 = 37$ nor $\frac{35}{3}\cdot 3 = 35$ exceeds 31 by 10, it follows that it is not worthwhile to make the call.
15. (a) he should send roses to avoid the possible -10 units pf appreciation; (b) he should send roses since he would have no chance of being asked again if he sent the candy.

CHAPTER 4

EVENTS

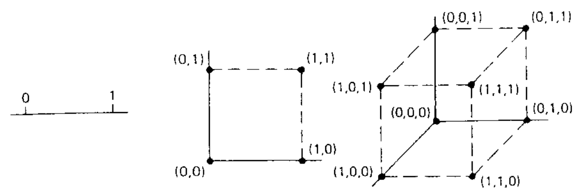
NOTES AND COMMENTS

When the set-oriented approach to probability was first introduced into elementary texts, it was feared that it might prove to difficult and too abstract. Happily, these fears turned out to be unfounded; in fact, many instructors found that the study of sample spaces, subsets, and their combinations is liked by students and leads to a sounder understanding of the whole subject. Thus, these concepts are studied in this chapter prior to the formal treatment of probability in Chapter 5.

DISCUSSION OF EXERCISES ON PAGES 94 THROUGH 98

Exercises 2, 3, 4, 5, 6, and 7 relate to material already discussed in the text; Exercises 8, 9, and 10 pertain to the same sample space, and so do Exercises 10, 11, and 12; Exercises 7, 10, 13, and 16 pertain to mutually exclusive events. Since much of the work in this problem set is done in one step, referring to lists of points or diagrams, it will not be given here in any detail; the answers to the odd-numbered exercises are given at the end of the text and the answers to the even-numbered exercises are given at the end of this manual.

1. (a)



Diagrams for parts (a), (b), and (c) of Exercise 1

8.

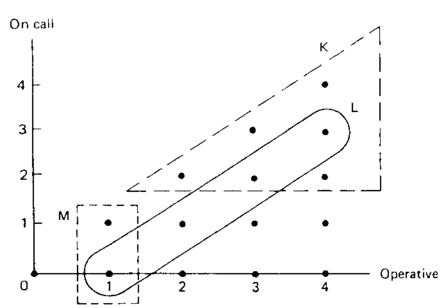


Diagram for Exercise 8

11.

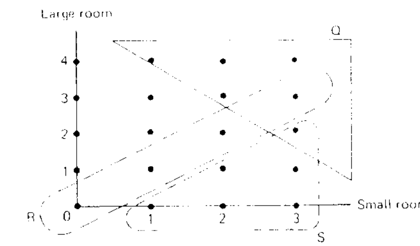


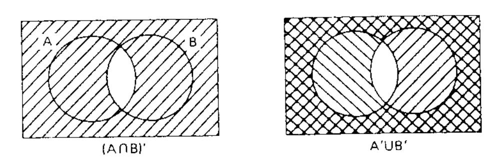
Diagram for Exercise 11

16. (a) Not mutually exclusive because there can be rain and sunshine on the same day; (b) mutually exclusive because when it is 11 p.m. in Los Angeles it is already the next day in Chicago; (c) mutually exclusive because the president must be at least 35; (d) not mutually exclusive since a person can speed through a red light; (e) not mutually exclusive, obviously; (g) not mutually exclusive as there are two black kings; (h) not mutually exclusive since he can get the walk and the home run in different at bats; (j) not mutually exclusive since the player can get an inside the park home run.

DISCUSSION OF EXERCISES ON PAGES 102 THROUGH 106

Exercises 1, 2, and 3 pertain to set notation; Exercises 4, 15, and 16 refer to examples discussed in the text; Exercises 5 through 11 refer to the sample spaces in the exercises of the preceding problem set; Exercises 12 through 14 are theoretical exercises; Exercises 17 through 19 pertain to Venn diagrams involving three circles; and Exercise 20 introduces Euler diagrams, a name sometimes erroneously given to Venn diagrams.

12.



Diagrams for Exercise 12

The region shaded in the first Venn diagram is the same as that shaded one way or the other in the second Venn diagram.

13. (a)

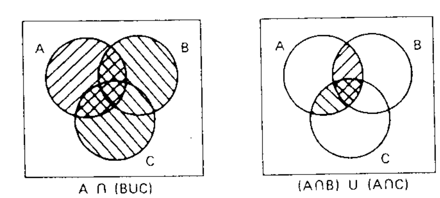
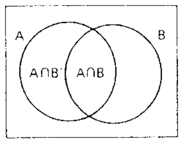
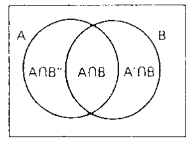


Diagram for part (a) of Exercise 13

The region shaded both ways in the first Venn diagram is the same as that shaded one way or the other in the second Venn diagram.

14. (c) and (d)





Diagrams for parts (c) and (d) of Exercise 14

20. (a) The two circles do not intersect, that is, they do not overlap and the corresponding events are mutually exclusive; (b) one circle is entirely contined in the other, so that all elements of B are also elements of A.

DISCUSSION OF EXERCISES ON PAGES 109 THROUGH 110

These exercises all pertain to the determination of the number of elements in various subsets, and they are fairly straightforward.

1. 145 - 50 = 95 got a raise but no promotion, 85 - 50 = 35 got a promotion but no raise, so that 240 - (95 + 50 + 35) = 60 get neither a raise nor a promotion, and the probability is 60/240 = 0.25.
2. If x students are enrolled in both courses, then 77 - x are enrolled in World History but not in World Geography, 64 - x are enrolled in World Geography but not in World History, so that (77 - x) + x + (64 - x) + 92 = 200, 233 - x = 200, and x = 33.
3. 312 - 173 = 139 regularly look at the food ads but do not read the "Dear Abby" column, 248 - 173 = 75 regularly read the "Dear Abby" column but not the food ads, so that there must be altogether 139 + 173 + 75 + 43 = 430 housewives. This does not agree with the figure that 400 housewives were interviewed in the survey.
4. Draw a Venn diagram with three circles and fill in first the 16% inside all three circles; then fill in the 23 - 16 = 7%, 34 - 16 = 18%, and 28 - 16 = 12% inside two circles but outside the third, and and then fill in the 37 - (7 + 16 + 12) = 2%, 54 - (7 + 16 + 18) = 13%, and 67 - (12 + 16 + 18) = 21% inside one circle but outside the other two; this leaves 100 - (16 + 7 + 18 + 12 + 2 + 13 + 21) = 11% outside all three circles.
5. Simply fill in the figures in the various regions of a Venn diagram with three circles.

CHAPTER 5

RULES OF PROBABILITY

NOTES AND COMMENTS

To "soften" the postulate approach to probability, the basic postulates are presented in this book in words as well as symbols, and they are immediately justified with reference to the various concepts of probability. The best way to teach this material is by means of examples, the more the better, and this is why the author has taken considerable pains to make the illustrations and the exercises cover a great variety of situations, not merely the usual games of chance.

DISCUSSION OF EXERCISES ON PAGES 119 THROUGH 126

In Exercises 1 through 10 the reader will be asked to check whether given probabilities are compatible with the postulates, or what they have to be in view of the postulates; among these, Exercises 5 through 7, in paricular, deal with subjective probabilities; Exercises 11 through 17 pertain to the generalization of Postulate 3 on page 115; and Exercises 19 through 26 pertain to the rule of determining probabilities on page 117, and its special case (on page 119) for equiprobable outcomes.

4. 0.58 + 0.22 = 0.80, which leads to odds of 4 to 1, but this argument would hold only if the two possibilities (filling the tank and looking under the hood) were mutually exclusive, which they are not.
6. The corresponding probabilities are $\frac{1}{8}$ and $\frac{1}{4}$, and since $\frac{1}{8} + \frac{1}{4} = \frac{3}{8}$, the odds are 3 to 5, namely, 5 to 3 against the novel becoming either kind of success.
7. The corresponding probabilities are $\frac{1}{3}$ and $\frac{1}{4}$, and since $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$, the odds are 7 to 5, which is better than an even chance.
8. The probabilities are 1/5, p, and 4/5 - p, so that 12,400 = $20{,}000\cdot\frac{1}{5} + 12{,}000\cdot p + 8{,}000\cdot(\frac{4}{5} - p) = 10{,}400 + 4{,}000p$, and it follows that $p = \frac{1}{2}$.
9. The corresponding probabilities are $\frac{2}{3}$ and $\frac{5}{6}$, whose sum exceeds 1.
10. The probabilities are $\frac{1}{5}$ and $\frac{2}{7}$, and since $\frac{1}{5} + \frac{2}{7} = \frac{17}{35}$, the odds should be 17 to 18 and not 3 to 9.
11. (a) 0.23 + 0.39 + 0.15 = 0.77; (b) 0.39 + 0.16 + 0.07 = 0.62; (c) 1 - (0.23 + 0.15) = 1 - 0.38 = 0.62.
12. (a) 0.36 + 0.23 + 0.09 = 0.68; (b) 0.36 + 0.23 + 0.09 + 0.18 = 0.86 (c) 0.23 + 0.36 + 0.18 = 0.77; (d) 0.36 + 0.18 + 0.14 = 0.68.
13. (a) 0.01 + 0.24 = 0.25; (b) 0.36 + 0.39 = 0.75; (c) 0.24 + 0.36 + 0.39 = 0.99.
15. (a) 0.22 + 0.09 = 0.31; (b) 0.10 + 0.04 = 0.14; (c) 0.15 + 0.10 + 0.08 = 0.33; (d) 0.09 + 0.10 + 0.04 = 0.23; (e) 0.15 + 0.22 + 0.09 + 0.08 = 0.54.
18. P(A) = 0.10 + 0.15 + 0.25 = 0.50; P(B) = 0.15 + 0.09 + 0.06 = 0.30;

$P(C) = 0.15 + 0.25 + 0.15 + 0.09 = 0.64$.

19. Simply multiply by $\frac{1}{15}$ the number of points in each of the subsets.
20. (a) $P(K) = 0.09 + 0.16 + 0.05 + 0.08 + 0.03 + 0.01 = 0.42$, $P(L) = 0.04 + 0.16 + 0.16 + 0.03 = 0.39$, and $P(M) = 0.04 + 0.04 = 0.08$; (b) $P(N) = 0.04 + 0.16 + 0.15 + 0.03 = 0.38$, $P(O) = 0.09 + 0.05 + 0.15 + 0.01 + 0.03 + 0.05 = 0.38$, and $P(P) = 0.01 + 0.04 + 0.04 = 0.09$.
23. (a) $0.17 + 0.10 + 0.04 + 0.13 = 0.44$; (b) $0.15 + 0.08 + 0.10 + 0.13 = 0.46$; (c) $0.08 + 0.21 + 0.04 + 0.13 = 0.46$.
25. Write the probabilities in the various regions of the Venn diagram and then add the probabilities of the respective regions.

COMMENTS ON EXERCISES ON PAGES 129 THROUGH 133

Exercises 1 through 4 are theoretical exercises, and some of this theory is applied in Exercise 5; Exercises 6 through 15 are applications of the general addition rule, and Exercises 16 through 20 deal with the extension of this rule to three events.

1. $S \cup \emptyset = S$ implies that $P(S \cup \emptyset) = P(S)$, and since S and $\emptyset$ are mutually exclusive, Postulates 2 and 3 lead to $P(S) + P(\emptyset) = P(S)$, and it follows that $P(\emptyset) = 0$.
2. Making use of the fact that $A \cap B$ and $A \cap B'$ are mutually exclusive (after all, the elements of the first set are all element of B while the elements of the second set are all elements of B'), it follows that $P(A) = P(A \cap B) + P(A \cap B')$, and since $P(A \cap B') \geq 0$, we conclude that $P(A) \geq P(A \cap B)$. To prove symbolically that $A \cap B$ and $A \cap B'$ are mutually exclusive, we have only to argue that $(A \cap B) \cap (A \cap B') = A \cap B \cap A \cap B' = A \cap A \cap (B \cap B') = A \cap A \cap \emptyset = \emptyset$.
3. $P(A \cup B) = P(A \cap B') + P(A \cap B) + P(A' \cap B) = P(A \cap B') + P(B)$, and since $P(A \cap B'$ cannot be negative, it follows that $P(A \cup B) \geq P(B)$. To prove the second inequality, we have only to interchange A and B.
4. $P(B) = P(B \cap A) + P(B \cap A') = P(B \cap A) + P(B' \cap A) + P(B \cap A') - P(B' \cap A) = P(A) + P(B \cap A') - P(B' \cap A)$, and since $P(B \cap A')$ cannot be negative and it is given that $P(B' \cap A) = 0$, it follows that $P(B) \geq P(A)$.
5. The probability that he will pass the psychology examination must equal the sum of the probabilities that he will pass both examinations and that he will pass in psychology but fail in economics, but for the given data $0.38 \neq 0.23 + 0.16$; (b) the second probability cannot be less than the first; in fact, it violates the rule of Exercise 3; (c) the probability that the winner will be a native of San Francisco cannot exceed the probability that he is a native California, since San Francisco is in California; this violates the rule of Exercise 4; (d) the general addition rule leads to a probability of $0.63 + 0.84 - 0.45$ for the probability that the team will win either game, and this is impossible since this quantity is greater than 1; (e) the second probability cannot exceed the first; in fact, this violates the rule of Exercise 2; (f) the three probabilities should add up to 1, which they don't.
8. (a) $0.23 - 0.08 = 0.15$; (b) $(0.23 - 0.08) + 0.08 + (0.18 - 0.08) = 0.33$; (c) $1 = 0.33 = 0.67$; (d) $0.33 - 0.08 = 0.25$.
10. $0.18 + 0.23 - 0.14 = 0.27$.
12. The corresponding probabilities are 0.40 that she will get a promotion, 0.50 that she will get a raise, and 0.20 that she will get both; thus, the probability that she will get either is $0.40 + 0.50 - 0.20 = 0.70$, and the odds are 7 to 3.

14. $P(X) = 0.09 + 0.16 + 0.05 + 0.08 + 0.03 = 0.41$, $P(Y) = 0.08 + 0.16 + 0.15 + 0.05 = 0.44$, $P(X \cap Y) = 0.08 + 0.16 = 0.24$, and $P(X \cup Y) = 0.41 + 0.44 - 0.24 = 0.61$.
18. Subtract from 0.64, the value obtained in the text, the sum of the probabilities that he will have two of things done to him, namely, $0.09 + 0.11 + 0.06$, but since we are, thus, subtracting the probability that he will have three things done to him three times, we have to add it back twice, that is, we have to add $2(0.03)$; it follows that the answer is $0.64 - (0.09 + 0.11 + 0.06) + 2(0.03) = 0.44$.
19. $0.62 + 0.69 + 0.49 - (0.37 + 0.39 + 0.34) + 0.28 = 0.98$.
20. $(0.70 + 0.64 + 0.58 + 0.58) - (0.45 + 0.42 + 0.41 + 0.35 + 0.39 + 0.32) + (0.23 + 0.26 + 0.21 + 0.20) - 0.12 = 0.94$.

CHAPTER 6

CONDITIONAL PROBABILITIES

NOTES AND COMMENTS

The concept of a conditional probability is a very important one, for all probabilities are conditional probabilities in the sense that they are meaningful only with regard to the sample space for which they are defined. Also, they lead to the multiplication rules for probabilities and the concept of independence. Finally, Bayes' rule, which for many years has been shunned for being too controversial, has recently gained a position of prominence in statistics, and it is thus a prerequisite for understanding many of the methods and concepts of modern statistics.

DISCUSSION OF EXERCISES ON PAGES 144 THROUGH 152

Exercise 1, like the illustration on page 134, serves to illustrate what can happen when we forget about the sample spaces with which we are dealing; Exercises 2 through 6 deal with the notation used for conditional probabilities; Exercises 7 through 16 are fairly straightforward problems dealing with conditional probabilities; Exercises 24 through 27 pertain to the independence of events, and Exercises 28 and 29 pertain to sampling with and without replacement; Exercises 30 through 37 deal with the extended multiplication rules for more than two events, and Exercises 38 and 39 are theoretical exercises dealing with the concepts of indepence and pairwise independence.

9. If each person with less than 5 years experience has the probability p of getting the job and each person with at least 5 years teaching experience has the probability 2p, then 42p + 18(2p) = 1 and p = 1/78. Thus, the various probabilities of parts (a) through (e) can be obtained by multiplying the respective number of candidates by 1/78. Actually, it is easiest to multiply the entries of the first row by 2, getting 24 and 12 instead of 12 and 6, and then proceeding as in Exercise 7, simply reading off the probabilities.
11. Use the same "trick" as in Exercise 9.
18. (a) $\frac{5}{8}\cdot\frac{4}{7} = \frac{5}{14}$; (b) $\frac{3}{8}\cdot\frac{2}{7} = \frac{3}{28}$; (c) $1 - \frac{5}{14} - \frac{3}{28} = \frac{15}{28}$.
19. (a) $\frac{0.72}{0.80} = 0.90$; (b) $\frac{0.72}{0.75} = 0.96$.
20. (a) (0.60)(0.90) = 0.54; (b) $\frac{0.54}{0.72} = 0.75$.
24. $P(L) = 0.39$, $P(M) = 0.08$, $P(L \cap M) = 0.04$, so that $P(L|M) = \frac{0.04}{0.08} = 0.50$, and since this does not equal P(L), the two events are not independent.
25. (a) $P(Q) = 6/20 = 0.30$, $P(R) = 4/20 = 0.20$, $P(Q \cap R) = 1/20 = 0.05$, so that $P(Q|R) = \frac{0.05}{0.20} = 0.25$, and since this does not equal P(Q), the two events are not independent; (b) note that R and S are mutually exclusive, and hence they are dependent; after all, if one happens the other cannot happen and vice versa.

26. $P(M) = \frac{4}{5}$, $P(N) = \frac{7}{10}$, $P(M \cap N) = \frac{3}{5}$, so that $P(N|M) = \frac{3/5}{4/5} = \frac{3}{4}$, and since this does not equal P(N), the two events are not independent.
27. (a) dependent, since a tired driver is more likely to have an accident that a driver who is not tired; (b) obviously independent; (c) clearly independent; (d) dependent, since wealthy persons have more money to spend on works of art; (e) any two mutually exclusive events are dependent since the occurrence of one makes the occurrence of the other impossible; (f) independent; (g) dependent, provided the person has the flat tire while on his way to work; (h) dependent, since the vast majority of persons under 20 (including small babies) do not smoke pipes; (i) dependent, since a person who cannot afford a meal is more likely to be hungry.
28. (a) $\frac{12}{30}\cdot\frac{11}{29} = \frac{22}{145}$; (b) $\frac{2}{5}\cdot\frac{2}{5} = \frac{4}{25}$.
29. (a) $\frac{26}{52}\cdot\frac{25}{51} = \frac{25}{102}$; (b) $\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$.
30. (a) $\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{16}$; (b) $\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6} = \frac{1}{7,776}$; (c) $\frac{1}{4}\cdot\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{4} = \frac{81}{4096}$.
31. (a) $(0.7)^3 = 0.343$; (b) $(0.58)^4 = 0.113$ (approx.) (c) $(0.8)^6 = 0.262$ (approximately).
32. (a) $\frac{13}{52}\cdot\frac{12}{51}\cdot\frac{11}{50} = \frac{11}{850}$; (b) $(\frac{1}{4})^3 = \frac{1}{64}$.
35. (a) (0.2)(0.4)(0.6) = 0.048; (b) $(0.2)(0.4)^3(0.6) = 0.00768$.
36. (a) (0.6)(0.8)(0.8)(0.8) = 0.3072; (b) (0.4)(0.3)(0.8) = 0.096; (c) (0.6)(0.2)(0.7)(0.3)(0.8) = 0.02016.
37. $P(A|B) = P(A) = \frac{1}{2}$, $P(A|C) = P(A) = \frac{1}{2}$, but $P(A|B \cap C) = 1 \neq P(A)$.
38. $P(A) = 0.6$, $P(B) = 0.8$, $P(C) = 0.5$, $P(A \cap B \cap C) = 0.24 = P(A)\cdot P(B)\cdot P(C)$, but $P(A \cap B) = 0.54 \neq P(A)\cdot P(B)$

DISCUSSION OF EXERCISES ON PAGES 162 THROUGH 166

Exercises 1 through 7 pertain to the rule of elimination, and Exercises 8, 9, 10 pertain to an extension of this rule which applies when there are more than two steps; Exercise 11 is a theoretical exercise; Exercises 12 through 22 deal with applications of Bayes' rule, and Exercises 23 and 24 pertain to the use of the summation sign.

1. (0.35)(0.82) + (0.65)(0.44) = 0.573.
2. (a) (0.50)(0.72) + (0.50)(0.84) = 0.78; (b) (0.50)(0.28)(0.84) + (0.50)(0.72)(0.72) + (0.50)(0.84)(0.72) + (0.50)(0.16)(0.84) = 0.746
3. (0.80)(0.90) + (0.20)(0.78) = 0.876.
4. $\frac{3}{5}(0.18) + \frac{2}{5}(0.66) = 0.372$.
5. (0.60)(0.80) + (0.40)(0.3) = 0.60; using the result, the answer to the second question is also (0.60)(0.80) + (0.40)(0.30) = 0.60.
6. (0.45)(0.004) + (0.30)(0.006) + (0.25)(0.010) = 0.0061.
7. (0.45)(0.62) + (0.30)(0.35) + (0.10)(0.26) + (0.15)(0.12) = 0.428.
8. (a) $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$; (b) $\frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{8} = \frac{5}{16}$; the individual terms in these sums are the probabilities associated with the various branches of the tree diagram shown at the top of the next page.

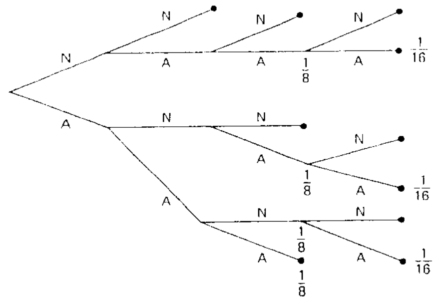


Diagram for Exercise 8

9. (a) 0.12 + 0.12 = 0.24; (b) 0.0075 + 0.0025 = 0.01; the terms in these sums are the probabilities associated with the respective branches of the following tree diagram

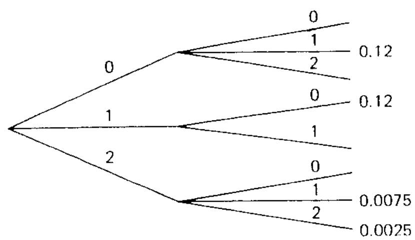


Diagram for Exercise 9

10. There are 4 possibilities where he studies 2 hours each on three days and 0 hours on the other day, and 6 possibilities where he studies 2 hours on two days and 1 hour on two days; the respective probabilities are (0.5)(0.1)(0.1)(0.1) = 0.0005 and (0.4)(0.4)(0.1)·(0.1) = 0.0016, so that the answer is 4(0.0005) + 6(0.0016) = 0.0116.

11. Making use of the fact that $A = (A\cap B)\cup(A\cap B')$, we get $P(A) = P(A\cap B) + P(A\cap B') = P(B)\cdot P(A|B) + P(B')\cdot P(A|B')$.

12. $\frac{(0.35)(0.82)}{(0.35)(0.82) + (0.65)(0.44)} = 0.501$.

13. $\frac{0.360}{0.780} = 0.468$ (approximately).

15. $\frac{0.264}{0.372} = 0.71$ (approximately), and the odds are about 71 to 29.

16. $\frac{(2/3)(3/4)}{(2/3)(3/4) + (1/3)(1/2)} = 0.75$.

17. $\frac{(0.25)(0.80)}{(0.75)(0.12) + 0.25)(0.80)} = \frac{20}{29}$; $\frac{(0.75)(0.88)}{(0.75)(0.88) + (0.25)(0.2)} = \frac{66}{71}$.

18. $\frac{(0.6)(0.8)}{(0.6)(0.8) + (0.4)(0.25)} = \frac{48}{58} = 0.828$ (approximately).

19. $\frac{0.279}{0.428} = 0.652$ (approximately).

20. $\frac{0.00250}{0.00610} = 0.410$, $\frac{0.00180}{0.00610} = 0.295$, and $\frac{0.00180}{0.00610} = 0.295$, respectively, for assembly lines C, A, and B.

21. $\frac{3/16}{5/16} = \frac{3}{5}$, where 3/16 is the probability that the American League team wins in seven games, and 5/16 is the probability that the American League team wins.

22. The probability that none of the donuts is filled with mustard is

$$\frac{\frac{1}{7}\cdot\frac{6}{6}}{\frac{1}{7}\cdot\frac{6}{6} + \frac{1}{7}\cdot\frac{5}{6} + \frac{1}{7}\cdot\frac{4}{6} + \frac{1}{7}\cdot\frac{3}{6} + \frac{1}{7}\cdot\frac{2}{6} + \frac{1}{7}\cdot\frac{1}{6} + \frac{1}{7}\cdot 0} = \frac{6}{21},$$

the probability that at least one of the donuts is filled with mustard is $1 - \frac{6}{21} = \frac{15}{21}$, so that the correct odds are 15 to 6, or 5 to 2. The tree diagram for this problem is shown below:

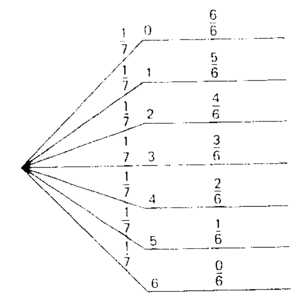


Diagram for Exercise 22

CHAPTER 7

PROBABILITY FUNCTIONS

NOTES AND COMMENTS

This chapter contains an introduction to random variables and their distributions. Our definition of a random variable is fairly intuitive, and it should be pointed out that the somewhat ill-chosen term "random variable" is neither random nor a variable; as has been pointed out, it is like an alligator pear (or avocado) which is neither an alligator nor a pear.

Our discussion of probability functions, or probability distributions, and their descriptions is limited to the binomial, hypergeometric, geometric, and multinomial distributions, although some extensions are given in the last set of exercises.

Although the formula for the mean of the binomial distribution, given on page 180, may seem intuitively obvious, some of the better students may appreciate seeing the following rigorous proof. According to the definition of μ on page 172 we have

$$\mu = \sum_{x=0}^{n} x\cdot\binom{n}{x}p^x(1-p)^{n-x} = \sum_{x=1}^{n} x\cdot\frac{n!}{x!(n-x)!}\cdot p^x(1-p)^{n-x}$$

where the summation starts with 1, since $x\cdot f(x)$ equals zero for $x = 0$. Then, if we make use of the fact that $n! = n(n-1)!$, $x! = x(x-1)!$, cancel the x's, and factor out n and p, we get

$$\mu = np\cdot\sum_{x=1}^{n}\frac{(n-1)!}{(x-1)!(n-x)!}\cdot p^{x-1}(1-p)^{n-x}$$

and if we let $y = x - 1$, this becomes

$$\mu = np\cdot\sum_{y=0}^{n-1}\binom{n-1}{y}\cdot p^y(1-p)^{n-1-y}$$

and, hence, $\mu = np$ since the last summation is the sum of the probabilities of 0, 1, 2, ..., and $n - 1$ successes in $n - 1$ trials, which equals 1.

DISCUSSION OF EXERCISES ON PAGES 172 THROUGH 175

Exercises 1 through 8 are designed to illustrate the concept of a probability function; Exercises 10 through 14 pertain to the mean of a probability distribution.

5. $f(0) = \frac{\binom{3}{0}}{8} = \frac{1}{8}$, $f(1) = \frac{\binom{3}{1}}{8} = \frac{3}{8}$, $f(2) = \frac{\binom{3}{2}}{8} = \frac{3}{8}$, $f(3) = \frac{\binom{3}{3}}{8} = \frac{1}{8}$.
6. (a) $f(0) = \frac{1}{5}$, $f(1) = \frac{1}{5}$, $f(2) = \frac{1}{5}$, $f(3) = \frac{1}{5}$, $f(4) = \frac{1}{5}$, $f(5) = \frac{1}{5}$ and the sum is $\frac{6}{5}$, which exceeds 1; (b) $f(1) = \frac{1}{10}$, $f(2) = \frac{2}{10}$, $f(3) = \frac{3}{10}$, $f(4) = \frac{4}{10}$, and the sum of these probabilities is 1; (c) $f(0) = 0$ $f(1) = \frac{1}{14}$, $f(2) = \frac{4}{14}$, $f(3) = \frac{9}{14}$, and the sum of the probabilities is 1. (d) $f(1) = -2/3$ and $f(2) = -1/3$.

9. (a) $0(0.49) + 1(0.42) + 2(0.09) = 0.60$; (b) $-2(0.09) - 1(0.12) + 0(0.34) + 1(0.20) + 2(0.25) = 0.40$. The mean of the distribution of the differences between two random variables equals the difference between the means of their respective distributions.
11. $2\cdot\frac{10}{28} + 1\cdot\frac{15}{28} + 0\cdot\frac{3}{28} = \frac{35}{28} = 1\frac{1}{4}$.
12. (a) $0\cdot\frac{5}{15} + 1\cdot\frac{4}{15} + 2\cdot\frac{3}{15} + 3\cdot\frac{2}{15} + 4\cdot\frac{1}{15} = \frac{4}{3}$; (b) $0\cdot\frac{1}{15} + 1\cdot\frac{2}{15} + 2\cdot\frac{3}{15} + 3\cdot\frac{4}{15} + 4\cdot\frac{5}{15} = 2\frac{2}{3}$; (c) $0\cdot\frac{5}{15} + 1\cdot\frac{4}{15} + 2\cdot\frac{3}{15} + 3\cdot\frac{2}{15} + 4\cdot\frac{1}{15} = \frac{4}{3}$.
13. (a) $0(0.25) + 1(0.25) + 2(0.25) + 3(0.25) = 1.5$; (b) $0(0.20) + 1(0.20) + 2(0.20) + 3(0.20) + 4(0.20) = 2$; (c) $0(0.05) + 1(0.10) + 2(0.15) + 3(0.20) + 4(0.20) + 5(0.15) + 6(0.10) + 7(0.05) = 3.5$. The mean of the distribution of the sum of two random variables equals the sum of the means of their respective distributions.
14. $0(0.01) + 1(0.08) + 2(0.34) + 3(0.44) + 4(0.13) = 2.60$; (b) $0(0.20) + 1(0.38) + 2(0.30) + 3(0.11) + 4(0.01) = 1.35$; (c) $0(0.23) + 1(0.39) + 2(0.29) + 3(0.08) + 4(0.01) = 1.25$. The mean of the distribution of the differences between two random variables equals the difference between the means of their respective distributions.

DISCUSSION OF EXERCISES ON PAGES 183 THROUGH 189

Exercises 1 through 19 are applications of the binomial distribution, based either on the formula or the table; among these Exercise 15 concerns so-called confidence limits and is a bit more difficult, while Exercises 16 through 19 are statistical decision problems; Exercises 20 through 23 pertain to the means of binomial distribution, and Exercises 24 and 25 are Bayesian applications.

1. (a) $\binom{5}{2}(\frac{1}{6})^2(\frac{5}{6})^3 = \frac{625}{3,888}$; (b) $\binom{5}{0}(\frac{1}{6})^0(\frac{5}{6})^5 + \binom{5}{1}(\frac{1}{6})^1(\frac{5}{6})^4 + \frac{625}{3,888} = \frac{3,125}{7,776} + \frac{3,125}{7,776} + \frac{625}{3,888} = \frac{7,500}{7,776}$; (c) $\binom{8}{6}(\frac{1}{2})^6(\frac{1}{2})^2 = \frac{7}{64}$; (d) $\frac{7}{64} + \binom{8}{7}(\frac{1}{2})^7(\frac{1}{2})^1 + \binom{8}{8}(\frac{1}{2})^8(\frac{1}{2})^0 = \frac{37}{256}$.
2. (a) $\binom{5}{1}(0.2)^1(0.8)^4 = 0.4096$; (b) 0.410.
3. (a) $\binom{6}{0}(0.25)^0(0.75)^6 = \frac{729}{4,096}$; (b) $\binom{6}{1}(0.25)^1(0.75)^5 = \frac{1,458}{4,096}$; (c) $\binom{6}{2}(0.25)^2(0.75)^4 = \frac{1,215}{4,096}$; (d) $1 - \frac{3,402}{4,096} = \frac{694}{4,096}$.
4. (a) $\binom{8}{0}(0.4)^0(0.6)^8 + \binom{8}{1}(0.4)^1(0.6)^7 + \binom{8}{2}(0.4)^2(0.6)^6 = 0.3156$; (b) 0.316.
5. (a) $\binom{6}{4}(0.7)^4(0.3)^2 = 0.3241$; (b) 0.324.
6. $\binom{8}{0}(0.3)^0(0.7)^8 = 0.576$, so that the outcome is not too surprising.
8. (a) $\binom{7}{5}(0.6)^5(0.4)^2 + \binom{7}{6}(0.6)^6(0.4)^1 + \binom{7}{7}(0.6)^7(0.4)^0 = 0.4199$; (b) 0.420.
14. (a) He will be wrong when x = 8, 9, or 10, and the probability is 0.233 + 0.121 + 0.028 = 0.382; (b) he will be wrong when x = 5, 6, 7, 8, 9, or 10, and the probability is 0.201 + 0.111 + 0.042 + 0.011 + 0.002 = 0.367.
15. (a) He will be wrong if x = 0, 4, 5, 6, 7, 8, 9, 10, 11, 12, or 13, and the probability is 0.055 + 0.154 + 0.069 + 0.023 + 0.066 + 0.001

= 0.308; he will be wrong if x = 0, 1, 2, 5, 6, 7, 8, 9, 10, 11, 12, or 13, and the probability is 0.010 + 0.054 + 0.139 + 0.180 + 0.103 + 0.044 + 0.014 + 0.003 + 0.001 = 0.548.

16. The claim is erroneously accepted when x = 7, 8, 9, 10, 11, 12, or 13, and the probability is 0.014 + 0.003 + 0.001 = 0.018.

17. (a) He will commit the error for x = 0 through 11, and the probability is 0.460; (b) he will commit the error for x = 12, and the probability is 0.069.

18. (a) They will commit the error for x = 0 through 6, and the probability is 0.001 + 0.006 + 0.026 + 0.088 = 0.121; (b) they will commit the error for x = 7, 8, 9, or 10, and the probability is 0.117 + 0.044 + 0.010 + 0.001 = 0.172; (c) they will commit the error for x = 7, 8, 9, or 10, and the probability is 0.009 + 0.001 = 0.010.

19. (a) They will commit the error if x = 0 through 5 or 12 through 15, and the probability is 0.002 + 0.007 + 0.024 + 0.063 + 0.022 + 0.005 = 0.123; (b) they will commit the error if x = 6 through 11, and the probability is 0.153 + 0.196 + 0.196 + 0.153 + 0.092 + 0.042 = 0.832; (c) they will commit the error if x = 6 through 11, and the probability is 0.001 + 0.003 + 0.014 + 0.043 + 0.103 + 0.188 = 0.352

20. μ = 0(0.001) + 1(0.007) + 2(0.032) + 3(0.085) + 4(0.155) + 5(0.207) + 6(0.207) + 7(0.157) + 8(0.092) + 9(0.041) + 10(0.014) + 11(0.003) + 12(0.001) = 5.612; np = 14(0.4) = 5.6.

23. (a) Value will differ from mean by not more than 1.5 for x = 6, 7, 8, or 9, and the probability is 0.153 + 0.196 + 0.196 + 0.153 = 0.698; (b) value will differ from mean by less than 2.0 for x = 3, 4, 5, or 6, and the probability is 0.142 + 0.213 + 0.227 + 0.177 = 0.759; (c) value will differ from mean by more than 1.8 for x = 0 or 5 through 14, and the probability is 1 - (0.1154 + 0.250 + 0.250 + 0.172) = 0.174.

24. The probability for Mr. Butler is $\frac{(0.80)(0.017)}{(0.80)(0.017) + (0.20)(0.167)}$ = 0.289 and the odds have changed to 289 to 711, or approximately 2 to 5.

25. (a) If the prior probability for Mr. Charles is x, that of Mr. Brown is 2x, and that of Mr. Ames is 6x, and since x + 2x + 6x = 1, it follows that x = 1/9, and the three prior probabilities are 6/9, 2/9, and 1/9; (b) the posterior probability for Mr. Ames is

$$\frac{\frac{6}{9}(0.377)}{\frac{6}{9}(0.377) + \frac{2}{9}(0.341) + \frac{1}{9}(0.206)} = 0.717,$$

and the posterior probabilities for Mr. Brown and Mr. Charles, obtained in the same way, are 0.217 and 0.066.

DISCUSSION OF EXERCISES ON PAGES 197 THROUGH 202

Exercises 1 through 9 deal with the hypergeometric distribution, and among them Exercises 3, 6, and 8 deal with the mean of this distribution; Exercises 10 through 12 pertain to the binomial approximation of the hypergeometric distribution; Exercises 19 through 21 deal with an extension of the geometric distribution, which is called the negative binomial distribution, and Exercises 22 through 24 deal with a distribution analogous to the geometric distribution, which applies to sampling without replacement; Exercises 25 through 28 pertain to the multinomial distribution, and Exercises 29 and 30 deal with a distribution analogous to the multinomial distribution, which applies to sampling without replacement; Exercises 19, 22, and 29 are of a theoretical nature.

1. $\frac{\binom{8}{2}\binom{12}{4}}{\binom{20}{6}} = \frac{28 \cdot 495}{38,760} = 0.36.$

2. (a) $\frac{\binom{6}{0}\binom{18}{3}}{\binom{24}{3}} = \frac{1 \cdot 816}{2,024} = 0.403$; (b) $\frac{\binom{6}{1}\binom{18}{2}}{2,024} = \frac{6 \cdot 153}{2,024} = 0.454$;

(c) $\frac{\binom{6}{2}\binom{18}{1}}{2,024} = \frac{15 \cdot 18}{2,024} = 0.133$; (d) $\frac{\binom{6}{3}\binom{18}{0}}{2,024} = \frac{20 \cdot 1}{2,024} = 0.010.$

3. 0(0.403) + 1(0.454) + 2(0.133) + 3(0.010) = 0.750; $\mu = 3 \cdot \frac{6}{24} = 0.75.$

4. (a) $\frac{\binom{1}{0}\binom{11}{2}}{\binom{12}{2}} = \frac{1 \cdot 55}{66} = 0.833$; (b) $\frac{\binom{2}{0}\binom{10}{2}}{66} = \frac{1 \cdot 45}{66} = 0.682$;

(c) $\frac{\binom{5}{0}\binom{7}{2}}{66} = \frac{1 \cdot 21}{66} = 0.318$; (d) 1 - 0.833 = 0.167; (e) $1 - \frac{\binom{3}{0}\binom{9}{2}}{66}$ $= 1 - \frac{36}{66} = \frac{30}{66} = 0.455.$

7. (a) $\frac{\binom{4}{0}\binom{11}{2}}{\binom{15}{2}} = \frac{1 \cdot 55}{105} = 0.524$; (b) $\frac{\binom{4}{1}\binom{11}{1}}{105} = \frac{1 \cdot 44}{105} = 0.419$;

(c) $\frac{\binom{4}{2}\binom{11}{0}}{105} = \frac{6 \cdot 1}{105} = 0.057.$

8. 0(0.524) + 1(0.419) + 2(0.057) = 0.533; $\mu = 2 \cdot \frac{4}{15} = \frac{8}{15} = 0.533.$

10. $\frac{\binom{32}{0}\binom{48}{4}}{\binom{80}{4}} = 0.123$; the binomial approximation is $\binom{4}{0}(0.4)^0(0.6)^4 =$ 0.1296, so that the error is 0.1296 - 0.123 = 0.0066.

11. $\frac{\binom{80}{3}\binom{40}{2}}{\binom{120}{5}} = \frac{48 \cdot 47 \cdot 46 \cdot 45}{80 \cdot 79 \cdot 78 \cdot 77} = 0.336$; the binomial approximation is $\binom{5}{3}(\frac{2}{3})^3(\frac{1}{3})^2 = \frac{10 \cdot 8}{243} = 0.329$, and the error is 0.007.

12. $\frac{\binom{8}{0}\binom{392}{3}}{\binom{400}{3}} = \frac{392 \cdot 391 \cdot 390}{400 \cdot 399 \cdot 398} = 0.941$; the binomial approximation is $\binom{3}{0}(0.02)^0(0.98)^3 = 0.941$, and the error is less than 0.001.

13. $(0.10)(0.90)^5 = 0.059.$

14. $(0.12)(0.88)^3 = 0.082.$

17. (a) $(0.10)(0.90)^3 = 0.073$; (b) $(0.10)(0.90)^6 = 0.053$; (c) this is the probability of 10 failures in a row, namely, $(0.90)^{10} = 0.349.$

18. $1(\frac{1}{2}) + 2(\frac{1}{4}) + 3(\frac{1}{8}) + 4(\frac{1}{16}) + 5(\frac{1}{32}) + 6(\frac{1}{64}) + 7(\frac{1}{128}) + 8(\frac{1}{256}) + 9(\frac{1}{512}) + 10(\frac{1}{1024}) = 1.98$; $\mu = \frac{1}{1/2} = 2.$ (First answer depends on rounding, and a value closer to 2 will be obtained by carrying more digits.)

19. The probability that the rth success occurs on the xth trial can be obtained by multiplying the binomial probability of the $r - 1$ successes in the first $x - 1$ trials by p, the probability of a success on the xth trial. For $r = 4$ and $x = 15$, we find that the probability of 3 successes in 14 trials (with $p = 0.10$) is 0.114, and, hence, that the answer is $(0.114)(0.10) = 0.0114$.

20. For $r = 2$ and $x = 10$, we find that the probability of 1 success in 9 trials (with $p = 0.20$ is 0.302, and, hence, that the answer is $(0.302)(0.20) = 0.0604$.

21. For $r = 2$ and $x = 14$, we find that the probability of 1 success in 13 trials (with $p = 0.10$) is 0.367, and, hence, that the answer is $(0.367)(0.10) = 0.0367$.

22. The probability of 4 failures on the first 4 tries is $\frac{\binom{10}{4}}{\binom{12}{4}} = \frac{210}{495}$, the probability that the 5th try will be a success is $\frac{2}{8}$, and the product of these two probabilities is $\frac{210}{495}\cdot\frac{2}{8} = 0.106$.

23. The probability of 2 failures of the first 2 tries is $\frac{\binom{4}{2}}{\binom{6}{2}} = \frac{6}{15}$, the probability that the third try will be a success is $\frac{2}{4}$, and the product of these two probabilities is $\frac{6}{15}\cdot\frac{2}{4} = 0.20$.

24. The probability of 1 success in the first 7 trials is $\frac{\binom{3}{1}\binom{12}{6}}{\binom{15}{7}} = \frac{3\cdot 924}{6,435}$, the probability that the 8th trial will be a success is $\frac{2}{8}$, and the product of these two probabilities is $\frac{3\cdot 924}{6,435}\cdot\frac{2}{8} = 0.108$.

25. $\frac{8!}{2!5!1!}(0.3)^2(0.5)^5(0.2)^1 = 168(0.09)(0.03125)(0.2) = 0.0945$.

26. $\frac{10!}{7!2!1!}(0.6)^7(0.3)^2(0.1) = 360(0.028)(0.09)(0.1) = 0.091$.

27. $\frac{12!}{6!3!1!2!}(0.6)^6(0.2)^3(0.1)^1(0.1)^2 = 55,440(.046656)(0.000008) = 0.021$.

28. $\frac{11!}{5!2!3!1!}(\frac{9}{16})^5(\frac{3}{16})^2(\frac{3}{16})^3(\frac{1}{16})^1 = 27,720\cdot\frac{3^{15}}{16^{11}} = 0.02$.

29. The formula is given among the answers on page 244; (b) the probability is $\frac{\binom{10}{3}\binom{7}{2}\binom{3}{1}}{\binom{20}{6}} = \frac{120\cdot 21\cdot 3}{38,760} = 0.195$.

30. $\frac{\binom{7}{5}\binom{4}{1}\binom{6}{5}\binom{3}{1}}{\binom{20}{12}} = \frac{21\cdot 4\cdot 6\cdot 3}{125,970} = 0.012$; (b) $\frac{\binom{7}{4}\binom{4}{2}\binom{6}{6}\binom{3}{0}}{125,970} = \frac{35\cdot 6\cdot 1\cdot 1}{125,970} =$ 0.0017.

CHAPTER 8

THE LAW OF LARGE NUMBERS

NOTES AND COMMENTS

This chapter introduces the standard deviation of a probability distribution as a measure of the chance variation, or chance fluctuations, of the corresponding random variable. Chebyshev's theorem is introduced as a means of predicting such chance fluctuations, and in particular, it is applied to the binomial distribution, leading to the Law of Large Numbers.

DISCUSSION OF EXERCISES ON PAGES 208 THROUGH 211

Exercises 1 through 4 and 6 are straightforward exercises dealing with the standard deviation of given probability distributions; Exercise 7 introduces a short-cut formula for σ, which is applied in Exercises 9 and 10; Exercises 11 and 12 pertain to binomial distributions, Exercises 13 through 17 pertain to hypergeometric distribution, and Exercises 18 and 19 pertain to geometric distributions; Exercises 5 and 8 are theoretical.

1. $\mu = 0(\frac{1}{3}) + 1(\frac{4}{15}) + 2(\frac{1}{5}) + 3(\frac{2}{15}) + 4(\frac{1}{15}) = \frac{4}{3}$, so that $\sigma^2 = (-\frac{4}{3})^2(\frac{1}{3}) + (-\frac{1}{3})^2(\frac{4}{15}) + (\frac{2}{3})^2(\frac{1}{5}) + (\frac{5}{3})^2(\frac{2}{15}) + (\frac{8}{3})^2(\frac{1}{15}) = \frac{14}{9}$, and $\sigma = \sqrt{14/9} = \sqrt{1.5556}$, which is approximately 1.25.

2. (a) $\mu = 0(0.272) + 1(0.354) + 2(0.230) + 3(0.100) + 4(0.032) + 5(0.009) + 6(0.003) = 1.3$; (b) $\sigma^2 = (-1.3)^2(0.272) + (-0.3)^2(0.354) + (0.7)^2(0.230) + (1.7)^2(0.100) + (2.7)^2(0.032) + (3.7)^2(0.009) + (4.7)^2(0.003) = 1.316$, so that $\sigma = \sqrt{1.316}$, which is approximately 1.15.

4. (a) $\mu = 0(\frac{1}{64}) + 1(\frac{6}{64}) + 2(\frac{15}{64}) + 3(\frac{20}{64}) + 4(\frac{15}{64}) + 5(\frac{6}{64}) + 6(\frac{1}{64}) = 3$; (b) $|-3|\cdot\frac{1}{64} + |-2|\cdot\frac{6}{64} + |-1|\cdot\frac{15}{64} + |0|\cdot\frac{20}{64} + |1|\cdot\frac{15}{64} + |2|\cdot\frac{6}{64} + |3|\cdot\frac{1}{64} = \frac{15}{16}$; (c) $\sigma^2 = (-3)^2\cdot\frac{1}{64} + (-2)^2\cdot\frac{6}{64} + (-1)^2\cdot\frac{15}{64} + 0^2\cdot\frac{20}{64} + 1^2\cdot\frac{15}{64} + 2^2\cdot\frac{6}{64} + 3^2\cdot\frac{1}{64} = 1.5$; (d) $\sigma = \sqrt{1.5} = 1.225$; (e) $\mu = 6\cdot\frac{1}{2} = 3$ and $\sigma = \sqrt{6\cdot\frac{1}{2}\cdot\frac{1}{2}} = \sqrt{1.5}$.

5. $(x_1 - \mu)\cdot f(x_1) + (x_2 - \mu)\cdot f(x_2) + \ldots + (x_k - \mu)\cdot f(x_k) = x_1\cdot f(x_1) + x_2\cdot f(x_2) + \ldots + x_k\cdot f(x_k) - \mu\cdot f(x_1) - \mu\cdot f(x_2) - \ldots - \mu\cdot f(x_k) = \mu - \mu\cdot[f(x_1) + f(x_2) + \ldots + f(x_k)] = \mu - \mu\cdot 1 = 0$.

7. (a) $\mu_2 = 0(0.272) + 1(0.354) + 4(0.230) + 9(0.100) + 16(0.032) + 25(0.009) + 36(0.003) = 3.019$, so that $\sigma^2 = 3.019 - (1.3)^2 = 1.33$ and $\sigma = \sqrt{1.33} = 1.15$; (b) $\mu_2 = 0(0.050) + 1(0.149) + 4(0.224) + 9(0.224) + 16(0.168) + 25(0.101) + 36(0.050) + 49(0.023) + 64(0.008) + 81(0.003) = 11.956$, so that $\sigma^2 = 11.96 - 9.00 = 2.96$ and $\sigma = \sqrt{2.96}$

(c) $\mu_2 = 1\cdot\frac{6}{64} + 4\cdot\frac{15}{64} + 9\cdot\frac{20}{64} + 16\cdot\frac{15}{64} + 25\cdot\frac{6}{64} + 36\cdot\frac{1}{64} = 10.5$, so that $\sigma^2 = 10.5 - 9 = 1.5$, and $\sigma = \sqrt{1.5} = 1.225$.

8. $\sigma^2 = \Sigma (x_i - \mu)^2\cdot f(x_i) = \Sigma (x_i^2 - 2x_i\mu + \mu^2)\cdot f(x_i) = \Sigma x_i^2\cdot f(x_i) - 2\mu\cdot\Sigma x_i\cdot f(x_i) + \mu^2\cdot\Sigma f(x_i) = \mu_2' - 2\mu\cdot\mu + \mu^2\cdot 1 = \mu_2' - \mu^2$.

9. (a) $\mu = 1\cdot\frac{12}{125} + 2\cdot\frac{48}{125} + 3\cdot\frac{64}{125} = 2.4$ and $\mu_2 = 1\cdot\frac{12}{125} + 4\cdot\frac{48}{125} + 9\cdot\frac{64}{125} = 6.24$, so that $\sigma^2 = 6.24 - 5.76 = 0.48$ and $\sigma = \sqrt{0.48} = 0.69$.

(b) $\sigma = \sqrt{3\cdot\frac{4}{5}\cdot\frac{1}{5}} = \sqrt{0.48} = 0.69$.

12. (a) $\sqrt{436\cdot\frac{1}{2}\cdot\frac{1}{2}} = \sqrt{109}$ or approximately 10.5; (b) $\sqrt{45\cdot\frac{1}{6}\cdot\frac{5}{6}} = \sqrt{6.25} = 2.5$; (c) $\sqrt{400(0.32)(0.68)} = \sqrt{87.04} = 9.3$; (d) $\sqrt{120(0.85)(0.15)} = \sqrt{15.3} = 3.9$.

14. $\sigma^2 = (-0.75)^2\cdot(0.403) + (0.25)^2\cdot(0.454) + (1.25)^2\cdot(0.133) + (2.25)^2\cdot(0.010) = 0.514$, so that σ is approximately 0.72.

15. $\mu_2 = 0(0.071) + 1(0.381) + 4(0.429) + 9(0.114) + 16(0.005) = 3.203$, so that $\sigma^2 = 3.203 - 2.56 = 0.643$ and σ is approximately 0.8.

16. $\sigma^2 = (-0.533)^2\cdot(0.524) + (0.467)^2\cdot(0.419) + (1.467)^2\cdot(0.057) = 0.363$ and σ is approximately 0.6.

17. (a) $\sigma = \sqrt{\frac{2\cdot4\cdot11\cdot13}{15\cdot15\cdot14}} = \sqrt{0.363} = 0.6$; (b) $\sigma = \sqrt{\frac{3\cdot6\cdot18\cdot21}{24\cdot24\cdot23}} = \sqrt{0.514} = 0.71$; (c) $\sigma = \sqrt{\frac{4\cdot4\cdot6\cdot6}{10\cdot10\cdot9}} = \sqrt{0.64} = 0.8$; (d) $\sigma = \sqrt{\frac{2\cdot4\cdot11\cdot13}{15\cdot15\cdot14}} = \sqrt{0.363} = 0.6$.

18. $\mu_2 = 1\cdot\frac{1}{2} + 4\cdot\frac{1}{4} + 9\cdot\frac{1}{8} + 16\cdot\frac{1}{16} + 25\cdot\frac{1}{32} + 36\cdot\frac{1}{64} + 49\cdot\frac{1}{128} + 64\cdot\frac{1}{256} + 81\cdot\frac{1}{512} + 100\cdot\frac{1}{1024} = 5.86$, so that $\sigma^2 = 5.86 - 4 = 1.86$, and σ is approximately 1.4.

19. $\sigma = \sqrt{\frac{1 - \frac{1}{2}}{\frac{1}{2}}} = \sqrt{2}$ or approximately 1.4.

DISCUSSION OF EXERCISES ON PAGES 215 THROUGH 217

In Exercises 1 through 7, Chebyshev's theorem is applied to various distributions; Exercises 8 through 14 deal with the Law of Large Numbers, and among these, the last two deal with the problem of determining the number of trials that are needed to attain a desired precision (that is, to make the difference between the proportion of successes and the probability of a success as small as desired).

1. (a) $\mu = 64\cdot\frac{1}{2} = 32$ and $\sigma = \sqrt{64\cdot\frac{1}{2}\cdot\frac{1}{2}} = 4$; (b) the probability is at least $1 - 1/3^2 = 8/9 = 0.89$ that he will get between $32 - 3\cdot4 = 20$ and $32 + 3\cdot4 = 44$ correct answers; (c) $48 - 32 = 16 = 4k$, so that $k = 4$ and the probability is at least $1 - 1/16 = 15/16 = 0.938$.

2. (a) $k = \frac{204 - 144}{12} = 5$, and the probability is at least $1 - 1/5^2 = 0.96$; (b) $k = \frac{174 - 144}{12} = 2.5$, and the probability is at most $1/2.5^2 = 0.16$.

3. (a) The probability is at least $1 - 1/16 = 0.938$ that between $124 - 4(8.5) = 90$ and $124 + 4(8.5) = 158$ marriage licenses will be issued; (b) $175 - 124 = 51 = 8.5k$, so that $k = 6$ and the probability is at most $1/36 = 0.028$.

4. $\mu = 9\cdot\frac{1}{2} = 4.5$ and $\sigma = \sqrt{9\cdot\frac{1}{2}\cdot\frac{1}{2}} = 1.5$, so that $4.5 - 2(1.5) = 1.5$, $4.5 + 2(1.5) = 7.5$, and according to Chebyshev's theorem the probability of getting x = 0, 1, 8, or 9 is at most 0.25; the corresponding exact probability is 0.002 + 0.018 + 0.018 + 0.002 = 0.04.

5. The probability is at least $1 - 1/3.3^2 = 0.908$ that there will be between 1.3 - 3.3(1.15) = -2.495 and 1.3 + 3.3(1.15) = 5.095 bank robberies, namely, that there will be at most 5 bank robberies.

6. The probability is at least $1 - \frac{1}{36} = 0.972$ that the person will catch between 3 - 6(1.73) = -7.38 and 3 + 6(1.73 = 13.38 trout, namely, that he will catch less than 14 trout.

8. For n = 10,000, p = 1/2, μ = 5,000, and σ = 50, we have .025(10,000) = 250 = 50k, so that k = 5 and the probability is at least 1 - 1/25 = 0.96; for n = 1,000,000, p = 1/2, μ = 500,000, and σ = 500, 0.025(1,000,000) = 25,000 = 500k, so that k = 50 and the probability is at least $1 - 1/50^2 = 0.9996$.

9. (a) $\mu = 450$ and $\sigma = 15$, so that $k = \frac{90}{15} = 6$ and the probability is at least $1 - \frac{1}{36} = \frac{35}{36}$; (b) $\mu = 1800$ and $\sigma = 30$, so that $k = \frac{180}{30} = 6$ and the probability is at least $\frac{35}{36}$; (c) $\mu = 45{,}000$ and $\sigma = 150$, so that $k = \frac{900}{150} = 6$ and the probability is at least $\frac{35}{36}$.

10. (a) $\mu = 75$ and $\sigma = 7.5$, so that $k = \frac{30}{7.5} = 4$ and the probability is at least $1 - 1/4^2 = \frac{15}{16}$; (b) $\mu = 1{,}875$ anf $\sigma = 37.5$, so that $k = \frac{150}{37.5} = 4$ and the probability is at least $\frac{15}{16}$; (c) $\mu = 30{,}000$ and $\sigma = 150$, so that $k = \frac{600}{150} = 4$ and the probability is at least $\frac{15}{16}$.

11. $0.01 = k\cdot\sqrt{\frac{\frac{1}{36}\cdot\frac{35}{36}}{56000}}$, from which it follows that k = 14.4 and, hence, the probability is at least $1 - 1/14.4^2 = 0.995$.

12. $0.025 = k\sqrt{\frac{\frac{1}{2}\cdot\frac{1}{2}}{10000}}$, from which it follows that k = 5 and, hence, the probability is at least $1 - 1/5^2 = 0.96$.

13. $\frac{1}{240} = \sqrt{\frac{\frac{1}{6}\cdot\frac{5}{6}}{n}}$, so that $\frac{1}{240^2} = \frac{\frac{1}{6}\cdot\frac{5}{6}}{n}$, $n = 240^2\cdot\frac{1}{6}\cdot\frac{5}{6} = 8{,}000$.

14. $1 - 1/k^2 = 0.99$ so that k = 10 and $0.04 = 10\sqrt{\frac{\frac{1}{2}\cdot\frac{1}{2}}{n}}$, $(0.004)^2 = \frac{\frac{1}{2}\cdot\frac{1}{2}}{n}$, $n = \frac{1/4}{(0.004)^2} = 125^2 = 15{,}625$.

ANSWERS TO EVEN-NUMBERED EXERCISES

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2. (a) 25%; (b) 0.25%; we cannot conclude that the two are equally likely, for all we know is that there are equally many possibilities.

4. No; so long as we have no information about the likelihood of the various possibilities, we cannot conclude that there is a better than fifty-fifty chance that there will be a seventh game.

6. There are six ways.

10. (a) 60; (b) 20%; (c) 30%.

12. (a) 20; (b) 60%; (c) 30%.

14. (a) 100; (b) 42; (c) 504.

16. 864. 18. 6,561.

20. 31. 22. (a) 80; (b) 81.

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2. (a) 43,680; (b) 5,040; (c) 27,300.

4. (a) 40,320; (b) 1,440. 6. (a) 12,144; (b) 12.5%.

8. (a) 24; (b) 6.

10. (a) 24; (b) 60; (c) 840; (d) 180.

12. 5,040.

14. (a) 6,720; (b) 60; (c) 840; (d) 1,680.

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2. (a) 120; (b) 30%. 4. 70.

6. 1,365. 8. 280.

10. (a) 4,330,260; (b) 40,095; (c) there are 1,732,104 and 673,596 possibilities, respectively, and hence more for five correct answers.

12. (a) 715; (b) 286; (c) 28.57%.

14. (a) 4,845; (b) 784; (c) 384.

16. 8. 18. (d) 1,120; (e) 3,360.

20. (a) 8,008; (b) 6,188; (c) 77,520; (d) 560; (e) 15,504.

24. (a) 4,096; (b) 16,777,216.

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2. (a) 1/26; (b) 4/13; (c) 11/13; (d) 1/2; (e) 5/13.

4. 1/8, 3/8, 3/8, and 1/8.

6. (a) 1/2; (b) 2/5; (c) 9/100. 8. (a) 5/33; (b) 35/66.

10. (a) 1/221; (b) 3/51; (c) 8/663.

12. 125/216, 75/216, and 1/216. 14. 3/4 and 1/22.

16. (a) 2/3, (b) 238/537, 59/537, and 240/537.

18. (a) 1/59,049; (b) 1,024/59,049; (c) 15,360/59,049; (d) 8,064/59,049.

20. 5/8. 22. (a) 1/3; (b) 14/33; (c) 1/3.

24. (a) 6 to 5; (b) 1 to 21; (c) 9 to 13.

26. (a) 15 to 49; (b) 1 to 25; (c) 7 to 3.

28. (a) 7/9; (b) 33/58; (c) 3/4; these are the probabilities of not rolling "7 or 11," drawing two black balls in succession, and not getting 1 heads and 3 tails.

32. If something is known about which novels are more likely than others to be on the test.

34. This kind of argument can lead to almost any result; for instance, we could argue that the probability should be 1/3 because there may be human life, non-human life forms only, or no life at all.

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2. 1,436/1,771 = 0.81. 4. 0.63.

6. (a) 1/3; (b) 1 to 2; (c) we would be favored.

8. (a) 0.85; (b) 3 to 17; (c) 17 to 3; (d) we would be favored.

10. 0.20. 12. 7/8.

14. Less than 0.25. 16. $4/5 \leq p < 13/15$.

18. $0.10 \leq p < 0.12$.

20. The assertion is valid; whatever happens in a single case cannot prove him right or wrong.

22. Yes, one might pertain only to novels published in the United States while the other might include also novels published in other countries. (This is only one of many possible answers.)

24. Although many persons seem to be more willing to bet under condition (b) than under condition (a), the odds should in each case be one to one. If we base our call on the flip of a balanced coin, there is a fifty-fifty chance of being right regardlesss of how many red beads and how many white beads there are in the box, and there is no way in which these odds can be improved without some further information about the beads.

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2. \$0.20. 4. (a) \$1.00; (b) \$1.50.

6. \$0.35, which exceeds the cost of the gasoline.

8. (a) \$12,000 and \$12,000; (b) \$13,000 and \$11,000.

10. \$0.40.

12. (a) 93/16; (b) 3/8, 5/16, and 5/16; 15/16 games.

14. 5/8; $p = \frac{a}{a + b}$.

16. The expected profit is \$60, but whether this makes it worthwhile to bid on the job is another matter.

18. 68.8 cents.

20. 41.25 cents.

22. 1.48 fires.

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2. $p < 2/3$.

4. $p = 1/3$.

6. (a) $p \leq 1/4$; (b) $p > 1/4$; (c) $p = 1/4$.

8. (a) $p > 1/6$; (b) $p \leq 1/6$; (c) $p = 1/6$.

10. (a) $p \leq 1/2$; (b) $p \geq 1/2$; (c) $p = 1/2$.

12. 4 utiles.

14. Approximately, (a) 1.8, (b) -2.4, (c) 2.25, and (d) -0.4.

16. (a) Even if minor medical expenses would "hurt," namely, if the utility of the cost of the insurance is preferable to the utility of expected medical expenses; (b) only if medical expenses exceeding \$500 would "hurt," namely, if the utility of expected minor medical expenses is preferable to the cost of corresponding insurance, but the utility of the cost of major medical insurance is preferable to the utility of expected major medical expenses; (c) if a person has enough money to provide his own insurance, that is, if the utility of the cost of insurance is not preferable to the utility of expected mecical expenses.

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2. (a) \$1,525,000; (b) -\$200,000.

4. (a) Since \$1,400,000 exceeds -\$210,000, the decision would be the same; (b) since \$100,000 exceeds -\$95,000, the decision would be reversed.

6. (a) The entries in the table are 11, 15, 13, and 9; (b) La Jolla; (c) Mission Beach; (d) does not matter; (e) Mission Beach, since 13 is preferable to 15; (f) Mission Beach, since 9 is preferable to 11; (g) Mission Beach, since 2 is preferable to 6.

8. In either case the expectation is 12 miles.

10. (a) 5; (b) 3.

14. (a) Send candy; (b) send roses; (c) 6 units of appreciation, or \$4.80, and this makes the call worthwhile; (d) send roses; (e) it is better not to send either present.

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2. (a) (1,1,1), (2,2,2), and (3,3,3); (b) (2.2.1), (2,2,2), and (2,2,3); (c) (1,2,2) and (1,3,3); (d) (1,3,3), (2,3,3), (3,1,3), (3,2,3), (3,3,1), and (3,3,2); (e) (2,2,2), (2,2,3), (2,3,2), (3,2,2), (2,3,3), (3,2,3), (3,3,2), and (3,3,3).

4. (a) (3,1), (3,2), and (3,3); (b) (1,1), (1,2), (1,3), (2,1), and (3,1); (c) (2,2) and (3,3); (d) (1,2), (1,3), (2,1), (2,3), (3,1), and (3,2).

6. (a) The baker will not sell any of the pies on Friday; (b) the baker will sell at least one pie on each day; (c) the baker will sell at least two of the pies on Friday.

8. (a) (2,2), (3,2), (3,3), (4,2), (4,3), and (4,4); (b) (1,0), (2,1), (3,2), and (4,3); (c) (1,0) and (1,1).

10. (a) Mutually exclusive; (b) not mutually exclusive; (c) not mutually exclusive; (d) not mutually exclusive; (e) mutually exclusive; (f) mutually exclusive.

12. (a) Altogether four tables are being used; (b) four tables are used in the larger dining room; (c) two more tables are used in the larger dining room than in the smaller dining room; (d) at least one of the two dining rooms is empty.

14. (a) $\{B, C, E, F\}$; (b) $\{D, F\}$; (c) $\{A\}$; (d) $\{A, B, C, E\}$.

16. (a) Not mutually exclusive; (b) mutually exclusive; (c) mutually exclusive; (d) not mutually exclusive; (e) not mutually exclusive; (f) mutually exclusive; (g) not mutually exclusive; (h) not mutually exclusive; (i) mutually exclusive; (j) not mutually exclusive.

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2. (a) E'; (b) $E \cap K$; (c) $E' \cup K$; (d) $E \cap K'$; (e) $E \cup K'$; (f) $E' \cap K'$.

4. (a) (1,1), (1,3), (2,1), (2,3), (3,1), (3,3); (b) (1,3), (2,3), (3,1), (3,2), and (3,3); (c) (1,1), (1,2), (1,3), and (3,1); (d) (1,2); (e) (2,3), (3,2), and (3,3); (f) (1,1) and (2,1).

6. (a) He sells at least as many pies on Saturday as on Friday; (b) he does not sell any of the pies on at least one of the days; (c) altogether he sells either one pie or all of the pies; (d) he sells one pie on Saturday and two or three on Friday; (e) he does not sell exactly one pie on Friday; (f) he sells all the pies and at least one each day; (g) he sells three on Saturday and none on Friday; (h) anything is possible; (i) he sells at least three of the pies on Saturday.

8. (a) (0,0), (1,0), (1,1), (2,0), (2,1), (3,0), (3,1), (4,0), and (4,1); (b) (0,0), (1,0), (1,1), (2,1), (2,2), (3,2), (3,3), (4,3), and (4,4); (c) (1,0), (2,1), (3,2), (4,3), (1,1), (3,1), and (4,1); (d) (1,0) and (1,1); (e) (2,2), (3,2), (3,3), (4,2), (4,3), (4,4), (1,1), (2,1), (3,1), and (4,1); (f) (1,0); (g) (2,0), (3,0), (3,1), (4,0), (4,1), and (4,2); (h) all but (0,0); (i) (0,0), (1,1), (2,2), (3,3), and (4,4).

10. (a) (0,0), (0,1), (0,3), (0,4), (1,0), (1,1), (1,2), (1,4), (2,0), (2,1), (2,2), (2,3), (3,0), (3,1), (3,2), (3,3), and (3,4); (b) (0,0), (0,1), (1,0), (0,2), (1,1), (2,0), (0,3), (1,2), (2,1), (3,0), (0,4), (1,3), (2,2), and (3,1); (c) (0,0), (1,0), (1,1), (2,0), (2,1), (2,2), (3,0), (3,1), (3,2), and (3,3); (d) (0,4); (e) (1,4), (2,4), and (3,4); (f) (0,0), (0,1), (0,2), (0,3), (0,4), (1,0), (2,0), (3,0), (1,3), (2,2), and (3,1); (g) all but (2,2);

(h) (1,1), (1,2), (1,3), (2,1), (2,2), and (3,1); (i) (0,4), (1,4), (2,4), and (3,4).

12. A person is not a wealthy resident of New York; a person is not wealthy and/or not a resident of New York.

16. (a) The movie is a financial success, an artistic success, and GP rated; (b) the movie is a financial success, an artistic success, but not GP rated; (c) the movie is GP rated, but neither a financial success nor an artistic success; (d) the movie is neither a financial success, nor an artistic success, nor is it GP rated; (e) the movie is an artistic success and GP rated; (f) the movie is an artistic success but not a financial success; (g) the movie is a financial success and/or an artistic success, but it is not GP rated (h) the movie is GP rated; (i) the movie is not a financial success.

18. (a) The program will appeal to teenagers, be on network television, and get a high rating; (b) the program will appeal to teenagers, get a high rating, but will not be on network television; (c) the program will be on network television and get a high rating; (d) the program will appeal to teenagers but not get a high rating; (e) the program will be on network television and/or get a high rating, but will not appeal to teenagers; (f) the program will not be on network television; (g) the program will be on network television and/or get a high rating; (h) the program will not appeal to teenagers and/or get a high rating.

20. $A \cap B = \emptyset$ and $A \cap B = B$ (or $A \cup B = A$); (a) intelligent persons are not prominent citizens (and vice versa); (b) all prominent citizens are intelligent.

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2. 33.

4. (a) 12%; (b) 13%; (c) 87%; (d) 33%; (e) 11%.

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2. (a) No, the sum of the probabilities is 5/6, which violates Postulate 2; (b) yes, the sum of the probabilities is one and none are negative; (c) yes, the sum of the probabilities is one and none are negative; (d) no, P(T) is negative, which violates Postulate 1; (e) no, the sum of the probabilities exceeds one, which violates Postulate 2.

4. The two percentages cannot be added since having the tank filled and having the car checked are not mutually exclusive events.

6. The odds would have to be 5 to 3 against a great or a modest success.

8. 0.50.

10. Wrong, the odds should be 17 to 18.

12. (a) 0.68; (b) 0.86; (c) 0.77; (d) 0.68.

14. (a) 0.43; (b) 0.68; (c) 0.11; (d) 0.75.

16. (a) 0.36; (b) 0.39; (c) 0.73; (d) 0.84.

20. (a) 0.42, 0.39, and 0.08; (b) 0.38, 0.38, and 0.09; (c) 0.58, 0.62,

0.61, 0.08, 0.80, 0.04, 0.38, 0.99, and 0.23.

22. (a) 4/5; (b) 3/10; (c) 1/10; (d) 7/10.

24. (e) 1/6; (f) 1/6; (g) 1/3; (h) 1/2; (i) 1/2.

26. (a) 1/3; (b) 7/15; (c) 2/3; (d) 2/15; (e) 2/3; (f) 1/5; (g) 1/3; (h) 13/15.

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6. (a) 0.73; (b) 0.54; (c) 0.73; (d) 0; (e) 0.27; (f) 0.46; (g) 0.27; (h) 1.

8. (a) 0.15; (b) 0.33; (c) 0.67; (d) 0.25.

10. 0.27.

12. 7 to 3.

14. 0.41, 0.44, 0.24, and 0.41 + 0.44 - 0.24 = 0.61.

16. (a) a + b + d + g, a + b + c + e, a + c + d + f, a + b, a + d, a + c, and a.

18. 0.44.

20. 0.94.

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2. (a) The probability that a job with a high starting salary has a good future; (b) the probability that a job with a good future does not have a high starting salary; (c) the probability that a job which does not have a good future has a high starting salary; (d) the probability that a job which does not have a high starting salary does not have a good future.

4. (a) $P(A|T)$; (b) $P(W \cap T'|A)$; (c) $P(T|W')$; (d) $P(W|A \cap T')$.

6. (a) The probability that a letter supporting the teacher was written by a student, 4/15; (b) the probability that a letter written by a parent will support the teacher, 11/24; (c) the probability that a letter supporting the superintendent was written by a parent, 13/15.

10. (a) 1/4; (b) 5/16; (c) 9/80; (d) 1/5; (e) 11/80; (f) 9/25; (g) 9/20; (h) 1/5; (i) 4/15.

12. (a) 8/23; (b) 4/9; (c) 15/82; (d) 67/77.

14. (a) 2/7; (b) 2/31; (c) 3/4.

16. (a) 12/23; (b) 12/67; (c) 5/7; (c) 2/3.

18. (a) 5/14; (b) 3/28; (c) 15/28.

20. (a) 0.54; (b) 0.75.

22. (a) 3/5; (b) 4/5; (c) 0.92; (d) 0.48; (e) 0.12; (f) 0.20.

24. They are not independent.

26. They are not independent.

28. (a) 22/145; (b) 4/25.

30. (a) 1/16; (b) 1/7,776; (c) 81/4,096.

32. (a) 11/850; (b) 1/64.

34. 5/28.

36. (a) 0.3072; (b) 0.096; (c) 0.02016.

38. $P(A) = 0.60$, $P(B) = 0.80$, $P(C) = 0.50$, $P(A \cap B \cap C) = 0.24$, but $P(A \cap B) = 0.54 \neq 0.48$.

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2. (a) 0.780; (b) 0.7464.

4. 0.372.

6. 0.0061.

8. (a) 3/8; (b) 5/16.

10. 0.0116.

12. 0.501.

14. 0.822.

16. 3/4.

18. 0.828.

20. 0.410, 0.295 and 0.295.

22. The 5 to 2 odds are fair.

24. (a) $\sum_{i=1}^{8} y_i$; (b) $\sum_{i=1}^{4} (x_i - 2)$; (c) $\sum_{i=1}^{3} P(A_i \cup B_i)$; (d) $\sum_{i=1}^{k} P(C_i')$; (e) $\sum_{i=1}^{4} P(A_1 \cap B_i)$; (f) $\sum_{i=3}^{6} P(D|C_i)$

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2. (a) The probabilities that he will sell 0, 1, 2, 3, or 4 of the pies on Friday are, respectively, 1/3, 4/15, 1/5, 2/15, and 1/15; (b) the probabilities that he will sell 0, 1, 2, 3, or 4 pies altogether are, respectively, 1/15, 2/15, 1/5, 4/15, and 1/3; (c) the probabilities that 0, 1, 2, 3, or 4 of the pies remain unsold are, respectively, 1/3, 4/15, 1/5, 2/15, and 1/15.

4. (a) The probabilities that 0, 1, 2, 3, or 4 of the cabs are operative are, respectively, 0.01, 0.08, 0.34, 0.44, and 0.13; (b) the probabilities that 0, 1, 2, 3, or 4 of the cabs are out on a call are, respectively, 0.20, 0.38, 0.30, 0.11, and 0.01; (c) the probabilities that 0, 1, 2, 3, or 4 operative cabs are not out on a call are, respectively, 0.23, 0.39, 0.29, 0.08, 0.01.

6. (a) No, the sum of the values exceeds one; (b) yes, the sum is one and none of the values is negative; (c) yes, the sum is one and none of the values is negative; (d) no, f(1) and f(2) are negative.

8. (a) The probabilities of rolling a total of 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, or 12, are, respectively, 1/36, 2/36, 3/36, 4/36, 5/36, 6/36, 5/36, 4/36, 3/36, 2/36, and 1/36.

10. 2.

12. (a) 4/3; (b) 8/3; (c) 4/3.

14. The mean of the distribution of the differences equals the difference between the means of the respective distributions; (a) 2.60; (b) 1.35; (c) 1.25.

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2. (a) 0.4096; (b) 0.410.

4. (a) 0.3156; (b) 0.316.

6. The probability is 0.0576, and hence the person should not be too surprised.

8. (a) 0.4199; (b) 0.420.

12. (a) 0.001; (b) 0.059; (c) 0.851; (d) 0.944; (e) 0.012.

14. (a) 0.382; (b) 0.367.

18. (a) 0.121; (b) 0.172; (c) 0.010.

20. 5.612, np = 5.6.

22. (a) 218; (b) 7.5; (c) 128; (d) 102; (e) 210; (f) 160.

24. Odds change from 4 to 1 to about 2 to 5.

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2. (a) 0.403; (b) 0.454; (c) 0.133; (d) 0.010.

4. (a) 0.833; (b) 0.682; (c) 0.318; (d) 0.167; (e) 0.455.

6. 1.601 and 1.6.

8. 0.533 and 8/15 = 0.533.

10. 0.123, the error is 0.0066.

12. 0.941, the error is less than 0.001.

14. 0.082.

16. 0.0097.

18. 1.98, $\mu = 2$.

20. 0.60.

22. 0.106.

24. 0.108.

26. 0.091.

28. 0.02.

30. (a) 0.012; (b) 0.0017.

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2. (a) 1.3; (b) 1.15.

4. (a) 3; (b) 15/16; (c) 1.5; (d) 1.225; (e) 1.225.

6. 0.65.

10. (a) 1.84; (b) 1.86; (c) 1.83.

12. (a) 10.5; (b) 2.5; (c) 9.3; (d) 3.9.

14. 0.72.

16. 0.60.

18. 1.4.

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2. (a) 0.96; (b) 0.16.

4. 0.04.

6. The probability is at least 0.972 that the number of trout caught is less than 14.

12. 0.96.

14. n = 15,625.