FOREWORD
Possibilities

Chapter 1

Notes and Comments
16. When are speedbp cases included? There are \( \binom{3}{5} \) possibilities when one tree, since all three of the other cases must be selected, when one tree is included. However, when no speedbp cases are included, there are \( \binom{3}{5} \) possibilities.

The correct answer, thus, there are more possibilities for \( \binom{2}{5} = 5 \) of the correct answer and \( \binom{3}{5} = 1 \) of the other answer can be answered in the first place. Each of the other questions correctly can be chosen in the second stage, and each of the other questions are answered in the third etc., which is only \( \binom{2}{5} = 5 \).
The number of possible cases is \(^{12}C_1\) and the total number of cases is \(^{15}C_2\). The number of possible cases for the first part is \(^{15}C_2\) and the total number of cases for the first part is \(^{15}C_2\). The number of possible cases for the second part is \(^{10}C_7\) and the total number of cases for the second part is \(^{10}C_7\). The number of possible cases for the third part is \(^{5}C_3\) and the total number of cases for the third part is \(^{5}C_3\). The number of possible cases for the fourth part is \(^{1}C_1\) and the total number of cases for the fourth part is \(^{1}C_1\).

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The foundational principle of the logical framework is to identify the key concepts and principles that underpin the theories. In this context, we can examine the structure of the concepts and how they interrelate. The analysis of the logical framework reveals the underlying assumptions and the methodologies used to validate these assumptions.

2. When discussing the foundational principles, it is essential to understand the core concepts and how they are interrelated. This analysis helps in identifying the strengths and weaknesses of the framework and in formulating new theories.

3. The discussion on the foundational principles is crucial for developing a comprehensive understanding of the theories. It is important to consider the implications of these principles in various fields and to evaluate their applicability in practical situations.

4. The foundational principles are the backbone of the theories, and understanding them is essential for further research and development. By examining the foundational principles, we can gain insights into the potential areas for improvement and innovation.

5. The logical framework is a powerful tool for understanding complex systems and for making predictions. It is important to critically evaluate the foundational principles and to explore their implications in various contexts.

6. The logical framework is a flexible and adaptable tool that can be applied to a wide range of fields. By understanding the foundational principles, we can develop new theories that address emerging challenges and opportunities.

7. The foundational principles are the building blocks of the theories, and their understanding is crucial for developing new perspectives and paradigms. By exploring the foundational principles, we can identify new avenues for research and innovation.

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DISCUSSION OF EXPECTATIONS

The expectation for the 3,729,700 job is 1.75, and it is not unreasonable to spread the 2,098,000, and it is more or less a reasonable agreement that the difference is small, a deviation from box A should be.

1. Expectation is 0.292 for box A, and 0.284 for box B.
2. Expectation is 0.289 for box A, and 0.306 for box B.
3. Expectation is 0.256 for box A, and 0.266 for box B.
4. Expectation is 0.241 for box A, and 0.251 for box B.
5. Expectation is 0.200 for box A, and 0.214 for box B.

All the expectations in this are very slightly overestimated applications of

NOTES AND COMMENTS

CHAPTER 9
Discussions of Experiments and the Evaluation of Their Results

Since the second is smaller, the second data point is closest to the center of the distribution.

The expectations are
\[
\begin{align*}
\text{Expectation of Particle Information} & = \frac{\epsilon}{\xi} + \frac{\xi}{\epsilon} = \frac{1}{4} + \frac{1}{4} = 0.5 \\
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\end{align*}
\]

6. (a) (b) (c) (d) (e) (f) (g) (h) (i) (j) (k) (l) (m) (n) (o) (p) (q) (r) (s) (t) (u) (v) (w) (x) (y) (z)
13. (a) Find the region shaded in the figure. Draw a diagram if necessary.

The region shaded in the figure is the same as that of Exercise 12.

Diagram for Exercise 12

14. (a) Find the region shaded in the figure. Draw a diagram if necessary.

The region shaded in the figure is the same as that of Exercise 13.

Diagram for Exercise 13

Discussion of Exercise No. 10

Discussion of Exercise No. 9

Discussion of Exercise No. 8
### RULES OF PROBABILITY

**CHAPTER 5**

The region shaded both ways in the figure below is the same.
13. Let the proposition that the event will kill neither of the two, and the event will kill exactly one of the two, be denoted by P1 and P2, respectively. If P1 is true, then P2 is false. Conversely, if P2 is true, then P1 is false.

14. The second property of exponents is that the product of two powers with the same base is equal to the power of the product of the bases. This is expressed as:

\[ a^m \times a^n = a^{m+n} \]

15. Similarly, the quotient of two powers with the same base is equal to the power of the quotient of the bases:

\[ \frac{a^m}{a^n} = a^{m-n} \]

16. These properties are crucial in simplifying expressions involving exponents.

17. For example, consider the expression (2^3 \times 2^4). According to the product rule, this can be simplified as:

\[ 2^{3+4} = 2^7 \]

18. Similarly, when dividing powers with the same base, such as (2^5 \div 2^2), the quotient rule applies:

\[ \frac{2^5}{2^2} = 2^{5-2} = 2^3 \]

19. These properties are widely used in various fields of mathematics, including algebra, calculus, and physics, to simplify complex expressions and solve problems more efficiently.
20

Corollary 6. (20)

The other cannot happen and vice versa.

All the conditions, hence they are dependent, after all, it can happen.

Therefore, two are independent.

Notice any comments.

Conditional Probabilities
Diagram for Exercise 22

Diagram for Exercise 22

Diagram for Exercise 9

Diagram for Exercise 9

There are 4 possibilities each for studying 2 hours each on three days.

There are 4 possibilities each for studying 2 hours each on three days.

The probability that none of the options is filled with mustard is

Fractional representation of probabilities:

\[ \frac{9}{15}, \frac{9}{15}, \frac{9}{15}, \frac{9}{15}, \frac{9}{15}, \frac{9}{15}, \frac{9}{15}, \frac{9}{15}, \frac{9}{15} \]
The mean of the difference between two independent random variables is the difference of their means.

\[ E(X - Y) = E(X) - E(Y) \]

Example: Let \( X \) and \( Y \) be two independent random variables. If \( X \) has mean \( 10 \) and \( Y \) has mean \( 20 \), then the mean of \( X - Y \) is \( E(X - Y) = 10 - 20 = -10 \).

The variance of the difference between two independent random variables is the sum of their variances.

\[ \text{Var}(X - Y) = \text{Var}(X) + \text{Var}(Y) \]

Example: If \( X \) and \( Y \) are independent with variances \( 4 \) and \( 9 \), respectively, then the variance of \( X - Y \) is \( \text{Var}(X - Y) = 4 + 9 = 13 \).

Probability distributions are used to represent the distribution of a random variable. A common technique is to use empirical distributions, which are based on observed data.

**Empirical Distribution**

- **Cumulative Distribution Function (CDF)**: The probability that a random variable is less than or equal to a certain value.
- **Probability Mass Function (PMF)**: The probability that a discrete random variable takes on a specific value.
- **Probability Density Function (PDF)**: The probability that a continuous random variable falls within an interval.

**Common Probability Distributions**

- **Bernoulli Distribution**: Used for modeling a single binary outcome (e.g., success or failure). The probability mass function is given by:
  \[ P(X = k) = \binom{n}{k} p^k (1-p)^{n-k} \]

where \( n \) is the number of trials, \( p \) is the probability of success, and \( k \) is the number of successes.

- **Binomial Distribution**: An extension of the Bernoulli distribution for modeling the number of successes in a fixed number of trials. The probability mass function is:
  \[ P(X = k) = \binom{n}{k} p^k (1-p)^{n-k} \]

where \( n \) is the number of trials, \( p \) is the probability of success, and \( k \) is the number of successes.

- **Normal Distribution**: A continuous distribution that is symmetric around the mean and characterized by two parameters, \( \mu \) (mean) and \( \sigma \) (standard deviation). The probability density function is:
  \[ f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \]

where \( \mu \) and \( \sigma \) are the mean and standard deviation, respectively.

**Joint Distributions**

- **Joint Probability Mass Function (PMF)**: The probability that two or more discrete random variables take on specific values simultaneously.
- **Joint Probability Density Function (PDF)**: The probability that two or more continuous random variables take on specific values simultaneously.

**Independence**

Two random variables \( X \) and \( Y \) are independent if the joint PMF or PDF is the product of the marginal PMFs or PDFs:

\[ P(X, Y) = P(X) P(Y) \]

**Expectation and Variance**

- **Expectation**: The long-run average of a random variable.
- **Variance**: A measure of the spread or dispersion of a random variable.

**Covariance**

- **Covariance** measures how much two random variables change together. It is defined as:
  \[ \text{Cov}(X, Y) = E[(X - E(X))(Y - E(Y))] \]

**Correlation**

- **Correlation** is a normalized version of covariance, ranging from -1 to 1. It is defined as:
  \[ \rho_{X,Y} = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y} \]

**Central Limit Theorem**

- **Central Limit Theorem** states that the sum of a large number of independent and identically distributed random variables will be approximately normally distributed, regardless of the underlying distribution of the individual variables.

**Applications**

- **Quality Control**
- **Finance**
- **Epidemiology**
- **Economics**
- **Engineering**

**Further Reading**

CHAPTER 8
THE LAW OF LARGE NUMBERS

NOTES AND COMMENTS
<table>
<thead>
<tr>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>6. (a) 1/100, 9/100, 1/10, 19/100</td>
</tr>
<tr>
<td>8. (a) 1/1000, 19/1000, 9/1000, 1/1000</td>
</tr>
<tr>
<td>10. (a) 1/1000, 19/1000, 9/1000, 1/1000</td>
</tr>
</tbody>
</table>

**Exercises**

- For all the rows that are equally possible, we cannot conclude that the two are equal.
- Except for rows that are equally possible, we cannot conclude that the two are equal.

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**Answers to Even-Numbered Exercises**

- 22. (a) 3/11, 1/11, 7/11, 4/11
- 22. (a) 3/11, 1/11, 7/11, 4/11
- 22. (a) 3/11, 1/11, 7/11, 4/11
- 22. (a) 3/11, 1/11, 7/11, 4/11